North American Actuarial Journal

ISSN: 1092-0277 (Print) 2325-0453 (Online) Journal homepage: http://www.tandfonline.com/loi/uaaj20

Capital Forbearance, Ex Ante Life Insurance Guaranty Schemes, and Interest Rate Uncertainty

Ya-Wen Hwang, Shih-Chieh Chang & Yang-Che Wu

North American Actuarial Journal, 19(2), 94–115, 2015
Copyright © Society of Actuaries
ISSN: 1092-0277 print / 2325-0453 online
DOI: 10.1080/10920277.2014.1001911

Capital Forbearance, Ex Ante Life Insurance Guaranty Schemes, and Interest Rate Uncertainty

Ya-Wen Hwang,[1] Shih-Chieh Chang,[2] and Yang-Che Wu[3]
[1]*Department of Risk Management and Insurance, Feng Chia University, Taichung, Taiwan*
[2]*Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan*
[3]*Department of Finance, Feng Chia University, Taichung, Taiwan*

Insurance guaranty funds have been adopted in many countries to compensate policyholders for losses resulting from insurers' insolvencies. In this article we focus on the risk-based premiums in ex ante insurance guaranty schemes since a preassessment mechanism could reduce the shareholders' incentive to engage in risk-taking behavior. We derive the closed-form solutions of the risk-based premium charged by the insurance guaranty fund in a setting that incorporates financial leverage, asset allocation, early closure, and capital forbearance during the grace period. Most importantly, we assume that the interest rate is stochastic, and we find that the premium is underpriced if the uncertainty of the interest rate is neglected by the insurance guaranty fund. Moreover, the influence of stochastic interest rate for the premium is more significant when we consider the capital forbearance mechanism. The impacts of the key factors in our model that decide the fair premium of the guaranty fund are examined numerically. The results of our analysis could provide valuable insights for regulators in terms of revising regulatory policies and insurance guaranty schemes.

1. INTRODUCTION

Life insurance companies are highly leveraged financial institutions whose liabilities are formed by the policyholders' premiums that render the institutions responsible for meeting the claim obligations over a lengthy coverage period. The statutory reserves and the shareholder's fund on the balance sheet of the insurers are invested in marketable securities between the inception of an insurance contract after the initial premium is collected and the date on which a claim for insurance benefit should be paid. The reserves make up the principal proportion of the life insurer's liabilities. As much as deposit insurance is widely used to meet the bank's obligations to depositors when the bank fails, it is necessary to develop a similar guaranty scheme to protect the rights of policyholders, an approach that is referred to as the insurance guaranty fund established by the financial authority.[1] The essential purpose of the guaranty fund is to stabilize the insurance system in ways such as covering certain claim obligations of insolvent insurers. Cummins (1988) points out that the establishment of a guaranty fund means that the costs associated with an insolvent insurer should be spread throughout the insurance system.

Both preassessment and postassessment approaches are adopted to cover these insolvency costs. The postassessment scheme, which is applied by the United States and the United Kingdom, means that the obligations of the insolvent insurance company will be distributed among other insurers in the same financial market. Krogh (1972) mentions that postassessment schemes provide incentives for sound financial supervision of insurers; however, Han et al. (1997) show that the postassessment approach tends to foster insurers' risk-taking behaviors.

On the other hand, under the ex ante assessment approach, all insurers have to regularly pay a premium to the guaranty fund to deal with the obligation in the event that a certain insurance company fails. France, Germany, Japan, Taiwan, and other countries have adopted the ex ante scheme. Oxera (2007) indications that it is fair under the prefunding approach because the future insolvent insurers have to pay the premium to the guarantee costs. However, an ex ante funding scheme will levy higher contributions from the participating insurers, because the probability and severity of insolvent events might not be easy to predict.

Address correspondence to Yang-Che Wu, 100 Wenhwa Road, Seatwen, Taichung, 40724, Taiwan. E-mail: wuyangche@fcu.edu.tw
Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/uaaj.
[1]Most insurance guaranty funds are established by the financial authority, such as Taiwan, United States, the United Kingdom, Germany, and France.

The advantages and disadvantages of preassessment and postassessment schemes are discussed in the studies of Duncan (1987) and Oxera (2007).[2] Han et al. (1997) first discuss these two funding approaches from the aspect of agency cost. They found that the preassessment approach could reduce agency costs under the interstate insurance guaranty scheme, which suggests that the ex ante scheme gives shareholders less incentive to engage in risk-taking behavior than the actual ex post assessment scheme. They also found that the preassessment approach reduces the wealth-transfer problem caused by the postfunding approach operated under an interstate system. However, the adoption of prefunding schemes cannot eliminate agency costs completely. Pros and cons are found for both assessment approaches, and so no one funding scheme strictly dominates the others. The providers of the insurance guaranty fund have to consider the timing and level of expected guarantee costs and the financial capacity of insurers to decide the funding approach (Oxera 2007).

The Taiwan Insurance Guaranty Fund (TIGF) is the insurance guaranty scheme in Taiwan,[3] and the funding approach of the TIGF is preassessment according to the enacted regulations. Moreover, Oxera (2007) finds that the majority of insurance guaranty funds apply the ex ante funding approach, and the preassessment premium could reduce the agency cost. Thus, in this study, we focus on discussing the fair premium of the ex ante assessment approach based on the current TIGF scheme. Furthermore, we incorporate the regulatory forbearance to investigate the influence of the fair premium within the scheme.

The original premium pricing problem is discussed for deposit insurance schemes. For example, Merton (1977) applies the put option to price the premium of deposit insurance. Then Ronn and Verma (1986) use the market data to estimate the parameters of Merton's put option pricing model. Duan and Yu (1994, 1999), Duan et al. (1995), and Duan and Simonato (2002) perform a series of studies to improve the deposit insurance pricing model, such as considering stochastic interest rates, applying the maximum likelihood estimation to estimate the parameters, and using the GARCH model to describe the bank's asset return dynamic. Lee et al. (2005) illustrate the influence of capital forbearance on the fair premium of deposit insurance. Bernard et al. (2005a) discuss the premium of mutual insurance of bank insurance and apply the Parisian option method to evaluate the fair premium. Hwang et al. (2009) investigate the premium of deposit insurance incorporating the bankruptcy costs and closure policies.

The other line of literature regarding the pricing guaranty fund problem discusses how to measure the fair premium of pension insurance. Sharpe (1976) first discusses the economic premium of the Pension Benefit Guaranty Corporation (PBGC). Marcus (1987) models the PBGC's liability as a contingent forward. Pennacchi and Lewis (1994) incorporate the stochastic characteristics of the assets and liabilities into the pension insurance pricing model. Chen (2011) derives a closed-form pricing model to determine the risk-based premium for the PBGC and the sponsoring company based on a deterministic risk-free rate assumption.

Moving on to the insurance guaranty fund, Cummins (1988) extends the work on deposit insurance premiums to derive the risk-based insurance guaranty fund premium. Duan and Yu (2005) extend Cummins' one-period model into a multiperiod framework. Both conclude that the risk-based premium is necessary, and the impact of certain key factors, such as the leverage ratio, is enormous. However, in reality, when the assets of the insurer are below the required capital level but do not fall below the minimal standard upon maturity, the regulator will offer a grace period to the insurer in its capital restructuring. During this period, the insolvent insurer is allowed to continue its operations. The purpose of regulatory forbearance is to give them the chance to recover from the financial distress situation. Thus, Yang et al. (2012) apply the deposit insurance premium model of Lee et al. (2005) to incorporate the capital forbearance scheme into the insurance guaranty fund pricing problem. They find that the premium with regulatory forbearance is higher than the current premium rate of the TIGF. However, both the studies of Lee et al. (2005) and Yang et al. (2012) are based on the constant interest rate. As the impact of interest rate uncertainty is significant in our study, the model is extended to incorporate the uncertainty of the spot rates and find that the premium is underpriced if the uncertainty of the interest rate is neglected.

To evaluate the fair premium charged by an ex ante life insurance guaranty fund, the premium should properly reflect the default risk of the life insurance company and also incorporate the uncertainty of the interest rates into our pricing framework. This study first incorporates the risk-based concept into the asset allocation of the insurers, the early closure policy, the capital forbearance, and the grace period according to the regulatory actions of the regulators. The early closure can be regarded as a barrier option[4] on the underlying asset whose price is geared toward maintaining the working capital level. The regulatory forbearance during the grace period of the capital injection schedule can be regarded as an option on a put option. Therefore, the premium is summarized as the sum of the values of these embedded options.

---

[2]Since the pros and cons of ex ante and ex post assessment approaches are not the main topic in this study, we list only some advantages and drawbacks of both approaches. For a detailed comparison see Duncan (1987) and Oxera (2007).

[3]In 1992, the life and nonlife insurance guarantee funds were separately managed, and they were merged into the single-entity TIGF in 2009.

[4]Numerous papers discuss the valuation of participating life insurance contracts, such as Bacinello (2001), Tanskanen and Lukkarinen (2003), Ballotta (2005), Bernard et al. (2005b), and Le Courtois and Quittard-Pinon (2008). On the other hand, many financial literatures investigate the insolvent problem of insurers, such as Briys and de Varenne (1994, 1997), Grosen and Jorgensen (2002), Bernard et al. (2005a), and Chen and Suchanecki (2007). Chesney et al. (1997) and Labart and Lelong (2009) performed surveys on the option. All of these provide useful option-pricing methodologies to derive the closed-form solution of the premium of the insurance guaranty fund.

TABLE 1
Capital Structure of the Life Insurer

Asset	Liability
$A(0)$	$L(0) \equiv l \cdot A(0)$ $E(0) \equiv (1-l)A(0)$

Our model and the numerical results illustrate that the premium is underpriced if the uncertainty of the interest rate is neglected by the insurance guaranty fund. We also find that the influence of stochastic interest rate for the premium is more significant when we consider the capital forbearance scheme. Moreover, the volatility of the risky asset return has a significant impact on the fair premium. The premium increases with higher volatility. Furthermore, the financial leverage ratio and asset allocation strategy are critical in determining the impact of the volatility of the risky asset on the fair premium. As expected, a higher leverage ratio and a more risky investment strategy will exacerbate the negative influence of the volatility of the risky asset. The regulatory forbearance scheme greatly increases the cost of the insurance guaranty fund in taking over the insolvent life insurers.

The remainder of this article is organized as follows. Section 2 introduces the financial structure and the contract specification of life insurers. This section derives the fair premium of the insurance guaranty fund under the conditions of regulatory forbearance. Section 3 presents numerical analyses of the premium of the insurance guaranty fund and demonstrates the sensitivity analysis of the key factors. Finally, Section 4 concludes the study.

2. A MODEL FOR PRICING THE INSURANCE GUARANTY FUND

2.1. Financial Structure of Life Insurers

We assume the financial markets are in a continuous-time frictionless economy with a perfect financial market, no tax effects, no transaction costs, and no other imperfections. Following the setup in Briys and Varenne (1994, 1997), Grosen and Jørgensen (2002), and Chen and Suchanecki (2007), at time $t = 0$ the insurer is assumed to have a capital structure of assets and liabilities as shown in Table 1.

To simplify the model, we assume that the contractual payment made by the representative policyholder, the unique liability holder, at the beginning of the contract, which constitutes the liability of the insurance company, is denoted by $L(0) \equiv l \cdot A(0)$, where $l \in [0, 1]$ is the leverage ratio, and that the equity of the representative equity holder is accordingly denoted by $E(0) \equiv (1-l)A(0)$. Through their initial investments in the company, they each acquire a claim on the firm's assets for a payoff at maturity (or before maturity).

Insurers could invest their assets into a variety of investment instruments such as stocks, corporate bonds, government bonds, and bank deposits. To focus on studying the financial impact due to risky behaviors of the insurers, a simplified model is constructed, representing the economic scenarios in the market. Basically the underlying assets could be categorized into security assets, fixed income assets, and risk-free assets. For simplicity, in this study, we assume that the insurer's assets are invested in the risk-free cash equivalents $C(t)$, rolling bond $B_R(t)$[5] with a constant maturity R, and the stock index fund $S(t)$.

$S(t)$, $B_R(t)$, and $C(t)$ are defined on the probability space $(\Omega, \mathbf{F}, \mathrm{P})$ equipped with a filtration $\mathbf{F}$ and a physical measure P. T is a fixed time horizon. Here $\sigma(S(u), t \geq u \geq 0)$ is the smallest $\sigma-$algebrawith respect to $S(t)$, and $\sigma(r(u), t \geq u \geq 0)$ is the smallest $\sigma-$algebrawith respect to $r(t)$, which is the instantaneous short rate. Also, $\mathbf{F}(t) = \sigma(S(u), t \geq u \geq 0) \vee \sigma(r(u), t \geq u \geq 0)$ contains all information on $S(t)$, $B_R(t)$,and $C(t)$. Let a filtration $(\mathbf{F}(t), 0 \leq t \leq T)$ be $\mathbf{F}(T) = \mathbf{F}$. The price processes of assets evolve according to the following:

$$\frac{dC(t)}{C(t)} = r(t)dt, \tag{1}$$

$$\frac{dS(t)}{S(t)} = \mu(t)dt + \sigma_1 dW_r(t) + \sigma_2 dW_S(t), \tag{2}$$

$$\frac{dB_R(t)}{B_R(t)} = r(t)dt + \sigma_R(dW_r(t) + \lambda_r dt). \tag{3}$$

[5]The introduction of a rolling bond refers to Rutkowski (1999). The duration of the rolling bond maintains constant, hence the computational complex can be reduced.

In this study, we assume that the stochastic interest rate $r(t)$ follows the dynamic process $dr(t) = \kappa(\theta - r(t))dt + \sigma_r dW_r(t)$ (Vasicek 1977) with the force of mean reversion κ, the level of mean reversion θ, and the interest rate volatility σ_r. Here λ_r denotes the premium of interest rate risk. The constant local volatility of the rolling bond is $\sigma_R = \frac{1-e^{-\kappa R}}{\kappa}\sigma_r$. $W_r(t)$ and $W_S(t)$ are two independent Weiner processes under the physical probability measure P. The stock index fund $S(t)$ follows Black-Scholes dynamics with an instantaneous rate of return $\mu > 0$ and a constant volatility $\sqrt{\sigma_1^2+\sigma_2^2} > 0$. By letting the correlation coefficient of $dS(t)$ and $dr(t)$ be ρ, then ρ can describe the correlation of the risky asset and interest rate:

$$\rho = corr(dS(t), dr(t)) = \frac{\sigma_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}.$$

Subsequently the underlying asset A is assumed to consist of the stock index fund$S$, the rolling bond B_R, and the risk-free asset C. We use w_1 to denote the fraction of the asset invested in the stock index fund, w_2 to denote the fraction invested in the rolling bond, and the remaining $1 - w_1 - w_2$ to denote the fraction invested in the risk-free underlying asset. The price dynamics of the life insurer asset evolves as follows:

$$\begin{aligned} dA(t) &= w_1 A(t)\frac{dS(t)}{S(t)} + w_2 A(t)\frac{dB_R(t)}{B_R(t)} + (1 - w_1 - w_2)A(t)\frac{dC(t)}{C(t)} \\ \Rightarrow \frac{dA(t)}{A(t)} &= w_1\frac{dS(t)}{S(t)} + w_2\frac{dB_R(t)}{B_R(t)} + (1 - w_1 - w_2)\frac{dC(t)}{C(t)}. \end{aligned}$$

We substitute equations (1), (2), and (3) into the equation above, and then obtain equation (4) as follows:

$$\frac{dA(t)}{A(t)} = (r(t) + w_1(\mu - r(t)) + w_2\sigma_R\lambda_r)dt + \sigma_{A,r}dW_r(t) + \sigma_{A,S}dW_S(t), \tag{4}$$

where the notations are defined below. Since $W_r(t)$ and $W_S(t)$ are two independent Weiner processes under the physical probability measure, $\frac{dA(t)}{A(t)}$ is a normal random variable. For simplicity, we use $\sigma_{A,r}$ to denote $w_1\sigma_1 + w_2\sigma_R$ and $\sigma_{A,S}$ to denote $w_1\sigma_2$. Thus, the degree of the insurer's risk preference increases with values w_1 and w_2.

According to Cummins (1988) and Duan and Yu (2005), the risk of the insurer's asset is a vital parameter under risk-based premiums, which means that the investment strategy and the volatility of the asset are important factors when measuring the premium. The asset mix held by the insurer could be modeled by formulating a given typical asset allocation according to the insurer's risk preference and tolerance. In our model setting, (w_1, w_2) could be used to describe the risk-taking attitude of the insurers. The numerical analysis of the influence of (w_1, w_2) is given in Section 3.1.

Moving on to the contract framework, we set the insurers' reserve $L(t)$ for the enforced policies increases in $e^{\int_0^t r(s)ds}$. Notice that we assume that the liability increases with the stochastic interest rate in reflecting the fair valuation. Let T be the maturity date, then we have

$$L(T) = L(0)\exp\left(\int_0^T r(s)ds\right).$$

2.2. The Default Time, Regulatory Forbearance Mechanism, and the Payoff of the Insurance Guaranty Fund

From the regulator's point of view, the supervisory authority will concern the policyholder's protection in the event of a life insurance company being unable to meet its liabilities, as the collapse of the life insurer might give rise to negative social perceptions and result in financial instability. The financial supervisory authority must therefore find a suitable mechanism through which to intervene to reduce the losses to policyholders once an insurer faces insolvency or liquidity problems. In this article we consider a regulatory structure as shown in Figure 1.

The general assumption is that the regulatory authority cannot continuously monitor the insurer's balance sheet due to the audit cost. The audit is repeated at regular intervals T to examine the insurer's balance sheet. As the soundness of the financial structure of an insurance company is vital, the regulatory authority usually sets the minimum capital requirement α,which means that $A(T)$

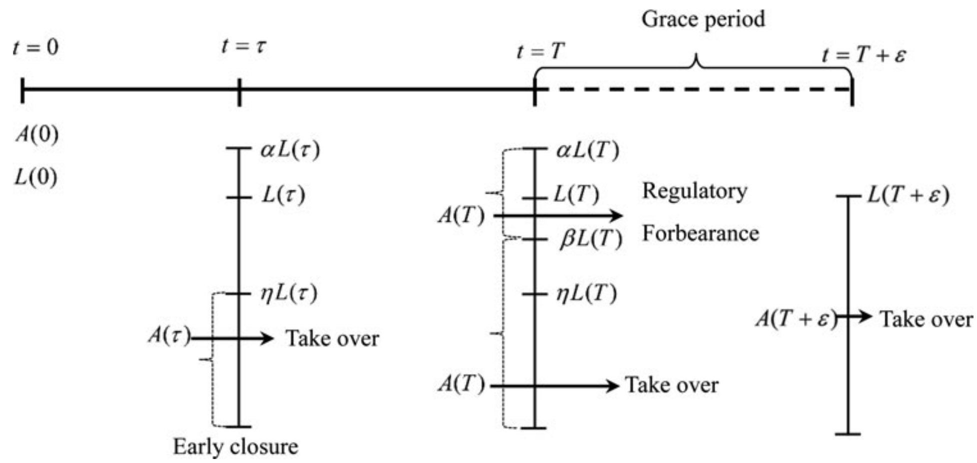

FIGURE 1. Illustration of Early Closure and Regulatory Forbearance.

has to be larger than $\alpha L(T)$ at the audit time T.[6] However, the insurer in the financial crisis may become insolvent during the audit window period. Thus, in this study, we have two cases: (1) the insurer's default occurs during the time intervals T and (2) the insurer is under the insolvent problem at time T.

First, the insurer's default is modeled on the basis that the insurer is not able to pay back the working capital, including the premium income. For simplicity, we assume that a solvency minimal capital requirement or mandatory asset level, $\eta L(t)$, is maintained. When the insurer's investment underperforms at τ, which is the default time, the insurer already defaults when $A(t) < \eta L(t)$. The default time τ can be constructed as the first time at which the insurer's assets fall below or cross the level for maintaining the working capital:

$$\tau = \inf \{ t \, | A(t) < \eta L(t) \} . \tag{5}$$

Note that if τ is not greater than the auditing time, T, the insurer goes out of business or is taken over. From the point of view of the TIGF, regardless of whether the insurer closes down or is taken over, the amount of the insurer's premium liquidated needs to be used to restore the asset values. Moreover, we assume that γ is the compensation ratio covered by the insurance guaranty fund once the insurer is taken over by the authority. We assume that P represents the payoff of the insurance guaranty fund. In this early insolvency situation, $P(\tau)$ can be formulated as $(\gamma - \eta)L(\tau)$.

Second, if the insurer is under the financially distressed problem at time T, then there are two kinds of scenarios. The capital forbearance β is the government intervention (similar to Duan and Yu 1999) and is defined as when an insurer fails to meet the regulatory capital standard but is allowed to continue its operations. In other words, when insurers face financial distress but the assets of the insurer have not fallen below the capital standard ($\beta L(T)$) upon maturity, the regulator might offer a grace period (ε) to the insurer during its capital restructuring. During this period, the insolvent insurer is allowed to continue its operations. On the other hand, if $A(T) < \beta L(T)$ at time T, then the TIGF will take over the insolvent insurer immediately. The purpose of regulatory forbearance is to ease the financial distress of those insurers and give them the chance to recover from the financial distress situation. Once the regulator takes the capital forbearance action, the grace period ε is considered if the asset value cannot meet the minimal capital standard $\alpha L(T)$ but does not fall below the capital forbearance $\beta L(T)$. Clearly, β should be greater than η, otherwise the capital forbearance might not be fair and effective. Therefore, $\alpha \geq 1 > \beta > \eta > 0$. According to the above

[6]The risk-based capital ratio (hereinafter referred to as the RBC ratio) is used as the capital supervision standard in most countries. For example, in Taiwan, the minimum requirement of the RBC ratio is 200%. Where the RBC ratio of an insurance company is lower than 200%, the insurance company shall not buy back its stock shares and distribute the net profit of the year for which the RBC Ratio Report is filed. For an insurance company with an RBC ratio between 150% and 200%, the competent authority may enact several regulatory actions. However, the RBC requirement is difficult to incorporate into the model; in this study, we adopt the proxy parameter αas in the study by Lee et al. (2005). Clearly $\alpha \geq 1$.

description, the cost of the insurance guaranty fund at time T, $P(T)$, can be expressed as

$$P(T) = \begin{cases} 0 & \alpha L(T) \le A(T) \\ F(T) & \beta L(T) \le A(T) < \alpha L(T) \\ \gamma L(T) - A(T) & \text{otherwise} \end{cases}. \tag{6}$$

When the grace period ε is applied to the insolvent insurers, the duration of the grace period is heavily dependent on the regulators. Regarding the regulatory forbearance for financially distressed insurers in Taiwan, the grace period set by the regulators might vary by firm size and the effects of contagion of the individual insurer. The normal grace period ε is usually set to be six months and can sometimes be extended to several years.[7] In Section 3.1 we show that a longer grace period will increase the bankruptcy cost of the insurance guaranty fund.

If the insurer facing financial distress is able to extend its operation until another time of auditing $T+\varepsilon$, then the insurer has to try to inject capital to satisfy the capital requirement within the grace period. At the end of grace period $T+\varepsilon$, the regulator would not adopt any action, and the insurer continues to operate the company when the asset value exceeds the minimal capital standard $\alpha L(T)$. If the financial distress becomes worse when the insurers' asset value $A(T+\varepsilon)$ falls between $\gamma L(T+\varepsilon)$ and $\alpha L(T+\varepsilon)$, the situation reaches the regulator action level (or the authorized control level), that is, in the U.S. RBC system. The regulator may issue a correction order such as to limit the insurer's business (or may reform or liquidate the company), hence, but the insurance guaranty fund might not be required to pay. Once the financial distress becomes worse, when the insurer's asset value $A(T+\varepsilon)$ is below $\gamma L(T+\varepsilon)$, the insurance guaranty fund are required to take over or liquidate the insolvent insurance company to prevent a possible larger loss. The insurance guaranty fund actually pays is the claim amount of the difference between $\gamma L(T+\varepsilon)$ and $A(T+\varepsilon)$. Therefore, $F(T)$ can be regarded as a put option in which the time to maturity is the grace period ε and the strike price is $\gamma L(T+\varepsilon)$. The option's payoff at $T+\varepsilon$ can be characterized as

$$F(T+\varepsilon) = \begin{cases} 0 & \text{if } \alpha L(T+\varepsilon) \le A(T+\varepsilon) \\ 0 & \text{if } \gamma L(T+\varepsilon) \le A(T+\varepsilon) < \alpha L(T+\varepsilon). \\ \gamma L(T+\varepsilon) - A(T+\varepsilon) & \text{otherwise} \end{cases} \tag{7}$$

Therefore, the payoff of the insurance guaranty fund, *P*, can be characterized as

$$P = (\gamma - \eta)^{+} L(\tau) I_{\{\tau \le T\}} + (\gamma L(T+\varepsilon) - A(T+\varepsilon))^{+} I_1 + (\gamma L(T) - A(T))^{+} I_2, \tag{8}$$

where $I_1 = I_{\{T<\tau, \beta L(T) \le A(T) < \alpha L(T), A(T+\varepsilon) < L(T+\varepsilon)\}}$ and $I_2 = I_{\{T<\tau, A(T)<\beta L(T)\}}$. I_D is the indicator function, which is 1 when event D occurs and 0 otherwise.

2.3. The Fair Premium of the Insurance Guaranty Fund

For simplicity, we assume that the insurance guaranty fund receives an upfront premium for providing security to the insurance company. The upfront premium corresponds to the present value of the insurance claim on the shortage of cash flows for the insolvent insurer. In this study, we take the expected discounted value under the risk-neutral probability measure Q to determine the present price of such claims. All stochastic processes mentioned hereafter are defined on the probability space $(\Omega, \mathbf{F}, \mathrm{P})$. From the Girsanov theorem for Brownian motions, we obtain the Radon-Nykodym density:

$$\frac{dQ}{d\mathrm{P}}\Big|_{\mathrm{F}} = \exp\left\{ -\int_0^T \lambda_r dW_r(t) - \int_0^T \lambda_S dW_S(t) - \frac{1}{2}\left(\int_0^T \lambda_r^2 dt + \int_0^T \lambda_S^2 dt \right) \right\},$$

[7]Recently, the Taiwan Financial Supervisory Commission (FSC) ordered the TIGF to take over two troubled insurers, Global Life Insurance Co. and Singfor Life Insurance Co., in 2014. The FSC indicates that the financial conditions of these two insurers have been deteriorating rapidly since 2005, and they lack progress toward improvement. Global Life's net worth declined from minus NT$100 million in 2006 to minus NT$8.3 billion in 2008 and minus NT$25.2 billion as of the end of June 2014, while Singfor Life's net worth has deteriorated from minus NT$100 million in 2005 to minus NT$16.4 billion in 2008, and to minus NT$23.9 billion in June 2014. The Taiwan FSC stated that the two insurers had met the regulatory criteria for government intervention in 2005. Even though regulatory forbearance has been adopted since then, these insurers continued to perform below expectations and received numerous penalizations over inadequacies in their operations, i.e., capital management, corporate governance, internal auditing processes, and overseas investment planning. In 2014 the Taiwan government made amendments to the Insurance Act that authorize regulators to take prompt correction action (PCA) against financially distressed insurers before their condition deteriorates further.

where λ_r denotes the market price of the interest rate process and $\lambda_s = \frac{w_1(\mu - r(t)) + w_2\sigma_R\lambda_r - \sigma_{A,r}\sigma_r\lambda_r}{\sigma_{A,S}}$ represents the market price of the market risk.

Then, the risk-neutral processes of the interest rate and the insurer's assets evolve according to

$$dr(t) = \bar{\kappa}(\bar{\theta} - r(t))dt + \sigma_r dW_r^Q(t), \tag{9}$$

$$\frac{dA(t)}{A(t)} = r(t)dt + \sigma_{A,r} dW_r^Q(t) + \sigma_{A,S} dW_S^Q(t), \tag{10}$$

where $\bar{\kappa} = \kappa + \lambda_r \sigma_r$, $\bar{\theta} = \frac{\kappa\theta}{\kappa + \lambda_r\sigma_r}$, $dW_r^Q(t) = dW_r(t) + \sigma_r\lambda_r dt$, and $dW_S^Q(t) = dW_S(t) + \frac{w_1(\mu - r(t)) + w_2\sigma_R\lambda_r - \sigma_{A,r}\sigma_r\lambda_r}{\sigma_{A,S}} dt$.

The solution to the stochastic differential equation formulated in equation (10) is expressed as $A(t) = A(0)\exp\{\int_0^t r(s)ds - \frac{1}{2}(\sigma_{A,r}^2 + \sigma_{A,S}^2)t + \sigma_{A,r} W_r^Q(t) + \sigma_{A,S} W_S^Q(t)\}$. The dynamics of the insurer's asset $A(t)$ can be rewritten as

$$A(t) = A(0)M(t)\exp X(t), \tag{11}$$

where $M(t) = e^{\int_0^t r(s)ds}$, $X(t) = at + \sigma_A W_A^Q(t)$, in which $a = -\frac{1}{2}(\sigma_{A,r}^2 + \sigma_{A,S}^2)$ and $\sigma_A^2 = \sigma_{A,r}^2 + \sigma_{A,S}^2$ such that $\sigma_A W_A^Q(t) \underline{\underline{d}} \sigma_{A,r} W_r^Q(t) + \sigma_{A,S} W_S^Q(t)$, where $\underline{\underline{d}}$means equal in distribution.

The risk-based premium paid by the insurer to the insurance guaranty fund is the expected discounted insurance payoff under the Q measure:

$$P(0) = E^Q\left[B(\tau)^{-1}P(\tau)I_{\{\tau \le T\}}\right] + E^Q\left[B(T)^{-1}P(T)I_{\{\tau > T\}}\right], \tag{12}$$

where $E^Q[\cdot]$ denotes the expected value under the Q measure. The closed-form solution can be expressed as follows:

$$\begin{aligned} P(0) = &\left\{(\gamma - \eta)L(0)\left[\Phi(c_2) + e^B\Phi(c_6)\right]\right\} \\ &+ \left\{\begin{array}{l} \gamma L(0)\left[(\Phi(c_1) - \Phi(c_2)) - e^{-B}\left(\Phi(c_3) - \Phi(c_4)\right)\right] \\ -A(0)\left[(\Phi(c_5) - \Phi(c_6)) - e^{-B}\left(\Phi(c_7) - \Phi(c_8)\right)\right] \end{array}\right\} \\ &+ \left\{\begin{array}{l} \gamma L(0)\left[(N(d_1, e_1, \delta) - N(d_5, e_1, \delta)) - e^{-B}\left(N(d_3, e_2, \delta) - N(d_6, e_2, \delta)\right)\right] \\ -A(0)\left[(N(d_7, e_3, \delta) - N(d_{11}, e_3, \delta)) - e^{-B}\left(N(d_9, e_4, \delta) - N(d_{12}, e_4, \delta)\right)\right] \end{array}\right\}, \end{aligned} \tag{13}$$

where $\Phi(x)$ represents a standard normal cumulative density function. $N(x, y, z)$ denotes standard bivariate normal cumulative density function, and $B = \ln\frac{\eta L(0)}{A(0)}$, $B_1 = \ln\frac{\alpha L(0)}{A(0)}$, $B_2 = \ln\frac{\beta L(0)}{A(0)}$, and $B_3 = \ln\frac{\gamma L(0)}{A(0)}$. Here $\delta = \sqrt{\frac{T}{T+\varepsilon}}$, $c_1 = \frac{B_2 - aT}{\sigma_A\sqrt{T}}$, $c_2 = \frac{B - aT}{\sigma_A\sqrt{T}}$, $c_3 = \frac{B_2 - 2B - aT}{\sigma_A\sqrt{T}}$, $c_4 = \frac{-B - aT}{\sigma_A\sqrt{bT}}$, $c_5 = \frac{B_2 + aT}{\sigma_A\sqrt{T}}$, $c_6 = \frac{B + aT}{\sigma_A\sqrt{T}}$, $c_7 = \frac{B_2 - 2B + aT}{\sigma_A\sqrt{T}}$ and $c_8 = \frac{-B + aT}{\sigma_A\sqrt{T}}$, $d_1 = \frac{B_1 - aT}{\sigma_A\sqrt{T}}$, $d_2 = \frac{B - aT}{\sigma_A\sqrt{T}}$, $d_3 = \frac{B_1 - 2B - aT}{\sigma_A\sqrt{T}}$, $d_4 = \frac{-B - aT}{\sigma_A\sqrt{T}}$, $d_5 = \frac{B_2 - aT}{\sigma_A\sqrt{T}}$, $d_6 = \frac{B_2 - 2B - aT}{\sigma_A\sqrt{T}}$, $d_7 = \frac{B_1 + aT}{\sigma_A\sqrt{T}}$, $d_8 = \frac{B + aT}{\sigma_A\sqrt{T}}$, $d_9 = \frac{B_1 - 2B + aT}{\sigma_A\sqrt{T}}$, $d_{10} = \frac{-B + aT}{\sigma_A\sqrt{T}}$, $d_{11} = \frac{B_2 + aT}{\sigma_A\sqrt{T}}$, and $d_{12} = \frac{B_2 - 2B + aT}{\sigma_A\sqrt{T}}$; and $e_1 = \frac{B_3 - a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$, $e_2 = \frac{B_3 - 2B - a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$, $e_3 = \frac{B_3 + a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$, and $e_4 = \frac{B_3 - 2B + a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$.

The detailed derivation is provided in Appendix A. There is no existing closed-form pricing formula with a stochastic interest rate in the literature that can be straightforwardly employed to evaluate the risk-based premium of the insurance guaranty fund. The closed-form solution in equation (13) can in fact be regarded as a general form of the formula used to calculate the insurance guaranty fund premium when considering the audit window period and capital forbearance.

The structure shown by equation (13) allows us to analyze the risk-based premium in a manner similar to that of Lee et al. (2005). The risk-based premium that takes into consideration the insurer's bankruptcy can be decomposed into an audit window component (P^a), a capital forbearance component (P^c) and a grace period component (P^ε), i.e., $P = P^a + P^c + P^\varepsilon$. The first term P^a of equation (13) is regarded as the audit window component, because the insurer reaches the default barrier $\eta L(\tau)$ at τ before the auditing time T. The insurance guarantee scheme thus cannot but implement the bankruptcy process. The insurer is taken over by the authority, and the insurance guaranty fund compensates for the difference between the insured asset level $\gamma L(\tau)$and the basic maintained asset level $\eta L(\tau)$. P^a describes this premium based on early closure. Its closed-form presentation is defined as

$$P^a = (\gamma - \eta)L(0)\left[\Phi(c_2) + e^B\Phi(c_6)\right]$$

The second term of equation (13) can be identified as the capital forbearance component, because its value is like the value of a down-and-out put option whose strike price is based on capital forbearance $\beta L(T)$ and the maturity time, the time of auditing T. Although the insurer may not go bankrupt, it touches the regulatory closure point $\beta L(T)$. The insurance company is taken over and its asset has an infusion of a rebate based on the difference between the insured asset level $\gamma L(T)$ and the real asset level $A(T)$. The rebate should be reflected in the premium as P^c. Its closed-form presentation is expressed as

$$P^c = \gamma L(0)\left[(\Phi(c_1) - \Phi(c_2)) - e^{-B}(\Phi(c_3) - \Phi(c_4))\right] \\ - A(0)\left[(\Phi(c_5) - \Phi(c_6)) - e^{B}(\Phi(c_7) - \Phi(c_8))\right].$$

The third term P^ε of equation (13) can be identified as the risk premium of regulatory delay. The premium results from the fact that the insurer is allowed to operate during the grace period ε to improve its asset level. Its closed-form presentation is identified as

$$P^\varepsilon = \gamma L(0)\left[(N(d_1, e_1, \delta) - N(d_5, e_1, \delta)) - e^{-B}(N(d_3, e_2, \delta) - N(d_6, e_2, \delta))\right] \\ - A(0)\left[(N(d_7, e_3, \delta) - N(d_{11}, e_3, \delta)) - e^{B}(N(d_9, e_4, \delta) - N(d_{12}, e_4, \delta))\right].$$

3. NUMERICAL ANALYSES

Here Section 3.1 discusses the influences on the risk-based premium under different settings of the leverage ratios, asset allocation strategies, the interest rate uncertainty, and volatilities of the risky asset. Moreover, we also analyze the effects of the capital forbearance and grace periods by comparison with Merton's put option cost (Merton 1977). Section 3.2 demonstrates the sensitivity and signs of partial derivatives of these main parameters (α, β, ε, γ, w_1, w_2, σ_r). The results clearly illustrate the impact of the insurance premium under regulatory forbearance.

3.1. The Sensitivity of Parameter

Table 2 shows the basic assumption of these vital parameters. Several countries apply the risk-based capital (RBC) approach, which is used to measure the minimum amount of capital that an insurer needs to support its overall business operations. For example, the RBC ratio is required to exceed 200% according to the current capital regulations in Taiwan. However, the formula for the RBC ratio might not be directly translated to the minimum capital requirement α. We might indirectly adopt the proxy parameters as in the study by Lee et al. (2005) and set α equal to 1.087 and β equal to 0.95 without losing generality. Moreover, we assume that the maintenance ratio, η, is set to be 0.5.[8] To compare with the premium of Merton's put model, we set γ to be 100%, which means that the insurance guaranty fund has to cover the whole insured liability of the insolvent insurer to its policyholders. According to the current contract setting, the maturity might be assumed to be one year and the grace period (ε) is 6 months, as the insurer has to report the RBC ratio twice a year. The parameters of the financial instruments follow the assumptions of Boulier et al. (2001), and we set the maturity of the rolling bond (R) to be 10 years according to the trading information of Taiwan's government bond.

Table 3 displays the insurance guaranty fund premium under different leverage ratios and investment strategies (w_1 and w_2). In this article, we fix the investment proportion of cash as 10% according to the public disclosed information of Taiwan's life insurers. In other words, $w_1 + w_2 = 90\%$. Notice that we also compute the premium under Merton's framework and decompose our insurance guaranty fund premium into three components, which are the early closure component, the capital forbearance component and the grace period component. The basic assumptions are as the same as shown in Table 2. To analyze the cost of the insurance liability, we set $L(0)$ to be 100.

Generally speaking, premium rates under the forbearance consideration are greater than those of Merton's put. For example, the premium under Merton's put is 0.0268 when the debt-to-asset ratio is 100/120 and w_1 is 10%, but the value (0.0981) is over threefold under the regulatory forbearance. The excess premium can be regarded as the cost of regulatory forbearance. Moreover, the main contribution of the excess premium results from the grace period when the debt-to-asset ratio is below 1.

Table 3 also indicates that the costs of bankruptcy are different under various parameter settings. First, the premium increases with the debt-to-asset ratio. More specifically, the premiums are 0.3830, 1.5800, and 4.8368 when the debt-to-asset ratios are

[8]The National Association of Insurance Commissioners (NAIC) made some references to the exit mechanism for insurance industries in 2008, Model Regulation to Define Standards and Commissioner's Authority for Companies Deemed to be in Hazardous Financial Condition, which has 20 criteria. According to one of these criteria (E regulation in section 3 of Model Regulation), the insurer's operating loss in the last 12-month period or any shorter period of time including net capital gain or loss is greater than 50% of the insurer's remaining surplus. Thus, we take η to be 0.5.

TABLE 2
Parameter Definition and Base Values

Name	Symbol	Value
Initial asset value	$A(0)$	110
Investment proportion of stock index fund	w_1	0.3
Investment proportion of rolling bond	w_2	0.6
Maturity the rolling bond	R	10
Initial rate	$r(0)$	2.67%
Force of mean reversion	κ	0.2
Mean of interest rate	θ	2%
Volatility of interest rate	σ_r	0.02
Impact coefficient of $W_r(t)$ on $dS(t)/S(t)$	σ_1	0.06
Impact coefficient of $W_S(t)$ on $dS(t)/S(t)$	σ_2	0.1908
Maturity date	T	1 year
Grace period	ε	0.5 year
Parameter in triggering the intervention	η	0.5
Capital standard parameter	α	1.087
Forbearance parameter	β	0.95
Initial liability value	$L(0)$	100
Compensation ratio	γ	1

100/120, 100/110, and 100/100 under w_1 at 40%. A higher leverage ratio reflects a greater risk of deficit between assets and liabilities, and thus the insurance guaranty fund naturally charges a higher premium. Moving on to the asset allocation strategies, we find that the cost of the insurance guaranty fund rises with the investment proportions of the stock index fund. To be precise, the premium is 4.8368 when w_1 is 40% and goes down to 3.7381 when w_1 is 10% for a debt-to-asset ratio of 100/100. The reason

TABLE 3
Insurance Guaranty Fund Premium under Different Asset Allocation Strategies

	Proportion of Risky Asset w_1			
	0.1	0.2	0.3	0.4
Debt-to-asset ratio = 100/100				
Merton's put	3.0933	3.2685	3.6024	4.0559
Under forbearance	3.7381	3.9409	4.3238	4.8368
Early closure component	0	0	0	0
Capital forbearance component	2.4859	2.6874	3.0664	3.5720
Grace period component	1.2522	1.2535	1.2574	1.2648
Debt-to-asset ratio = 100/110				
Merton's put	0.4297	0.5197	0.7094	0.9997
Under forbearance	0.7999	0.9313	**1.1962**	1.5800
Early closure component	0	0	0	0
Capital forbearance component	0.2501	0.3241	0.4896	0.7584
Grace period component	0.5499	0.6072	0.7066	0.8215
Debt-to-asset ratio = 100/120				
Merton's put	0.0268	0.0409	0.0799	0.1619
Under forbearance	0.0981	0.1250	0.2234	0.3830
Early closure component	0	0	0	0
Capital forbearance component	0.0104	0.0180	0.0424	0.1018
Grace period component	0.0877	0.1070	0.1811	0.2811

Note: 0 denotes any value less than 0.0001.

TABLE 4
Insurance Guaranty Fund Premium under Different σ_r

	σ_r				
	0.01	0.02	0.03	0.04	0.05
Debt-to-asset ratio = 100/100					
Merton's put	2.8782	3.6024	4.4505	5.3634	6.3128
Under forbearance	3.4874	4.3238	5.2772	6.2789	7.2997
Early closure component	0	0	0	0	0
Capital forbearance component	2.2359	3.0664	4.0049	4.9880	5.9901
Grace period component	1.2515	1.2574	1.2723	1.2908	1.3090
Debt-to-asset ratio = 100/110					
Merton's put	0.3295	0.7094	1.2771	1.9851	2.7898
Under forbearance	0.6472	**1.1962**	1.9298	2.7768	3.6922
Early closure component	0	0	0	0	0
Capital forbearance component	0.1724	0.4896	1.0255	1.7284	2.5420
Grace period component	0.4747	0.7066	0.9044	1.0484	1.1502
Debt-to-asset ratio = 100/120					
Merton's put	0.0146	0.0799	0.2622	0.5952	1.0732
Under forbearance	0.0624	0.2234	0.5555	1.0552	1.6891
Early closure component	0	0	0	0	0
Capital forbearance component	0.0047	0.0424	0.1825	0.4776	0.9307
Grace period component	0.0578	0.1811	0.3730	0.5776	0.7584

Note: 0 denotes any value less than 0.0001.

is similar to the leverage ratio. The portfolio shares (w_1) of the stock index fund increase the risk of the life insurer. Then the guaranty fund has to ask for a higher premium. Furthermore, in Figure 2, we present the relationship between the asset allocation and the debt-to-asset ratio. We find that the slope of the blue solid line (debt-to-asset ratio = 100/100) is larger, which suggests that the influence of the risky asset is significant when the leverage ratio increases.

Subsequently, we illustrate the effects of the volatility of interest rates (σ_r) in Table 4 and Figure 3. We set the basic scenario as being that $w_1 = 30\%$, the debt-to-asset ratio = 100/120 (red dashed line), 100/110 (green dotted line), and 100/100 (blue solid line). In Table 4, σ_r is from 0.01 to 0.05, and we find that when the volatility of the interest rate increases, the cost of the ex ante insurance guarantee fund goes up dramatically. For example, the premium is 1.1962 under the basic scenario (as in Table 3),

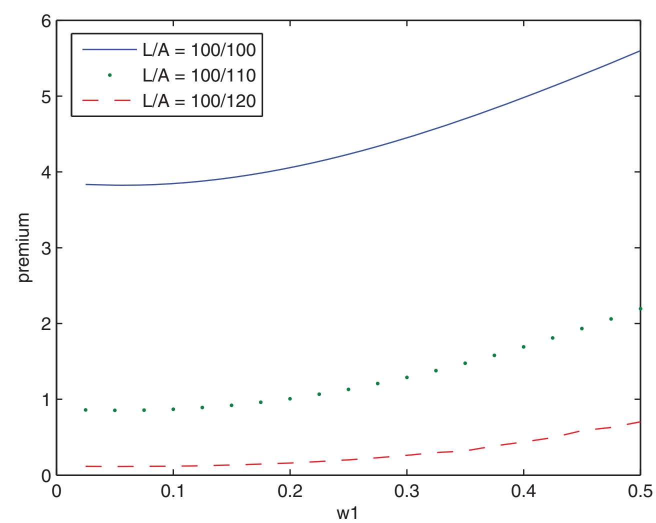

FIGURE 2. Relationship between w_1 and Debt-to-Asset Ratio.

TABLE 5
Insurance Guaranty Fund Premium under Different σ_1 and σ_2

	(σ_1, σ_2)				
	(0.03,0.1908)	(0.06,0.1908)	(0.09,0.1908)	(0.06,0.3)	(0.06,0.4)
Debt-to-asset ratio = 100/100					
Merton's put	3.3327	3.6024	3.8866	4.5432	5.5354
Under forbearance	4.0148	4.3238	4.6460	5.3801	6.4648
Early closure component	0	0	0	0	0
Capital forbearance component	2.7608	3.0664	3.3843	4.1059	5.1709
Grace period component	1.2541	1.2574	1.2618	1.2741	1.2939
Debt-to-asset ratio = 100/110					
Merton's put	0.5544	0.7094	0.8873	1.3452	2.1265
Under forbearance	0.9809	**1.1962**	1.4340	2.0138	2.9408
Early closure component	0	0	0	0	0
Capital forbearance component	0.3535	0.4896	0.6527	1.0920	1.8707
Grace period component	0.6274	0.7094	0.7813	0.9218	1.0700
Debt-to-asset ratio = 100/120					
Merton's put	0.0471	0.0799	0.1272	0.2897	0.6726
Under forbearance	0.1498	0.2234	0.3183	0.6002	1.1625
Early closure component	0	0	0	0	0
Capital forbearance component	0.0216	0.0424	0.0757	0.2056	0.5494
Grace period component	0.1282	0.1811	0.2426	0.3946	0.6131

Note: 0 denotes any value less than 0.0001.

but rises to 3.6922 when σ_r is 5%. This is because the liability is volatile when the volatility of the interest rate goes up, and it increases the probability of a mismatch between liability and asset. The results of Table 4 show the underpricing problem of insurance guaranty funds when the uncertainty of interest rates is not fully incorporated.

Table 5 and Figure 4 present the influence of the volatility of the risky asset return (σ_1 and σ_2). We find that the premium of the insurance guaranty fund increases when the risky asset becomes more volatile. These numerical results show that the premium of the insurance guaranty fund should be adjusted according to the asset allocation strategy of the insurer.

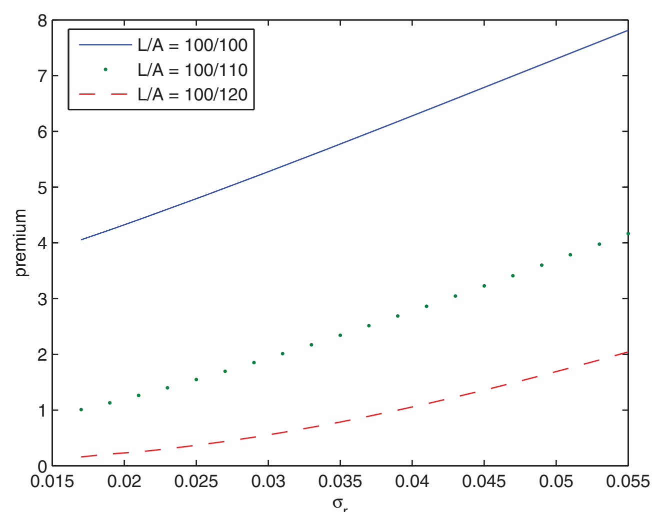

FIGURE 3. Relationship between σ_r and Debt-to-Asset Ratio.

TABLE 6
Insurance Guaranty Fund Premium under Different β

	Capital Forbearance Threshold (β)				
	1	0.97	0.95	0.9	0.8
Debt-to-asset ratio = 100/100					
Under forbearance	3.9905	4.2390	4.3238	4.3893	4.3946
Early closure component	0	0	0	0	0
Capital forbearance component	3.6024	3.4035	3.0664	1.8055	0.1708
Grace period component	0.3881	0.8355	1.2574	2.5638	4.2238
Debt-to-asset ratio = 100/110					
Under forbearance	1.0356	**1.1642**	1.1962	1.2140	1.2148
Early closure component	0	0	0	0	0
Capital forbearance component	0.7094	0.6146	0.4896	0.1889	0.0055
Grace period component	0.3262	0.5496	0.7066	1.0251	1.2094
Debt-to-asset ratio = 100/120					
Under forbearance	0.1916	0.2184	0.2234	0.2255	0.2254
Early closure component	0	0	0	0	0
Capital forbearance component	0.0799	0.0616	0.0424	0.0103	0
Grace period component	0.1117	0.1568	0.1811	0.2152	0.2253

Note: 0 denotes any value less than 0.0001.

The premiums of the insurance guaranty fund for an alternative capital forbearance threshold are presented in Table 6. We set the basic scenario as in Table 2, and the threshold β varies in the sequence 1, 0.97, 0.95, 0.9, and 0.8. The numerical results show that the premiums rise as the value of the forbearance threshold drops. As the forbearance threshold extends the insurance coverage to the undercapitalized insurers, the probability of the insurer surviving at auditing time T increases and thus the grace period component significantly increases. Moreover, this also shows that increases in the debt-to-asset ratio raise the sensitivities of the grace period and capital forbearance components to the forbearance threshold. Moreover, in Figure 5, we display the cross-influence of the grace period (ε) and the threshold (β). As the value of the forbearance threshold become lower, the cost of the grace period component increases, while the cost of the capital component decreases. Thus, the premium increases significantly from 0.9 to 1, while it is almost flat below 0.9. Table 7 illustrates the influence of η. Notice that the cost of early closure is zero from Table 3 to Table 6. This is because the low level $\eta = 0.5$ of early closure results in the cost being than 0.0001. In Table 7 we compare η from 0.5 to 0.8. The cost of early closure increases as η increases.

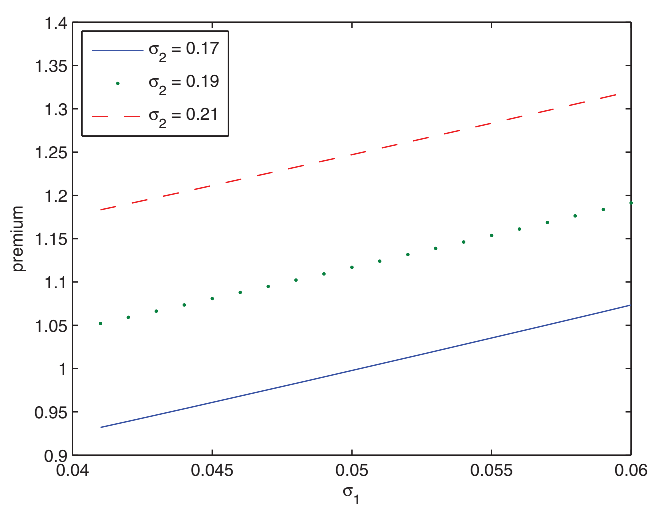

FIGURE 4. Relationship between σ_1 and σ_2.

TABLE 7
Insurance Guaranty Fund Premium under Different η

	η			
	0.5	0.6	0.7	0.75
Debt-to-asset ratio = 100/100				
Under forbearance	4.3238	4.3238	4.3230	4.3147
Early closure component	0	0	0.0021	0.0328
Capital forbearance component	3.0664	3.0664	3.0636	3.0246
Grace period component	1.2574	1.2574	1.2574	1.2574
Debt-to-asset ratio = 100/110				
Under forbearance	1.1962	1.1962	1.1962	1.1961
Early closure component	0	0	0	0.0004
Capital forbearance component	0.4896	0.4896	0.4896	0.4890
Grace period component	0.7066	0.7066	0.7066	0.7066
Debt-to-asset ratio = 100/120				
Under forbearance	0.2234	0.2234	0.2434	0.2434
Early closure component	0	0	0	0
Capital forbearance component	0.0424	0.0424	0.0424	0.0424
Grace period component	0.1811	0.1811	0.1811	0.1811

Note: 0 denotes any value less than 0.0001.

Lee et al. (2005) and Yang et al. (2012) assume that the interest rate is fixed and incorporate the capital forbearance to derive the closed-form solution of the premium. However, stochastic interest rate is a vital assumption under fair pricing problem. Thus, the capital forbearance scheme is incorporated into the pricing problem under the stochastic interest rate environment. Table 8 shows the cross-influence of stochastic interest rate and capital forbearance mechanism. In Table 8 the basic benchmark is the premium under σ_r is 0%, which means that the interest rate is fixed. We list the premium increment induced by the interest rate volatility (from 0% to 3% and 5%). Under the case of Merton's put, there is no capital forbearance mechanism. First, we find that the increments under capital forbearance are larger than those under Merton's put. In other words, when the interest rate is more volatile, the influence for the premium is more significant when we consider the capital forbearance mechanism. Second, a high leverage ratio will deteriorate the influence of the stochastic interest rate. For example, when σ_r increases to 3% under the capital

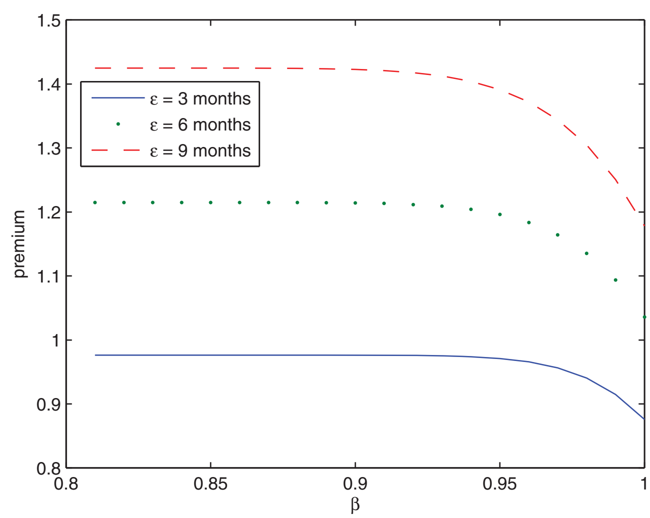

FIGURE 5. Relationship between β and ε.

TABLE 8
Premium Increment Induced by the Volatility of Interest Rates ($w_1 = 0.2$)

	Premium Increment	
	$\sigma_r : 0 \to 3\%$	$\sigma_r : 0 \to 5\%$
Debt-to-asset ratio = 100/100		
Merton's put	2.7726	5.0836
Under forbearance	3.2316	5.7355
Debt-to-asset ratio = 100/110		
Merton's put	1.2080	3.1048
Under forbearance	1.8083	4.0045
Debt-to-asset ratio = 100/120		
Merton's put	0.2401	1.2902
Under forbearance	0.5189	1.9603

forbearance case, then the premium increment is 0.5189 under debt-to-asset ratio is 100/120; however, the premium increment goes up to 3.2316 under debt-to-asset ratio is 100/100.

3.2. The Sensitivity of the Greeks

In this section, we discuss how the value of the insurance premium under forbearance varies in response to changes in the main parameters ($\alpha, \beta, \varepsilon, \gamma, w_1, w_2, \sigma_r$). The partial derivatives of these parameters are too complicated to determine directly their

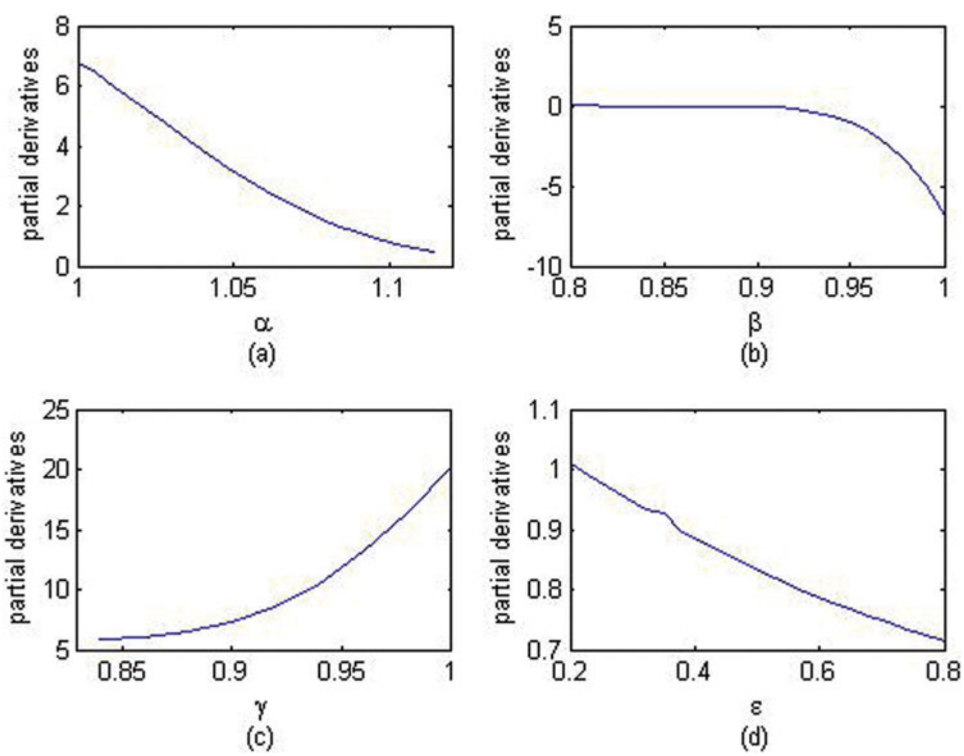

FIGURE 6. The Partial Derivatives with Respect to α, β, γ, and ε.

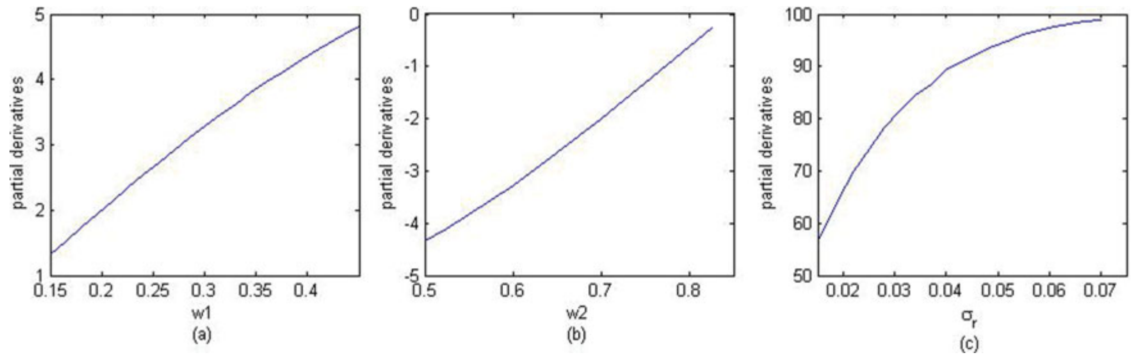

FIGURE 7. The Partial Derivatives with Respect to w_1, w_2, and σ_r.

signs according to their partial derivative formulas.[9] We ascertain the signs of the partial derivatives of these parameters through numerical experiments.

Figure 6 presents the results of partial derivatives with respect to α, β, γ, and ε. We find that $\frac{\partial P(0)}{\partial \alpha}$ drops and the sign is still positive as α rises, as shown in Figure 6(a). This means that the premium increases with the capital requirement but its impact on the premium decreases. Figure 6(b) shows that $\frac{\partial P(0)}{\partial \beta}$ drops and the sign is still negative as β rises. This indicates a loose-fitting capital forbearance setting, which means that the lower β increases the premium owing to the cost of capital forbearance component, but the impact of forbearance on the premium decreases. Figure 6(c) helps to easily understand the sign and trend of $\frac{\partial P(0)}{\partial \gamma}$. As γ increases, $\frac{\partial P(0)}{\partial \gamma}$ rises and is positive, suggesting that the cost of the insurance guaranty fund increases when γ rises. The increased premium is owing to the cost of compensation, and the impact on the premium increases. Moving on to the grace period, Figure 6(d) helps to easily understand the sign and trend of $\frac{\partial P(0)}{\partial \varepsilon}$. As ε increases, $\frac{\partial P(0)}{\partial \varepsilon}$ drops but is still positive. This illustrates that ε increases the premium owing to the cost of the grace period component, while the impact on the premium decreases.

Figure 7 illustrates the partial derivatives with respect to w_1, w_2, and σ_r, which are related to the investment strategy. The formula of the partial derivatives is listed in Appendix B. From Figure 7(a), we find that as w_1 increases, $\frac{\partial P(0)}{\partial w_1}$ is positive and rises. It indicates that w_1 increases the bankruptcy cost of the insurance guaranty fund owing to the increment of the portfolio's volatility, and the impact on the premium increases. On the other hand, as w_2 increases, $\frac{\partial P(0)}{\partial w_2}$ rises, as shown in Figure 7(b). However, the sign remains negative, which means that w_2 decreases the premium. In our model, we set the sum of w_1 and w_2 to be 90%; thus, a greater w_2 results in a lower w_1 and then decreases the volatility of the whole investment portfolio, while the impact of w_2 on the premium lessens. Figure 7(c) helps us to explicitly understand the sign and trend of $\frac{\partial P(0)}{\partial \sigma_r}$. As σ_r increases, $\frac{\partial P(0)}{\partial \sigma_r}$ is positive and rises. This demonstrates that σ_r increases the premium owing to the increment of the portfolio's volatility, and the impact on the premium increases.

4. CONCLUSION

This study is devoted to deriving the pricing formula to determine a fair premium. The early closure, financial leverage, and risky asset allocation are each taken into account in our risk-based model. Compared with the study in Yang et al. (2012), in this study the uncertainty of interest rates is incorporated in premium rating. The numerical experiments show the influence of the vital parameters. First, we find that the premium of the insurance guaranty fund will be underestimated if we omit the influence of the uncertainty of interest rates. Second, the premium is larger under regulatory forbearance than the traditional Merton put, which means that the cost of regulatory forbearance is enormous. Third, when the interest rate is more volatile, the influence for the premium is more significant under the capital forbearance mechanism case than that under Merton's put case. Finally, we find that the premium increases with higher financial leverage and more risky asset allocation. As the financial leverage and risky asset allocation strategy play important roles in the solvency risk of the insurance company, the higher risk is reflected in a higher insurance premium. Moreover, the volatility of the return on the risky asset is an important parameter under fair premium pricing. The premium increases with higher volatility. Thus, we suggest that the regulator should charge a different premium for the insurance guaranty fund according to the life insurer's financial structure and risky investment strategy. Finally, premiums will rise as the values of the forbearance threshold drop, and this reminds the regulator to set a reasonable forbearance threshold.

[9] The detailed derivations of the Greeks of the main parameters are listed in Appendix B. In Section 3.2, we present the analysis through conducting numerical experiments.

There are a variety of assets in the investment portfolio of the life insurance industry, including equities, bonds, and real estate. In this article, the model is simplified to allow only equities, bonds, and cash in our investment portfolio to improve our understanding on premium increment induced by the volatility of interest and other vital parameters. Also the portfolio composition on the liability side of life insurance might have significant influences on the financial stability of the life insurers. Therefore, in future research, our model could be extended to allow a more realistic asset portfolio and liability structure.

REFERENCES

Bacinello, A. R. 2001. Fair Pricing of Life Insurance Participating Policies with a Minimum Interest Rate Guaranteed. *ASTIN Bulletin* 31: 275–297.

Ballotta, L. 2005. A Levy Process-Based Framework for the Fair Valuation of Participating Life Insurance Contracts. *Insurance Mathematics and Economics* 37: 173–196.

Bernard, C., O. Le Courtois, and F. Quittard-Pinon. 2005a. A Study of Mutual Insurance for Bank Deposits. *Geneva Risk and Insurance Review* 30: 129–146.

Bernard, C., O. Le Courtois, and F. Quittard-Pinon. 2005b. Market Value of Life Insurance Contracts under Stochastic Interest Rates and Default Risk. *Insurance Mathematics and Economics* 36: 499–516.

Boulier, J. F., S. J. Huang, and G. Taillard. 2001. Optimal Management under Stochastic Rates: The Case of a Protected Defined Contribution Pension Fund. *Insurance Mathematics and Economics* 28: 173–189.

Briys, E., and F. de Varenne. 1994. Life Insurance in a Contingent Claim Framework: Pricing and Regulatory Implications. *Geneva Papers on Risk and Insurance Theory* 19: 53–72.

Briys, E., and F. de Varenne. 1997. On the Risk of Insurance Liabilities: Debunking Some Common Pitfalls. *Journal of Risk and Insurance* 64: 673–694.

Chen, A. 2011. A Risk-Based Model for the Valuation of Pension Insurance. *Insurance Mathematics and Economics* 49: 401–409.

Chen, A., and M. Suchanecki. 2007. Default Risk, Bankruptcy Procedures and the Market Value of Life Insurance Liabilities. *Insurance Mathematics and Economics* 40: 231–255.

Cummins, J. D. 1988. Risk-Based Premiums for Insurance Guaranty Funds. *Journal of Finance* 43: 823–839.

Duan, J. C., A. F. Moreau, and C. W. Sealey. 1995. Deposit Insurance and Bank Interest Rate Risk: Pricing and Regulatory Implications. *Journal of Banking and Finance* 19: 1091–1108.

Duan, J. C., and J. G. Simonato. 2002. Maximum Likelihood Estimation of Deposit Insurance Value with Interest Rate Risk. *Journal of Empirical Finance* 9: 109–132.

Duan, J. C., and M. T. Yu. 1994. Assessing the Cost of Taiwan's Deposit Insurance. *Pacific-Basin Finance Journal* 2: 73–90.

Duan, J. C., and M. T. Yu. 1999. Capital Standard, Forbearance and Deposit Insurance Pricing under GARCH. *Journal of Banking and Finance* 23: 1691–1706.

Duan, J. C., and M. T. Yu. 2005. Fair Insurance Guaranty Premia in the Present of Risk-Based Capital Regulations, Stochastic Interest Rate and Catastrophe Risk. *Journal of Banking and Finance* 29: 2435–2454.

Duncan, M. P. 1987. Property-Liability Post-Assessment Guaranty Funds. In *Issues in Insurance*, edited by E. D. Randall, vol. 2, pp. 239–302. Malvern, PA: American Institute for Property and Liability Underwriters.

Grosen, A., and P. L. Jørgensen. 2002. Life Insurance Liabilities at Market Value: An Analysis of Insolvency Risk, Bonus Policy, and Regulatory Intervention Rules in a Barrier Option Framework. *Journal of Risk and Insurance* 69: 63–91.

Han, L. M., G. C. Lai, and R. C. Witt. 1997. A Financial-Economic Evaluation of Insurance Guaranty Fund System: An Agency Cost Perspective. *Journal of Banking and Finance* 21: 1107–1129.

Hwang, D. Y., F. S. Shie, K. Wang, and J. C. Lin. 2009. The Pricing of Deposit Insurance Considering Bankruptcy Costs and Closure Policies. *Journal of Banking and Finance* 33: 1909–1919.

Jeanblanc M., M. Yor, and M. Chesney. 2009. *Mathematical Methods for Financial Markets*. Springer Finance. London: Springer.

Krogh, H. C. 1972. Insurer Post-Insolvency Guaranty Funds. *Journal of Risk and Insurance* 39: 431–450.

Labart, C., and J. Lelong. 2009. Pricing Parisian Options Using Laplace Transforms. *Bankers, Markets and Investors* 99: 29–43.

Le Courtois, O., and F. Quittard-Pinon. 2008. Fair Valuation of Participating Life Insurance Contracts with Jump Process. *Geneva Risk and Insurance Review* 33: 106–136.

Lee, S. C., J. P. Lee, and M. T. Yu. 2005. Bank Capital Forbearance and Valuation of Deposit Insurance. *Canadian Journal of Administrative Sciences* 22: 220–229.

Marcus, A. 1987. Corporate Pension Policy and the Value of PBGC Insurance. In *Issues in Pension Economics*, edited by Z. Bodie, J. B. Shoven, and D. A. Wise, pp. 49–80. Chicago: University of Chicago Press.

Merton, R. C. 1977. An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees. *Journal of Banking and Finance* 1: 3–11.

Oxera. 2007. Insurance Guarantee Schemes in the EU: Comparative Analysis of Existing Schemes, Analysis of Problems and Evaluation of Options. A final report for European Commission DG Internal Market and Services, United Kingdom.

Pennacchi, G. G., and C. M. Lewis. 1994. The Value of Pension Benefit Guaranty Corporation Insurance. *Journal of Money, Credit and Banking* 26: 735–753.

Ronn, E. I., and A. K. Verma. 1986. Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model. *Journal of Finance* 41: 871–895.

Rutkowski, M. 1999. Self-financing Trading Strategies for Sliding, Rolling-Horizon, and Consol Bonds. *Mathematical Finance* 9: 361–385.

Sharpe, W. F. 1976. Corporate Pension Funding Policy. *Journal of Financial Economics* 3: 183–193.

Tanskanen, A. J., and J. Lukkarinen. 2003. Fair Valuation of Path-Dependent Participating Life Insurance Contracts. *Insurance Mathematics and Economics* 33: 595–609.

Vasicek, O. 1977. An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics* 5: 177–188.

Yang, S. Y., Y. W. Hwang, and S. C. Chang. 2012. The Bankruptcy Cost of Life Insurance Industry under Regulatory Forbearance: An Embedded Option Approach. *North American Actuarial Journal* 16: 513–523.

Discussions on this article can be submitted until January 1, 2016. The authors reserve the right to reply to any discussion. Please see the Instructions for Authors found online at http://www.tandfonline.com/uaaj for submission instructions.

APPENDIX A. EXPLICIT DERIVATION OF THE CLOSED-FORM PRICING FORMULA

The risk-based premium paid by the insurer to the insurance guaranty fund is the expected discounted insurance payoff under the risk-neutral probability measure Q and can be decomposed into three parts. As shown in equation (12), the first part is the premium that reflects the default risk from the insurer before the auditing time. The second part consists of the premium that reflects the expected claim resulting from the insurer's asset value below the capital forbearance at the auditing time. The final part reflects the premium of the expected claim caused by the insurer at the grace period:

$$P(0) = E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T\}}\right] + E^Q\left[\frac{P(T)}{M(T)}I_{\left\{T\le\tau,\frac{A(T)}{L(T)}<\beta\right\}}\right] + E^Q\left[\frac{P(T)}{M(T)}I_{\left\{T\le\tau,\beta<\frac{A(T)}{L(T)}<\alpha\right\}}\right].$$

For the derivation of the above equation, we exhibit the liability and asset price processes of the life insurer as follows:

$$\begin{aligned} L(T) &= L(0)\exp\left(\int_0^T r(s)ds\right) = L(0)M(T), \\ A(t) &= A(0)\exp\left\{\int_0^t r(s)ds - \frac{1}{2}\left(\sigma_{A,r}^2+\sigma_{A,S}^2\right)t + \sigma_{A,r}W_r^Q(t) + \sigma_{A,S}W_S^Q(t)\right\} \\ &= A(0)M(t)\exp X(t), \end{aligned}$$

where $M(t) = e^{\int_0^t r(s)ds}$, $X(t) = at + \sigma_A W_A^Q(t)$ in which $a = \frac{-1}{2}(\sigma_{A,r}^2 + \sigma_{A,S}^2)$ and $\sigma_A^2 = \sigma_{A,r}^2 + \sigma_{A,S}^2$ such that $\sigma_A W_A^Q(t)\underline{\underline{d}}\sigma_{A,r}W_r^Q(t) + \sigma_{A,S}W_S^Q(t)$, where $\underline{\underline{d}}$ means equal in distribution. So $\frac{2a}{\sigma_A^2}$ equals -1.

The insurance guaranty fund takes corresponding action when the insurer's asset value $A(t)$ reaches thresholds $(\eta L(t), \alpha L(t), \beta L(t), \gamma L(t))$ in the payoff scheme of equations (6) and (7). To shorten the expression in the equations, we hereafter define the following notation:

$$B = \ln\frac{\eta L(0)}{A(0)},\ B_1 = \ln\frac{\alpha L(0)}{A(0)},\ B_2 = \ln\frac{\beta L(0)}{A(0)},\ \text{and } B_3 = \ln\frac{\gamma L(0)}{A(0)}.$$

The inequality equation $B_1 > B_3 > B_2 > B$ holds as $\alpha > 1 > \beta > \eta$. Then some results from mathematical finance (Jeanblanc et al. 2009) are introduced for the derivation of formula. The default time τ is the first time at which the Brownian motion $X(t)$ with drift $a \cdot t$ reaches the barrier B. The notation $m(T)$ means the minimum of $X(t)$ from time 0 to date T, that is, $m(T) = \min\limits_{0\le s\le T} X(s)$. Therefore, the joint probability of $X(t)$ and $m(T)$ is

$$\begin{aligned} &\Pr(X(T) < B_1, m(T) \ge B) \\ &= \left[\Phi\left(\frac{-B+aT}{\sigma_A\sqrt{T}}\right) - \Phi\left(\frac{-B_1+aT}{\sigma_A\sqrt{T}}\right)\right] - e^{\frac{2aB}{\sigma_A^2}}\left[\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right) - \Phi\left(\frac{-B_1+2B+aT}{\sigma_A\sqrt{T}}\right)\right] \\ &= \left[\Phi\left(\frac{-B+aT}{\sigma_A\sqrt{T}}\right) - \Phi\left(\frac{-B_1+aT}{\sigma_A\sqrt{T}}\right)\right] - e^{-B}\left[\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right) - \Phi\left(\frac{-B_1+2B+aT}{\sigma_A\sqrt{T}}\right)\right], \end{aligned} \tag{A1}$$

and the probability of the default time τ before the time of auditing T is

$$\begin{aligned} \Pr(\tau < T) &= \Pr(m(T) < B) \\ &= \Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right) + e^{\frac{2aB}{\sigma_A^2}}\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right) = \Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right) + e^{-B}\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right), \end{aligned} \tag{A2}$$

where $\Phi(\cdot)$ stands for the cumulative function of the standard normal distribution. The joint probability density is obtained by taking the first derivative of the above equation with respect to B_1 and B,

$$f(x,m) = \frac{\partial^2 \Pr(X(T) < B_1, m(T) \ge B)}{\partial B_1 \partial B}\Big|_{B_1=x, m(T)=m} = \frac{2(x-2m)}{\sigma_A^2 T\sqrt{2\pi\sigma_A^2 T}}e^{-\frac{(x-2m)^2}{2\sigma_A^2 T} + \left(\frac{ax}{\sigma_A^2} - \frac{a^2T}{2\sigma_A^2}\right)},$$

where $x > 2m$.

Next, we are in a position to calculate the three parts of the premium formula, where Lemma 1 is for $E^Q[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T\}}]$, Lemma 2 for $E^Q[\frac{P(T)}{M(T)}I_{\{T\le\tau,\frac{A(T)}{L(T)}<\beta\}}]$, and Lemma 3 for $E^Q[\frac{P(T)}{M(T)}I_{\{T\le\tau,\beta<\frac{A(T)}{L(T)}<\alpha\}}]$.

Lemma 1

The premium of the audit window component P^a is given by

$$E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T\}}\right]=(\gamma-\eta)L(0)\left(\Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right)+e^{-B}\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right)\right),$$

where $\Phi(x)$ represents a standard normal cumulative density function.

Proof. Since the definition of early closure states that $\tau=\inf\{t\,|A(\tau)\le\eta L(\tau)\}$ comes before T and the theorem of the first passage time is as mentioned above, the probability of default before the auditing time is derived as follows:

$$\begin{aligned}E^Q\left[I_{\{\tau<T\}}\right]&=\overset{Q}{\Pr}(\tau<T|A(0)>\eta L(0))\\&=\overset{Q}{\Pr}\left(\min_{0\le s\le T}\frac{A(s)}{L(s)}<\eta\left|\frac{A(0)}{L(0)}>\eta\right.\right)\\&=\overset{Q}{\Pr}\left(\min_{0\le s\le T}\left(as+\sigma_A W_A^Q(s)\right)<\ln\left(\frac{\eta L(0)}{A(0)}\right)\left|\frac{A(0)}{L(0)}>\eta\right.\right)\\&=\Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right)+e^{-B}\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right).\text{ (according to (A2)).}\end{aligned}$$

Thus,

$$\begin{aligned}E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T\}}\right]&=E^Q\left[(\gamma-\eta)L(0)I_{\{\tau<T\}}\right]\\&=(\gamma-\eta)L(0)\left(\Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right)+e^{-B}\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right)\right).\end{aligned}$$

Lemma 2

The premium of the capital forbearance component P^c is given by

$$\begin{aligned}&E^Q\left[\frac{P(T)}{M(T)}I_{\left\{T\le\tau,\frac{A(T)}{L(T)}<\beta\right\}}\right]\\&=\gamma L(0)\left[(\Phi(c_1)-\Phi(c_2))-e^{-B}(\Phi(c_3)-\Phi(c_4))\right]-A(0)\left[(\Phi(c_5)-\Phi(c_6))-e^{B}(\Phi(c_7)-\Phi(c_8))\right],\end{aligned}$$

where $c_1=\frac{B_2-aT}{\sigma_A\sqrt{T}}$, $c_2=\frac{B-aT}{\sigma_A\sqrt{T}}$, $c_3=\frac{B_2-2B-aT}{\sigma_A\sqrt{T}}$, $c_4=\frac{-B-aT}{\sigma_A\sqrt{T}}$, $c_5=\frac{B_2+aT}{\sigma_A\sqrt{T}}$, $c_6=\frac{B+aT}{\sigma_A\sqrt{T}}$, $c_7=\frac{B_2-2B+aT}{\sigma_A\sqrt{T}}$ and $c_8=\frac{-B+aT}{\sigma_A\sqrt{T}}$.

Proof. The joint probability of the default event at the auditing time and the nondefault event before means that the joint probability of the asset-debt ratio $\frac{A(T)}{L(T)}$ is lower than β at maturity T and the minimum of the ratio $\min_{0\le s\le T}\frac{A(s)}{L(s)}$ is higher than η before maturity T:

$$\begin{aligned}&E^Q\left[I_{\left\{T\le\tau,\frac{A(T)}{L(T)}<\beta\right\}}\right]\\&=P_r^Q\left[\min_{0\le s\le T}\frac{A(s)}{L(s)}\ge\eta,\frac{A(T)}{L(T)}<\beta\left|\frac{A(0)}{L(0)}>\eta\right.\right]\\&=P_r^Q\left[\min_{0\le s\le T}\left(as+\sigma_A W_A^Q(s)\right)\ge\ln\left(\frac{\eta L(0)}{A(0)}\right),as+\sigma_A W_A^Q(T)<\ln(\frac{\beta L(0)}{A(0)})\left|\frac{A(0)}{L(0)}>\eta\right.\right]\\&=\left[\Phi\left(\frac{-B+aT}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{-B_1+aT}{\sigma_A\sqrt{T}}\right)\right]-e^{-B}\left[\Phi\left(\frac{B+aT}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{-B_1+2B+aT}{\sigma_A\sqrt{T}}\right)\right]\end{aligned}$$

(according to (A1))

Thus, we have

$$
\begin{aligned}
&E^Q\left[\frac{P(T)}{M(T)}I_{\left\{T\le\tau,\,\frac{A(T)}{L(T)}<\beta\right\}}\right]\\
&=E^Q\left[\frac{\gamma L(T)}{M(T)}I_{\left\{T\le\tau,\,\frac{A(T)}{L(T)}<\beta\right\}}\right]-E^Q\left[\frac{A(T)}{M(T)}I_{\left\{T\le\tau,\,\frac{A(T)}{L(T)}<\beta\right\}}\right]\\
&=\gamma L(0)E^Q\left[I_{\left\{T\le\tau,\,\frac{A(T)}{L(T)}<\beta\right\}}\right]-A(0)E^Q\left[e^{X(T)}I_{\left\{T\le\tau,\,\frac{A(T)}{L(T)}<\beta\right\}}\right]\\
&=\gamma L(0)\left[\Phi\left(\frac{B_2-aT}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{B-aT}{\sigma_A\sqrt{T}}\right)\right]-e^{-B}\left[\Phi\left(\frac{B_2-2B-aT}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{B-2B-aT}{\sigma_A\sqrt{T}}\right)\right]\\
&-A(0)\left(\begin{array}{l}\left[\Phi\left(\frac{B_2-(a+\sigma_A^2)T}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{B-(a+\sigma_A^2)T}{\sigma_A\sqrt{T}}\right)\right]\\-e^{2B(1+\frac{a}{\sigma_A^2})}\left[\Phi\left(\frac{B_2-2B-(a+\sigma_A^2)T}{\sigma_A\sqrt{T}}\right)-\Phi\left(\frac{-B-(a+\sigma_A^2)T}{\sigma_A\sqrt{T}}\right)\right]\end{array}\right)\\
&=\gamma L(0)\left[(\Phi(c_1)-\Phi(c_2))-e^{-B}(\Phi(c_3)-\Phi(c_4))\right]-A(0)\left[(\Phi(c_5)-\Phi(c_6))-e^{B}(\Phi(c_7)-\Phi(c_8))\right],
\end{aligned}
$$

where $c_1=\frac{B_2-aT}{\sigma_A\sqrt{T}},\ c_2=\frac{B-aT}{\sigma_A\sqrt{T}},\ c_3=\frac{B_2-2B-aT}{\sigma_A\sqrt{T}},\ c_4=\frac{-B-aT}{\sigma_A\sqrt{T}},\ c_5=\frac{B_2+aT}{\sigma_A\sqrt{T}},\ c_6=\frac{B+aT}{\sigma_A\sqrt{T}},\ c_7=\frac{B_2-2B+aT}{\sigma_A\sqrt{T}}$, and $c_8=\frac{-B+aT}{\sigma_A\sqrt{T}}$.

Lemma 3

The premium of the capital forbearance component P^ε is given by

$$
\begin{aligned}
&E^Q\left[\frac{P(T)}{M(T)}I_{\left\{T\le\tau,\beta<\frac{A(T)}{L(T)}<\alpha\right\}}\right]\\
&=\gamma L(0)\left[(N(d_1,e_1,\delta)-N(d_5,e_1,\delta))-e^{-B}(N(d_3,e_2,\delta)-N(d_6,e_2,\delta))\right]\\
&-A(0)\left[[N(d_7,e_3,\delta)-N(d_{11},e_3,\delta)]-e^{B}[N(d_9,e_4,\delta)-N(d_{12},e_4,\delta)]\right],
\end{aligned}
$$

where $N(d,e,\delta)$ denotes the standard bivariate normal cumulative density function $\int_{-\infty}^{d}\Phi(\frac{e-\delta z}{\sqrt{1-\delta^2}})f(z)dZ(T)$. Z$(T)$ denotes a normal variable with mean 0 and variance T. Here$\delta=\sqrt{\frac{T}{T+\varepsilon}}$; $d_1=\frac{B_1-aT}{\sigma_A\sqrt{T}},\ d_2=\frac{B-aT}{\sigma_A\sqrt{T}},\ d_3=\frac{B_1-2B-aT}{\sigma_A\sqrt{T}},\ d_4=\frac{-B-aT}{\sigma_A\sqrt{T}},\ d_5=\frac{B_2-aT}{\sigma_A\sqrt{T}},\ d_6=\frac{B_2-2B-aT}{\sigma_A\sqrt{T}},\ d_7=\frac{B_1+aT}{\sigma_A\sqrt{T}},\ d_8=\frac{B+aT}{\sigma_A\sqrt{T}},\ d_9=\frac{B_1-2B+aT}{\sigma_A\sqrt{T}},d_{10}=\frac{-B+aT}{\sigma_A\sqrt{T}},\ d_{11}=\frac{B_2+aT}{\sigma_A\sqrt{T}},\ d_{12}=\frac{B_2-2B+aT}{\sigma_A\sqrt{T}}$; and $e_1=\frac{B_3-a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}},e_2=\frac{B_3-2B-a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}},\ e_3=\frac{B_3+a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$, and $e_4=\frac{B_3-2B+a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$.

Proof. To obtain a closed form of the premium formula, we first express the form of the expected value as a form of double integrals, and then simplify these integrals. The detail derivations are given as follows:

$$
\begin{aligned}
&E^Q\left[\frac{P(T)}{M(T)}I_{\left\{\tau>T,\beta<\frac{A(T)}{L(T)}<\alpha\right\}}\right]\\
&=E^Q\left[\frac{1}{M(T)}E^Q\left[\frac{M(T)}{M(T+\varepsilon)}\max\{\gamma L(T+\varepsilon)-A(T+\varepsilon),0\}\right]\middle|\beta<\frac{A(T)}{L(T)}<\alpha,\ \min_{0\le s\le T}\frac{A(s)}{L(s)}>\eta\right]\\
&=E^Q\left[\frac{1}{M(T)}E^Q\left[\gamma L(T)\Phi(b_2)-A(T)\Phi(b_1)\right]\middle|\beta<\frac{A(T)}{L(T)}<\alpha,\ \min_{0\le s\le T}\frac{A(s)}{L(s)}>\eta\right]\\
&=E^Q\left[\gamma L(0)-A(0)e^{X(T)}\middle|\beta<\frac{A(T)}{L(T)}<\alpha,\ \min_{0\le s\le T}\frac{A(s)}{L(s)}>\eta\right]\\
&=\gamma L(0)\left(P_r^Q\left[X(T)<B_1,\ \min_{0\le s\le T}X(s)>B\right]-P_r^Q\left[X(T)<B_2,\ \min_{0\le s\le T}X(s)>B\right]\right)
\end{aligned}
$$

$$-A(0)\left(E^{Q}\left[e^{X(T)}\middle|X(T)<B_1,\ \min_{0\le s\le T}X(s)>B\right]-E^{Q}\left[e^{X(T)}\middle|X(T)<B_2,\ \min_{0\le s\le T}X(s)>B\right]\right)$$
$$=\gamma L(0)\left(\int_B^{B_1}\int_B^{x}\Phi(b_1)f(x,m)dm(T)dX(T)-\int_B^{B_2}\int_B^{x}\Phi(b_1)f(x,m)dm(T)dX(T)\right)$$
$$-A(0)\left(\int_B^{B_1}\int_B^{x}e^{x}\Phi(b_2)f(x,m)dm(T)dX(T)-\int_B^{B_2}\int_B^{x}e^{x}\Phi(b_2)f(x,m)dm(T)dX(T)\right),$$
$$=\gamma L(0)\,(A3-A4)-A(0)\,(A5-A6),$$

where ε denotes the length of grace period, and $b_1=\frac{\ln(\gamma L(T)/A(T))-a\varepsilon}{\sigma_A\sqrt{\varepsilon}}=\frac{B_3-X(T)-a\varepsilon}{\sigma_A\sqrt{\varepsilon}}$ and $b_2=\frac{\ln(\gamma L(T)/A(T))-(a+\sigma_A^2)\varepsilon}{\sigma_A\sqrt{\varepsilon}}=\frac{B_3-X(T)+a\varepsilon}{\sigma_A\sqrt{\varepsilon}}$. The four terms A3, A4, A5, and A6 denote $\int_B^{B_1}\int_B^{x}\Phi(b_1)f(x,m)dm(T)dX(T)$, $\int_B^{B_2}\int_B^{x}\Phi(b_1)g(x,m)dm(T)dX(T)$, $\int_B^{B_1}\int_B^{x}e^{x}\Phi(b_2)f(x,m)dm(T)dX(T)$, and $\int_B^{B_2}\int_B^{x}e^{x}\Phi(b_2)f(x,m)dm(T)dX(T)$, and respectively.

Second, we simplify the four double integrals of A3, A4, A5, and A6. The term A3 can be simplified by the following derivation:

$$\int_B^{B_1}\int_B^{x}\Phi(b_1)f(x,m)dm(T)dX(T)$$
$$=\int_B^{B_1}\int_B^{x}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\left(\frac{2(x-2m)}{bT\sqrt{2\pi bT}}e^{-\frac{(x-2m)^2}{2bT}+\left(\frac{ax}{b}-\frac{a^2T}{2b}\right)}\right)dm(T)dX(T)$$
$$=\int_B^{B_1}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\left(\frac{1}{\sqrt{2\pi bT}}e^{\left(\frac{ax}{b}-\frac{a^2T}{2b}\right)}\right)\int_B^{x}\left(\frac{2(x-2m)}{bT}e^{-\frac{(x-2m)^2}{2bT}}\right)dm(T)dX(T)$$
$$=\int_B^{B_1}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\left(\frac{1}{\sqrt{2\pi bT}}e^{\left(\frac{ax}{b}-\frac{a^2T}{2b}\right)}\right)e^{-\frac{(x-2m)^2}{2bT}}dX(T)-\int_B^{B_1}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\left(\frac{1}{\sqrt{2\pi bT}}e^{\left(\frac{ax}{b}-\frac{a^2T}{2b}\right)}\right)e^{-\frac{(x-2B)^2}{2bT}}dX(T)$$
$$=\int_B^{B_1}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi bT}}e^{-\frac{(x-aT)^2}{2bT}}dX(T)-e^{\frac{2aB}{b}}\int_B^{B_1}\Phi\left(\frac{B_3-x-a\varepsilon}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi bT}}e^{-\frac{(x-2B-aT)^2}{2bT}}dX(T)$$
$$=\int_{d_2}^{d_1}\Phi\left(\frac{B_3-a(T+\varepsilon)-\sqrt{bT}z}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dZ(T)-e^{-B}\int_{d_4}^{d_3}\Phi\left(\frac{B_3-2B-a(T+\varepsilon)-\sqrt{bT}z}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dZ(T)$$
$$=\int_{d_2}^{d_1}\Phi\left(\frac{e_1-\delta z}{\sqrt{1-\delta^2}}\right)f(z)dZ(T)-e^{-B}\int_{d_4}^{d_3}\Phi\left(\frac{e_2-\delta z}{\sqrt{1-\delta^2}}\right)f(z)dZ(T)$$
$$=[N(d_1,e_1,\delta)-N(d_2,e_1,\delta)]-e^{-B}[N(d_3,e_2,\delta)-N(d_4,e_2,\delta)],$$

where Z(T) denotes a normal variable with mean 0 and variance T. $f\ (T)$ is the probability density of Z(T). $N(d,e,\delta)$ denotes the bivariate normal probability density function $\int_{-\infty}^{d}\Phi(\frac{e-\delta z}{\sqrt{1-\delta^2}})f(z)dZ(T)$.

The other notations are $\delta=\sqrt{\frac{T}{T+\varepsilon}}$, $d_1=\frac{B_1-aT}{\sigma_A\sqrt{T}}$, $d_2=\frac{B-aT}{\sigma_A\sqrt{T}}$, $d_3=\frac{B_1-2B-aT}{\sigma_A\sqrt{T}}$, $d_4=\frac{-B-aT}{\sigma_A\sqrt{T}}$, $e_1=\frac{B_3-a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$, and $e_2=\frac{B_3-2B-a(T+\varepsilon)}{\sigma_A\sqrt{T+\varepsilon}}$.

The term A4 can be simplified in a similar way as term A3, with the following results:

$$\int_B^{B_2}\int_B^{x}\Phi\,(b_1)\,g\,(x,m)\,dm\,(T)\,dX\,(T)$$
$$=[N\,(d_5,e_1,\delta)-N\,(d_2,e_1,\delta)]-e^{-B}\,[N\,(d_6,e_2,\delta)-N\,(d_4,e_2,\delta)],$$

where $d_5=\frac{B_2-aT}{\sigma_A\sqrt{T}}$ and $d_6=\frac{B_2-2B-aT}{\sigma_A\sqrt{T}}$.

The term A5 can be simplified by the following derivation:

$$\int_B^{B_1}\int_B^x e^x\Phi(b_2)f(x,m)dm(T)dX(T)$$
$$=\int_B^{B_1}\int_B^x e^x\Phi\left(\frac{\ln(D/A(0))-x-(a+b)\varepsilon}{\sqrt{b\varepsilon}}\right)\frac{2(x-2m)}{bT\sqrt{2\pi bT}}e^{-\frac{(x-2m)^2}{2bT}+\left(\frac{ax}{b}-\frac{a^2T}{2b}\right)}dm(T)dX(T)$$
$$=\int_B^{B_1}\frac{e^x}{\sqrt{2\pi bT}}e^{\frac{ax}{b}-\frac{a^2T}{2b}-\frac{(x-2x)^2}{2bT}}\Phi\left(\frac{B_3-x-(a+b)\varepsilon}{\sqrt{b\varepsilon}}\right)dX(T)$$
$$-\int_B^{B_1}\frac{e^x}{\sqrt{2\pi bT}}e^{\frac{ax}{b}-\frac{a^2T}{2b}-\frac{(x-2B)^2}{2bT}}\Phi\left(\frac{B_3-x-(a+b)\varepsilon}{\sqrt{b\varepsilon}}\right)dX(T)$$
$$=\int_B^{B_1}\frac{1}{\sqrt{2\pi bT}}e^{-\frac{(x-(a+b)T)^2}{2bT}}\Phi\left(\frac{B_3-x-(a+b)\varepsilon}{\sqrt{b\varepsilon}}\right)dX(T)$$
$$-e^{2B(1+a/b)}\int_B^{B_1}\frac{1}{\sqrt{2\pi bT}}e^{-\frac{(x-2B-(a+b)T)^2}{2bT}}\Phi\left(\frac{B_3-x-(a+b)\varepsilon}{\sqrt{b\varepsilon}}\right)dX(T)$$
$$=\int_{\frac{B-(aT+bT)}{\sqrt{bT}}}^{\frac{B_1-(aT+bT)}{\sqrt{bT}}}\Phi\left(\frac{B_3-(a+b)(T+\varepsilon)-\sqrt{bT}z}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dZ(T)$$
$$-e^{2B(1+a/b)}\int_{\frac{B-(2B+aT+bT)}{\sqrt{bT}}}^{\frac{B_1-(2B+aT+bT)}{\sqrt{bT}}}\Phi\left(\frac{B_3-2B-(a+b)(T+\varepsilon)-\sqrt{bT}z}{\sqrt{b\varepsilon}}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dZ(T)$$
$$=\int_{d_8}^{d_7}\Phi\left(\frac{e_3-\delta z}{\sqrt{1-\delta^2}}\right)f(z)dZ(T)-e^{2B(1+a/b)}\int_{d_{10}}^{d_9}\Phi\left(\frac{e_4-\delta z}{\sqrt{1-\delta^2}}\right)f(z)dZ(T)$$
$$=[N(d_7,e_3,\delta)-N(d_8,e_3,\delta)]-e^B[N(d_9,e_4,\delta)-N(d_{10},e_4,\delta)],$$

where $d_7=\frac{B_1+aT}{\sqrt{bT}},\ d_8=\frac{B+aT}{\sqrt{bT}},\ d_9=\frac{B_1-2B+aT}{\sqrt{bT}},\ d_{10}=\frac{-B+aT}{\sqrt{bT}},\ e_3=\frac{B_3+a(T+\varepsilon)}{\sqrt{b(T+\varepsilon)}}$ and $e_4=\frac{B_3-2B+a(T+\varepsilon)}{\sqrt{b(T+\varepsilon)}}$.

The term (A6) can be simplified in a similar way as term (A5), with the following results:

$$\int_B^{B_2}\int_B^x e^x\Phi(b_2)f(x,m)dm(T)dX(T)$$
$$=[N(d_{11},e_3,\delta)-N(d_8,e_3,\delta)]-e^B[N(d_{12},e_4,\delta)-N(d_{10},e_4,\delta)],$$

where

$$d_{11}=\frac{B_2+aT}{\sqrt{bT}}\text{ and }d_{12}=\frac{B_2-2B+aT}{\sqrt{bT}}.$$

We sum up the three results from Lemmas 1, 2, and 3, and then obtain the closed-form pricing formula of the premium for the insurance guaranty fund.

APPENDIX B. THE CALCULATION OF THE GREEKS

In Section 3.2 we apply numerical analysis to discuss how the value of the premium under the forbearance mechanism varies in response to changes in the main parameters $(\alpha,\beta,\varepsilon,\gamma,\ \omega_1,\omega_2,\sigma_r)$. In this appendix, we present the detailed derivation of the first partial derivatives of the premium formula with respect to these parameters. As the first partial derivatives with respect to ε, w_1, w_2, and σ_r are complicated, thus, we discuss only the trends of their partial derivatives by the numerical analysis in Section 3.2.

(1) The first partial derivatives of $P(0)$ with respect to α is derived as follows:

$$\frac{\partial P(0)}{\partial \alpha} = \gamma L(0)\left[\frac{\partial N(d_1, e_1, \delta)}{\partial \alpha} - e^{-B}\frac{\partial N(d_3, e_2, \delta)}{\partial \alpha}\right] - A(0)\left[\frac{\partial N(d_7, e_3, \delta)}{\partial \alpha} - e^{B}\frac{\partial N(d_9, e_4, \delta)}{\partial \alpha}\right]$$

$$= \frac{\gamma L(0)}{\alpha\sigma_A\sqrt{T}}\left[\Phi\left(\frac{e_1 - \delta d_1}{\sqrt{1-\delta^2}}\right)\phi(d_1) - e^{-B}\Phi\left(\frac{e_2 - \delta d_3}{\sqrt{1-\delta^2}}\right)\phi(d_3)\right]$$

$$-\frac{A(0)}{\alpha\sigma_A\sqrt{T}}\left[\Phi\left(\frac{e_3 - \delta d_7}{\sqrt{1-\delta^2}}\right)\phi(d_7) - e^{B}\Phi\left(\frac{e_4 - \delta d_9}{\sqrt{1-\delta^2}}\right)\phi(d_9)\right]$$

(2) The first partial derivatives of $P(0)$ with respect to β is derived as follows:

$$\frac{\partial P(0)}{\partial \beta} = \frac{\partial \gamma L(0)\left[\Phi(c_1) - e^{-B}\Phi(c_3)\right] - A(0)\left[\Phi(c_5) - e^{B}\Phi(c_7)\right]}{\partial \beta}$$

$$+\frac{\gamma L(0)\left[(-N(d_5, e_1, \delta)) - e^{-B}(-N(d_6, e_2, \delta))\right] - A(0)\left[[-N(d_{11}, e_3, \delta)] - e^{B}[-N(d_{12}, e_4, \delta)]\right]}{\partial \beta}$$

$$= \frac{\gamma L(0)}{\beta\sigma_A\sqrt{T}}\left[\phi(c_1) - e^{-B}\phi(c_3) - \left[\Phi\left(\frac{e_1 - \delta d_5}{\sqrt{1-\delta^2}}\right)\phi(d_5) - e^{-B}\Phi\left(\frac{e_2 - \delta d_6}{\sqrt{1-\delta^2}}\right)\phi(d_6)\right]\right]$$

$$-\frac{A(0)}{\beta\sigma_A\sqrt{T}}\left[\phi(c_5) - e^{B}\phi(c_7) - \left[\Phi\left(\frac{e_3 - \delta d_{11}}{\sqrt{1-\delta^2}}\right)\phi(d_{11}) - e^{B}\Phi\left(\frac{e_4 - \delta d_{12}}{\sqrt{1-\delta^2}}\right)\phi(d_{12})\right]\right].$$

(3) The first partial derivatives of $P(0)$ with respect to γ is derived as follows:

$$\frac{\partial P(0)}{\partial \gamma} = \frac{\partial \gamma L(0)\left[(N(d_1, e_1, \delta) - N(d_5, e_1, \delta)) - e^{-B}(N(d_3, e_2, \delta) - N(d_6, e_2, \delta))\right]}{\partial \gamma}$$

$$-\frac{\partial A(0)\left[[N(d_7, e_3, \delta) - N(d_{11}, e_3, \delta)] - e^{B}[N(d_9, e_4, \delta) - N(d_{12}, e_4, \delta)]\right]}{\partial \gamma}$$

$$= L(0)\left[(N(d_1, e_1, \delta) - N(d_5, e_1, \delta)) - e^{-B}(N(d_3, e_2, \delta) - N(d_6, e_2, \delta))\right]$$

$$+\frac{L(0)}{\sigma_A\sqrt{T+\varepsilon}}\left[\begin{array}{l}\left(\int_{-\infty}^{d_1}\phi\left(\frac{e_1 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T) - \int_{-\infty}^{d_5}\phi\left(\frac{e_1 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T)\right) \\ -e^{-B}\left(\int_{-\infty}^{d_3}\phi\left(\frac{e_2 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T) - \int_{-\infty}^{d_6}\phi\left(\frac{e_2 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T)\right)\end{array}\right]$$

$$-\frac{A(0)}{\sigma_A\sqrt{T+\varepsilon}}\left[\begin{array}{l}\left(\int_{-\infty}^{d_7}\phi\left(\frac{e_3 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T) - \int_{-\infty}^{d_{11}}\phi\left(\frac{e_3 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T)\right) \\ -e^{B}\left(\int_{-\infty}^{d_9}\phi\left(\frac{e_4 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T) - \int_{-\infty}^{d_{12}}\phi\left(\frac{e_4 - \delta z}{\sqrt{1-\delta^2}}\right)\phi(z)dZ(T)\right)\end{array}\right].$$