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SPECIAL ISSUE: CONSTRUCTIVE AND COMPUTABLE ANALYSIS IN MATHEMATICAL FINANCE — THE FESTSCHRIFT ISSUE FOR PROF KUMARASWAMY (VELA) VELUPILLAI

EDITOR'S INTRODUCTION

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This special issue of *New Mathematics and Natural Computation* is published this year (2012) to celebrate and honour our friend, colleague and teacher, Prof Vela Velupillai's 65th year. We have all been touched and blessed with his personal friendship, intellectual inspiration and professional hosting, over many years, in many continents and at several universities — Sweden, Denmark, the US, UK, Ireland and Italy.

At 65, he is embarking on new intellectual adventures and we expect to be able to celebrate also the proverbial "three score years and ten", in half a decade, with a new report on the new frontiers of research he would, surely, try to open.

Six distinguished scholars, as it happens come from five continents[a] and five different countries — six, if my own "coordinates" are included[b] — have joined in this homage to Velupillai's work in the mathematizing and mathematics of economics, with constructive, computable, methodological and philosophical foundations, always seeking meaningful approximations and feasible computations of even run-of-the-mill economic propositions, particularly those with policy relevance.

It is particularly appropriate that this commemoration is being reported in *New Mathematics and Natural Computation*. In all his intellectual life, he has been working, and intellectually experimenting with, alternative mathematics for the formalization of economics, restlessly searching for the appropriate way to encapsulate the unruly

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[a] Including, of course, Australia — from which base Professor Cassey Lee is writing — in this set! Foely and Rosser, from North America, Doria from South America, Zambelli and Kinsella from Europe and Sarukkai from Asia, make up the other four continents.

[b] US, Brazil, Italy, Ireland, India and Taiwan.

contours and concepts of economics in a meaningful way, respecting the heritage of our past classical and neoclassical masters. Equally, computation has been a fulcrum for his mathematical foundations. Mathematical methods, utilized in the formalization of economics, devoid of computational content — in recent work he has come to refer to this aspect as an *epistemological deficit* — has never satisfied him. In personal conversations, he has attributed this particular emphasis as a legacy inherited from his most important teachers: Professors Ryoichiro Kawai at Kyoto University, Professor Björn Thalberg at the University of Lund and, above all, Professor Richard Goodwin at Cambridge University.

His own "reflections" on five of the six contributions summarize very well the general aims and scope of the articles in this special issue. The sixth, by Doria, is on themes that have engaged the two of them, in discussions, for over half a decade. Even if, ostensibly, the topic of Doria's contribution is not on the theme set out in Velupillai's thematic article, it is — indirectly — related to it in a deep methodological and philosophical sense. This is mainly because two themes that permeate the main article by Velupillai, *undecidability* and *incompleteness*, also form the foundations of Doria's contribution to this special issue, albeit in a different context than economics. The ubiquity of undecidability and incompleteness in any mathematical formalization is something Velupillai has been stressing, partly as a result of his serious engagement with Doria's own valuable contributions, stretching back over almost two decades.

Stefano Zambelli and Steve Kinsella were both students of Velupillai, but separated by almost two decades! Yet, common themes highlight the work they have been doing, inspired by their teacher. Their two contributions are, in a way, elegiacs to various aspects of computable economics, the field most clearly identified with Velupillai's work and is essentially a field he created, almost single-handedly. I use the word 'almost' most deliberately, simply because Stefano Zambelli may, in some serious senses, can be considered even a joint-cofounder of this fascinating field.

Barkley Rosser's contribution may well be the one that tackles the issues that are central to Velupillai's lead article — and in his "reflections" he — Velupillai — also responds to some of the issues that remain ambiguous. Perhaps, that is the way they will always be.

No one who ponders on the methodological and epistemological foundations of any computation — whether founded on computability theory, computable analysis, constructivity theory, interval analysis (*a la* Ramon Moore), scientific computation (numerical analysis) and its underpinning in varieties of computability theory — can, or should, be unserious about approximations. There is, after all, a well-defined, codified, theory of approximation, a field that is completely neglected in mathematical economics, except in trivial ways. Foley has given serious thought to the foundations of simulation and approximation in the mathematics of equilibrium economics. In his contribution here he breaks new ground in linking his earlier work to issues and concepts of computability and constructivity in characteristically original ways.

Sarukkai's contribution, and its place in Velupillai's work, is discussed in a fairly detailed way in the "reflections" mentioned above. Sarukkai, in suggesting new vistas

for the mathematization of economics, from the point of view of the mathematical philosopher, also admirably summarizes Velupillai's own philosophy on the topic. Moreover, Sarukkai's subtle and deep distinction between applicability and reducibility, defining one kind of divide between constructivity and non-constructivity may, in the years to come, lead to new speculations on the mathematization of economics in the search to reduce the epistemological deficit. Only a philosopher of mathematics could have brought out the need for epistemological clarity, as Sarukkai has done, in his fertile contribution.

Cassey Lee provides an admirable, yet concise, "survey" of the vast canvas that is the mathematization of economics, against which to try to place in relief, so to speak, Velupillai's own methodological ruminations on the subject — both historically and thematically. Cassey Lee underlines an aspect of Velupillai's work that is often misunderstood by unreflective readers of his writings: the constant, tireless, pleas he — Velupillai — has made for explorations in alternative modes of mathematizing economics. To be blind to alternative possibilities of mathematization is a worse vice than even the perpetuation of an epistemological deficit — partly because new and possible worlds are excluded by decree by those who are unimaginatively shackled by mathematical blinkers, no less than those who are monolithic in other aspects of economic theorizing.

This is not the first festschrift for Vela. Stefano Zambelli has done the first one (Zambelli, 2010). This first one was initially planned for Vela's 60th birthday. In his foreword, Zambelli gave a good story on why he thought that Vela deserves a festschrift. I completely concur with the points that he made, except *not just one* festschrift. Being a founder of computable economics, Vela has been constantly devoting to the promotion of this "new" mathematics to a large class of economists. Five years for him can mean a lot of changes, particularly, at an increasing rate with his age.

Therefore, 3 years ago, I talked to Paul Wang, the founder of this journal, about an issue on "new" mathematical economics as a festschrift for Vela's 65th birthday. Paul was overwhelmingly supportive of this idea. As Vela mentions at the very end of his milestone project *Computable Economics*,[1] "[P]erhaps the Gods did not lay out a path for the discovery of the laws of the economics by human mathematics we know it today. But, then, we cannot know whether that is the case until we try some of the other "human 20th-century mathematics" at our disposal." Having new mathematics as part of its name, this journal is a place for *other* human 20th-century mathematics and now, with the publication of this festschrift, it helps economists try different routes to mathematical economics.

References

1. V. Velupillai, *Computable Economics: The Arne Ryde Memorial Lectures* (Oxford University Press, 2000).
2. S. Zambelli, *Computable, Constructive and Behavioural Economic Dynamics: Essays in Honour of Kumaraswamy (Vela) Velupillai* (Routledge, 2010).