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The optimal pricing of a market maker in a heterogeneous agent economy

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A B S T R A C T

This paper extends some classical models, built upon the representative-agent and Walrasian market-clearing mechanism, into one characterized by a market-maker trading mechanism with investors having heterogeneous beliefs regarding the likely future payoff of a risky security. We show the optimal determination of the bid and ask prices and resultant trading volume. The endogenously-determined spread and volume are increasing with the degree of the heterogeneity of investors' beliefs. We analyze the market marker's risk exposure based on his inventory, under the condition in which he is fully informed of the investors' beliefs, and under the condition in which he is not.

1. Introduction

The representative agent paradigm which fails to take into consideration investors' heterogeneous beliefs and hence the market's buy and sell orders simultaneously is not suitable for the study of the *market-maker trading mechanism* in which the bid and ask prices are simultaneously set by the market marker. One essential element of this market mechanism is the interaction between the market maker and a group of heterogeneous investors. Recently, heterogeneous-agent modeling has played an

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increasingly important role in the literature on asset pricing, including the market-maker trading mechanism.[1] Heterogeneous-agent modeling (HAM) should be suitable for studying the nature of the market-maker trading mechanism in which the bid and ask prices need to be determined by effectively mapping a set of investors, with heterogeneous beliefs, into buyers and sellers. The existence of the bid and ask prices and the difference between the two, known as the spared, is, in effect, a common practice to make this mapping feasible. Nonetheless, most existing applications of HAM to the market-maker trading mechanism only consider one price, instead of considering both the bid and ask prices.

Outside HAM, there is a large body of literature addressing the bid and ask pricing, although with some stringent assumptions. For example, Garman (1976) and Amihud and Mendelson (1980) assume that the arrival of buy and sell orders in the market follow exogenously-given independent Poisson processes; Copeland and Galai (1983) introduce the exogenous probability density functions of market price and investors' orders; some seminal papers, such as Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987), and Easley and O'Hara (1992), regard the investors' demand as an exogenous independent variable; Shen and Starr (2002) even specify that the evolution of price is exogenous. These assumptions help keep the tractability of the model, but in reality the size and the frequencies of the buy and the sell orders and the bid and ask prices are not mutually independent. Neither is any of them exogenous to the other; they mutually affect each other and are determined endogenously together. HAM directly works on the heterogeneous structure of the investors and directly bridges the investors' trading behavior and the market maker's bid and ask price formation with little reliance on the above assumptions.

With the above research background, this paper attempts to study the optimal pricing behavior of the market maker and hence the determination of the bid and ask prices in a heterogeneous agent economy. Before getting into the technical details of this heterogeneous-agent model of the market-maker trading mechanism, we provide a sketch of the model with its theoretical underpinnings and insights.

We propose an extended version of the Grossman–Stiglitz model (Grossman and Stiglitz, 1980), in which there are two types of investors and one market maker trading simultaneously. The equilibrium means that *supply equals demand*. This setting is different from that of the classical *sequential trade model* used by Glosten and Milgrom (1985), etc., in which investors arrive one by one randomly. The two types of investors differ in their beliefs regarding the future payoff of a risky asset, which makes them have different positions in the market. As we shall see later, the one holding the long position is the buyer, and the one holding the short position is the seller. Given these two types of investors, the market maker determines a bid price and an ask price, and quotes the former to buy from the seller and quotes the latter to sell to the buyer.

Comerton-Forde et al. (2010) use an 11-year panel of daily NYSE market-maker inventory and revenues to show that after the market maker holds large positions, the effective spreads widen, which implies that the market maker widens the spread to reduce his inventory and to limit his risk exposure. This finding challenges the market maker's *zero-profit* assumption introduced by Glosten and Milgrom (1985), who point out that if the inventory carrying capacity of the market maker is large enough, the market maker earns zero profit. But it needs to be asked why the market maker is willing to hold excessive inventory. As an enterprise, the market maker makes profits from trading with investors rather than investing in the risky security like an investor. His goal is to minimize risk and to maximize profit. In this vein and for simplicity, we further assume that the market maker's targeted inventory equals 0, i.e., a zero-inventory target, which implies zero risk exposure. With this assumption, our research question can be formally presented as a constrained optimization problem: the market maker is assumed to maximize his profit by bearing zero risk. We call the solution to this problem the optimal bid and ask prices.

We obtain the solution of the optimal bid and ask prices. This article finds that, if the market maker has perfect information regarding all investors' beliefs, the zero-inventory target can be achieved, and the derived optimal bid and ask prices, volume, and market maker's profit are all positively related to the degree of the heterogeneity in the beliefs of investors about the likely future value of the security.

[1] See, e.g., Chiarella et al. (2006), Zhu et al. (2009), He and Zheng (2010), and Chiarella et al. (2011).

However, if the market maker does not have full information regarding investors' beliefs and has to estimate the payoff of the asset on his own, then, as we shall show in this paper, the zero-inventory target becomes difficult to achieve, unless the bid-ask spread is sufficiently wide.

The significance of our findings can shed light on some earlier studies. First, our results are comparable to those of Shen and Starr (2002), who show that, with balanced orders and zero-profit assumptions, the bid-ask spread is just wide enough to cover the market maker's expected cost. In our model, with the profit-maximization assumption, we show that, even though there is no expected cost, the bid-ask spread still exists with a balanced order. Its existence reflects the heterogeneity of investors' beliefs, an essential characteristic of a heterogeneous economy. In other words, heterogeneity, and not only adverse selection as emphasized by Glosten and Milgrom (1985), accounts for the bid-ask spread, which means that all investors can be uninformed and the prices may not contain any available information. However, the pursuit of the balanced-order or the zero-inventory target can fail, which reflects the market maker's misjudgment regarding the degree of the belief heterogeneity.

Second, the individual demand is decreasing in the bid-ask spread representing the inverse of liquidity. From another perspective, expanding the bid-ask spread is an efficient way of easing excessively imbalanced orders without the market maker incurring additional risk. History has shown us the ability of market markers to stabilize the market. During turbulent times (such as October 1987, August through October 1998, and April 2000), the bid-ask spread widened significantly, which can be regarded as an action taken by the market maker that enhances market stability.

The rest of the paper is organized as follows. Section 2 introduces the heterogeneous-agent model. Based on the proposed model, Section 3 presents the basic theoretical results of the model under the assumption that the heterogeneous structure of the investors' beliefs is fully known to the market maker. Specifically, we derive the asset demand schedule and the determination of the optimal bid and ask prices, i.e., the optimal spread, as well as the resultant market trading volume. Section 4 then relaxes the full-information assumption and analyzes what happens when the information regarding the heterogeneous beliefs is not fully known to the market maker. Specifically, we address the effect of imperfect information on the inventory. Section 5 then gives the concluding remarks.

2. The model

In this paper, we propose a simple heterogeneous-agent model, for which the investment horizon has only two periods, namely, period 0 and period 1.

2.1. The market structure

There are two assets, a risk-less bond in perfectly elastic supply with a constant interest rate r and an risky security that has a net supply of zero. All assets are traded in period 0. The risky security pays V in period 1.

$$V = \mu + \varepsilon, \tag{1}$$

where μ is a constant, and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, which implies that the risky asset is unlimited-liability. There is no short-sale constraint, i.e., the position of the market participant can be negative.

2.2. The market participants

There are two heterogeneous investors and a market maker. Investors must deal with the market maker. Each investor is not endowed with any risky security or risk-less bond, but is endowed with some initial cash.

2.3. Preferences and beliefs

Both investors have constant absolute risk-aversion utility over their wealth in period 1, and maximize their expected utility

$$E_i[-e^{-\gamma W_{i,1}}], \quad i = 1, 2, \tag{2}$$

where $W_{i,1}$ is the wealth of investor i in period 1; γ is a risk-aversion parameter which is identical for all investors; and $E_i[\cdot]$ denotes the expectation with respect to investor i's information set.

Assume for the moment that V is normally distributed conditional on investor i's information set. Here, investor i reckons the payoff of the risky security

$$V = \mu_i + \varepsilon_i, \tag{3}$$

where $\varepsilon_i \sim N(0, \sigma_i^2)$. Note that $E_i[V] = \mu_i$ and $\sigma_i^2(V) = \sigma_i^2$, which implies that investor i realizes that the shock ε_i will exist in period 1, but does not know its exact value in advance. The parameters μ_i and σ_i^2 are exogenously determined by investor i.[2] Without loss of generality, we assume that $\mu_1 > \mu_2$.

2.4. The bid and ask prices

The bid price and the ask price are the buying price and the selling price set by the market maker. As expected, the bid price must be lower than the ask price.

If investor i buys the risky security in period 0, then

$$E_i[W_{i,1}] = x_i(\mu_i - P_a(1+r)) + C_{i,0}(1+r), \tag{4}$$

where x_i is the risky security position of investor i; $C_{i,0}$ is the cash endowment of investor i in period 0; and P_a denotes the ask price. On the other hand, if investor i sells the risky security in period 0, then

$$E_i[W_{i,1}] = x_i(\mu_i - P_b(1+r)) + C_{i,0}(1+r), \tag{5}$$

where P_b denotes the bid price.

3. Optimal pricing under the known heterogeneity

In order to obtain the optimal pricing, we must first find each investor's position in relation to the risky security. Proposition 1 gives the result.

Proposition 1. *For the economy defined in Section 2, investor i's demand for the risky security is*

$$x_i = \begin{cases} \frac{\mu_i - P_a(1+r)}{\gamma\sigma_i^2} > 0, & \text{if } \frac{\mu_i}{1+r} > P_a \\ 0, & \text{if } P_b \leqslant \frac{\mu_i}{1+r} \leqslant P_a \\ \frac{\mu_i - P_b(1+r)}{\gamma\sigma_i^2} < 0, & \text{if } \frac{\mu_i}{1+r} < P_b \end{cases} \tag{6}$$

where i = 1, 2.

Fig. 1 shows investor i's demand for the risky security as a function of the present discounted value of his expected payoff. The dotted line represents the demand at a price P with no spread. As soon as the market maker decreases the bid price and increases the ask price from the spread-free price P, the demand function changes to the thick solid line. Hence, the bid-ask spread can reduce the absolute value of the investor's demand. The wider that the bid-ask spread is, the lower that the market trading volume is. This, therefore, reveals the existence of a negative relationship between the spread and the liquidity level.

Given all investors' demand functions, the market maker needs to find out the respective optimal pricing, which needs to take into account both profits and risks simultaneously.[3] We assume that the market marker will not bear any risk, and he will not maintain any position on the risky asset, or, to put it differently, he will target zero inventory. With this assumption, the optimal pricing for the market maker

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[2] In the literature, the belief of an investor can be exogenously given or endogenously formed. For example, Mendel and Shleifer (2012) introduce a rational but uninformed agent who endogenously determines his belief μ_i and σ_i. In our paper, we only consider the exogenously given beliefs.

[3] Since there is no operating cost for the market maker in this economy, the revenue and profit are equivalent.

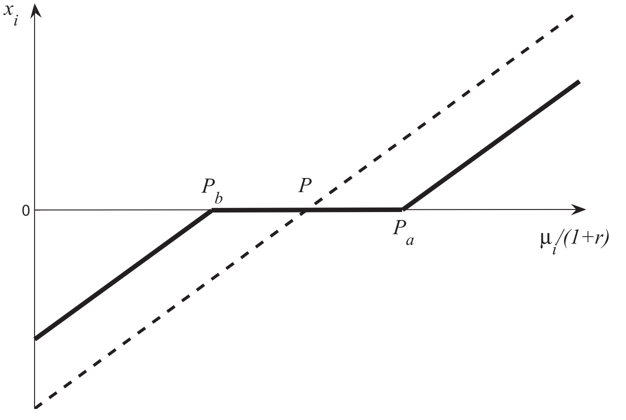

Fig. 1. The demand for the risky security. The horizontal axis represents the present discounted value of investor *i*'s expected payoff. The thick solid line denotes the demand for the risky security with a bid-ask spread, and the dotted line denotes the demand for the security when there is no spread, i.e., $P_a = P_b$.

can be characterized as the solution to the following constrained optimization problem and the optimal prices can be characterized as the solution to the problem

$$\max_{P_a,P_b} \pi = x_{long} \cdot P_a - x_{short} \cdot P_b, \tag{7a}$$

$$s.t. \quad x_{long} = x_{short} \geqslant 0, \tag{7b}$$

where π is the profit of the market maker from trading with investors; and x_{long} (x_{short}) is the long (short) position of the investors who buy (sell) the risky security. We first assume that the market maker knows the beliefs of all investors perfectly.[4] Then the solution to the above constrained optimization problem is shown in Proposition 2.

Proposition 2. *The optimal bid and ask prices are*

$$P_a = \frac{\mu_1 + \mu_\sigma}{2(1+r)}, \tag{8a}$$

$$P_b = \frac{\mu_2 + \mu_\sigma}{2(1+r)}; \tag{8b}$$

the trading volume is

$$Volume = x_{long} = x_{short} = \frac{\mu_1 - \mu_2}{2\gamma(\sigma_1^2 + \sigma_2^2)}; \tag{9}$$

and the market maker's profit π *is*

$$\pi = \frac{(\mu_1 - \mu_2)^2}{4\gamma(1+r)(\sigma_1^2 + \sigma_2^2)}, \tag{10}$$

where $\mu_\sigma = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$, *which is the variance-weighted expected payoff. Clearly,* $\mu_1 > \mu_\sigma > \mu_2$.

Fig. 2 depicts the investors' demand, volume, bid-ask spread and market maker's profit, all in relation to the heterogeneity of the investors' beliefs, $\mu_1 - \mu_2$. Proposition 2 basically says that the optimal spread determined by the market maker reflects the degree of the heterogeneity of the beliefs of investors about the likely future payoff of the security. This result is a consequence of the assumption of the fully-informed market maker, who happens to know how diverse the investors' beliefs are. In

[4] We relax this assumption in Section 4.

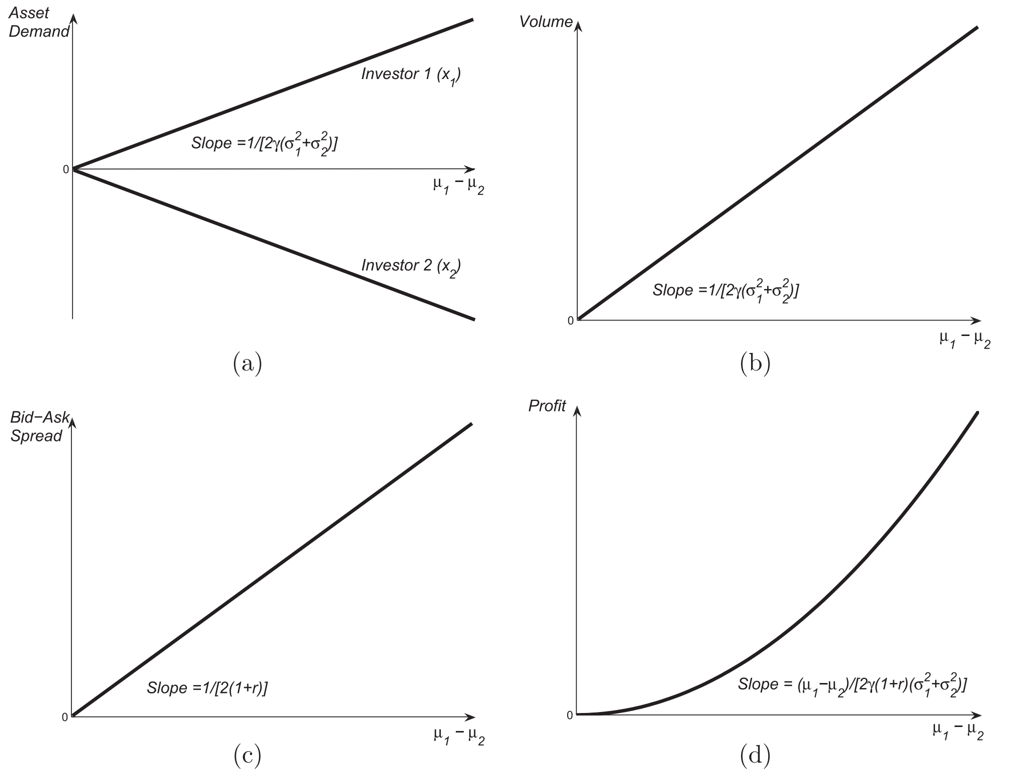

Fig. 2. The above four panels, from the left to the right and from the top to the bottom, show the investors' demand, market trading volume, and the market maker's bid-ask spread and profit as a function of the heterogeneity of the investors' beliefs, $\mu_1 - \mu_2$, under the assumption that the market maker knows the investors' beliefs perfectly and pursues a zero-inventory target.

addition, given the spread, the investors optimally determine their demand for the asset and the market trading volume as shown in (9). According to (9), the trading volume increases with the degree of the heterogeneity in the investors' belief. The relationship between the spread and the trading volume can then be directly shown in (11) simply by rearranging (8a), (8b), and (9).

$$V = \frac{1+r}{\gamma(\sigma_1^2 + \sigma_2^2)} \cdot S, \tag{11}$$

where V is the trading volume and S is the bid-ask spread, $P_a - P_b$. Eq. (11) shows that the trading volume increases with the spread, which may be a little counterintuitive. Nevertheless, one has to notice that Eq. (11) is only a static relationship between the optimal spread and volume, rather than a causal relationship between them. The optimal spread and market trading volume are simultaneously determined by the given structure.

Does the middle point of the bid and ask prices reflect the fundamentals? Not necessarily. After all, the market maker only cares about what the investors believe rather than what the market fundamentals are.

4. Extensions with unknown heterogeneities

If the market maker cannot fully trace the beliefs of all investors, then it is highly likely that his zero-inventory target may fail to be fulfilled. To show this, let us assume, for the purpose of simplicity, that the market maker knows nothing about the investors' beliefs except that $\mu_1 > \mu_2$. Proposition 3 shows how the inventory is determined under this assumption.

Proposition 3. *The market maker's inventory x_m is*

$$x_m = \begin{cases} \frac{\frac{1}{2}(\overline{\mu_1}+\overline{\mu_\sigma})-\mu_\sigma}{\gamma\Sigma^2} < 0, & \text{if } \mu_1 > \mu_2 > \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} > \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2}; \quad (12a) \\ \frac{\frac{1}{2}(\overline{\mu_1}+\overline{\mu_\sigma})-\mu_1}{\gamma\sigma_1^2} < 0, & \text{if } \mu_1 > \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} \geqslant \mu_2 \geqslant \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2}; \quad (12b) \\ \frac{\frac{1}{2}\overline{K}-\mu_\sigma}{\gamma\Sigma^2}, & \text{if } \mu_1 > \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} > \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2} > \mu_2; \quad (12c) \\ 0, & \text{if } \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} \geqslant \mu_1 > \mu_2 \geqslant \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2}; \quad (12d) \\ \frac{\frac{1}{2}(\overline{\mu_2}+\overline{\mu_\sigma})-\mu_2}{\gamma\sigma_2^2} > 0, & \text{if } \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} \geqslant \mu_1 \geqslant \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2} > \mu_2; \quad (12e) \\ \frac{\frac{1}{2}(\overline{\mu_2}+\overline{\mu_\sigma})-\mu_\sigma}{\gamma\Sigma^2} > 0, & \text{if } \frac{\overline{\mu_1}+\overline{\mu_\sigma}}{2} > \frac{\overline{\mu_2}+\overline{\mu_\sigma}}{2} > \mu_1 > \mu_2; \quad (12f) \end{cases}$$

where $\overline{\mu_1}, \overline{\mu_2}$ and $\overline{\mu_\sigma}$ are the estimates of the market maker for μ_1, μ_2 and μ_σ, respectively, and $\overline{\mu_1} > \overline{\mu_\sigma} > \overline{\mu_2}$; $\Sigma^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$, and $\overline{K} = \frac{(\overline{\mu_1}+\overline{\mu_\sigma})\sigma_2^2+(\overline{\mu_2}+\overline{\mu_\sigma})\sigma_1^2}{\sigma_1^2+\sigma_2^2}$.

To make it easier to harness Proposition 3, we present it in pictorial form in Fig. 3. To not make the figure too complicated, we only show the result of the inventory; however, the corresponding profit can be derived accordingly using (12). The basic message revealed by Fig. 3 is that the realized inventory and hence the risk exposure is a piecewise-linear binary function on μ_1 and μ_2. In Fig. 3, we use the coordinate (10, 5) as a reference in which the market maker's estimation coincides with the investors' beliefs, i.e., zero estimation biases. Starting from there, we can see that the inventory

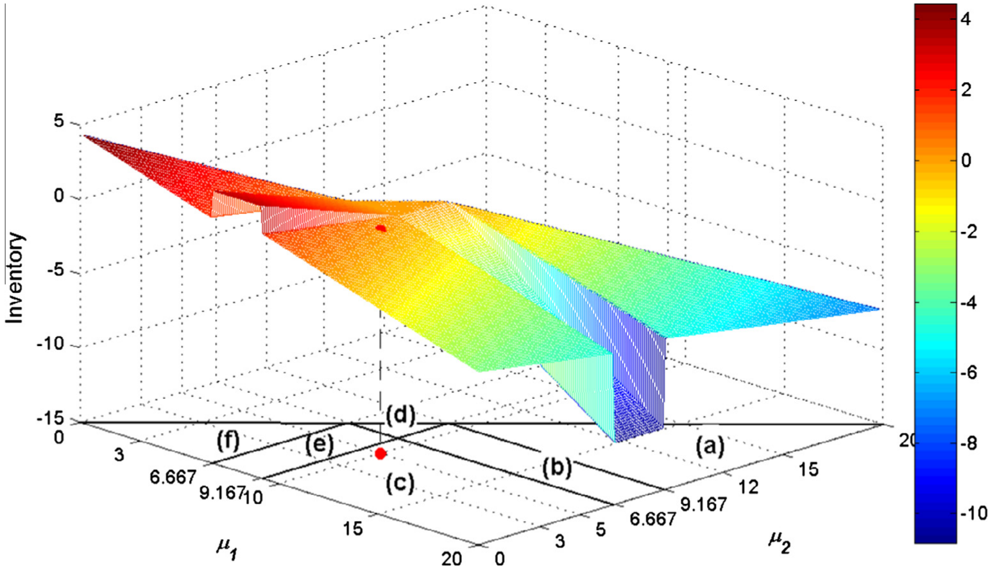

Fig. 3. The market maker's inventory. The diagram is drawn based on the following setting: $\mu_1 > \mu_2, \overline{\mu_1} = 10, \overline{\mu_2} = 5$, $\sigma_1^2 = 1, \sigma_2^2 = 2$, and $\overline{\mu_\sigma} = (\overline{\mu_1}\sigma_2^2 + \overline{\mu_2}\sigma_1^2)/(\sigma_1^2 + \sigma_2^2) = 8.333$. Basically, the entire demand surface is a piece-wise linear function, and based on Eq. (12) there are six linear segments (a–f), and two thresholds which generate the boundaries of these segments. The two thresholds are $(\overline{\mu_1} + \overline{\mu_\sigma})/2 = 9.167$ and $(\overline{\mu_2} + \overline{\mu_\sigma})/2 = 6.667$. We use the solid line to locate these two thresholds in the μ_1-μ_2 plane. The red point in the function projects to the red point in the μ_1-μ_2 plane corresponding to the coordinate (10, 5) where the market maker's estimation coincides with the investors' beliefs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

can deviate from zero in most cases when (μ_1, μ_2) leaves from (10, 5). However, in particular, there is a segment (12d), in which the inventory remains zero even with the presence of the bias, i.e., when the bias causes the spread to be wide enough to cover μ_1 and $\mu_2, P_a(1+r) = (\overline{\mu_1} + \overline{\mu_\sigma})/2 \geqslant \mu_1 > \mu_2 \geqslant (\overline{\mu_2} + \overline{\mu_\sigma})/2 = P_b(1+r)$, and results in a zero-trade outcome.

5. Concluding remarks

In this paper, a simple two-period heterogeneous-agent model is proposed to understand the optimal bid and ask prices and hence the optimal spread determined by the market maker. The main contribution of this paper is that it connects the heterogeneity of investors to pricing and the spread. This connection is built upon two key considerations: first, the heterogeneity of investors; second, information regarding the aforementioned heterogeneity. Given the specific kind of heterogeneity addressed in this paper, i.e., the belief in relation to the likely payoff in the future, we first show that when the market maker is well informed of the investors' heterogeneity he can actually find a profit-maximizing spread with zero risk exposure (a zero level of inventory), and that the spread is an increasing function of the degree of the heterogeneity.

Nevertheless, when the market maker is not well informed, the pricing and the spread based on the biased belief can lead to various possibilities so that the level of inventory deviating from a target of zero can be quite prevalent. The biased belief can even lead to an excessively large spread which essentially makes the market idle. In that case, although the zero-inventory target can still be maintained, with a zero-trade market it is no longer a sensible target to pursue.

Despite its simplicity, the results which we have obtained from this baseline model may encourage us to go further to examine the effect of the heterogeneity of investors on the spread in a more dynamic and realistic setting. First, in the case of imperfect information, it would be interesting to study the heterogeneity effect when the market maker can learn from his constant interactions with the investors, and to examine the spread as part of the market dynamics in an evolutionary manner. Second, in light of the recent progress in agent-based computational finance, the heterogeneity of the investors may not be exogenously given as we assumed in this paper and can be endogenously evolving as well (Hommes, 2006; LeBaron, 2006). After being integrated into our model, it can lead to an agent-based financial market with a market-marker mechanism and a series of empirical studies in this new line of research. Furthermore, in future work we plan to consider the more common limited-liability risky asset by the assumption that its payoff follows a lognormal distribution. To achieve this goal, we need to find a new framework applicable to the lognormal distribution assumption.

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Appendix

Proof of Proposition 1. For the economy defined in Section 2, the expected-utility maximization in the form of (2) can be shown to be equivalent to

$$\max_{x_i} \quad E_i[W_{i,1}] - \frac{1}{2}\gamma\sigma_i^2 x_i^2. \tag{A.1}$$

Based on (4) and (5), investor i can maximize (A.1) by first deciding his position, which in turn depends on which options, long or short, can lead to a higher return as demonstrated in (A.2).

$$\max_{x_i}\begin{cases} x_i(\mu_i - P_a(1+r)) + C_{i,0}(1+r) - \frac{1}{2}\gamma\sigma_i^2 x_i^2, & \text{if } x_i > 0, \\ x_i(\mu_i - P_b(1+r)) + C_{i,0}(1+r) - \frac{1}{2}\gamma\sigma_i^2 x_i^2, & \text{if } x_i < 0. \end{cases} \tag{A.2}$$

Proposition 1, including what is shown in Fig. 1, is proved by simply applying the first-order condition to each of the two options. □

Proof of Proposition 2. The proof starts from the planning problem (7). There are two steps. Step 1: rewrite the constraint condition (7a); step 2: solve the maximization problem (7a) through step 1.

Step 1 (Risk Minimization): investor 1 buys and investor 2 sells risky securities since $\mu_1 > \mu_2$. Therefore, $x_1 = x_{long}$ and $x_2 = -x_{short}$, which implies that

$$\begin{cases} \frac{\mu_1 - P_a(1+r)}{\gamma\sigma_1^2} = \frac{P_b(1+r) - \mu_2}{\gamma\sigma_2^2}, \\ P_a < \frac{\mu_1}{1+r}, \\ P_b > \frac{\mu_2}{1+r}. \end{cases} \tag{A.3}$$

Solving (A.3) gives

$$P_b = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2 - P_a(1+r)\sigma_2^2}{(1+r)\sigma_1^2}, \tag{A.4}$$

where $\frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{(1+r)(\sigma_1^2+\sigma_2^2)} < P_a < \frac{\mu_1}{1+r}$.

Step 2 (Profit Maximization): The market maker's profit

$$\pi = x_1 \cdot P_a + x_2 \cdot P_b. \tag{A.5}$$

Putting (6) and (A.4) into (A.5) gives

$$\pi = \frac{\mu_1 - P_a(1+r)}{\gamma\sigma_1^2}\left(P_a - \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2 - P_a(1+r)\sigma_2^2}{(1+r)\sigma_1^2}\right). \tag{A.6}$$

The first-order condition

$$\frac{d\pi}{dP_a} = -\frac{1+r}{\gamma\sigma_1^2}\left(P_a - \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2 - P_a(1+r)\sigma_2^2}{(1+r)\sigma_1^2}\right) + \frac{\mu_1 - P_a(1+r)}{\gamma\sigma_1^2}\left(1 + \frac{\sigma_2^2}{\sigma_1^2}\right) = 0. \tag{A.7}$$

The second-order condition

$$\frac{d^2\pi}{dP_a^2} = -\frac{2(1+r)}{\gamma\sigma_1^2}\left(1 + \frac{\sigma_2^2}{\sigma_1^2}\right) < 0. \tag{A.8}$$

Based on the first-order and second-order conditions, we obtain the optimal pricing of the market maker in Proposition 2.

The volume equals the demand of investor 1. Putting the optimal pricing into the demand function of investor 1 gives the volume. Putting the optimal pricing and equilibrium demand into (7a) gives the profit of the market maker. □

Proof of Proposition 3. Without loss of generality, we assume that $\mu_1 > \mu_\sigma > \mu_2$ and $\overline{\mu_1} > \overline{\mu_\sigma} > \overline{\mu_2}$. Based on this assumption, we prove the proposition separately for six segments covering the entire parameter space.

Case 1. If $\mu_1 > \mu_2 > (\overline{\mu_1} + \overline{\mu_\sigma})/2 > (\overline{\mu_2} + \overline{\mu_\sigma})/2$, then both investors buy the risky security, and $x_i = (\mu_i - (\overline{\mu_1} + \overline{\mu_\sigma})/2)/(\gamma\sigma_i^2) > 0, i = 1, 2$.

Case 2. If $\mu_1 > (\overline{\mu_1} + \overline{\mu_\sigma})/2 \geqslant \mu_2 \geqslant (\overline{\mu_2} + \overline{\mu_\sigma})/2$, then investor 1 buys and investor 2 does not trade, and $x_1 = (\mu_1 - (\overline{\mu_1} + \overline{\mu_\sigma})/2)/(\gamma\sigma_1^2) > 0, x_2 = 0$.

Case 3. If $\mu_1 > (\overline{\mu_1} + \overline{\mu_\sigma})/2 > (\overline{\mu_2} + \overline{\mu_\sigma})/2 > \mu_2$, then investor 1 buys and investor 2 sells, and $x_1 = (\mu_1 - (\overline{\mu_1} + \overline{\mu_\sigma})/2)/(\gamma\sigma_1^2) > 0, x_2 = (\mu_2 - (\overline{\mu_2} + \overline{\mu_\sigma})/2)/(\gamma\sigma_2^2) < 0$.

Case 4. If $(\overline{\mu_1} + \overline{\mu_\sigma})/2 \geqslant \mu_1 > \mu_2 \geqslant (\overline{\mu_2} + \overline{\mu_\sigma})/2$, then neither of the investors trades, and $x_1 = x_2 = 0$.

Case 5. If $(\overline{\mu_1} + \overline{\mu_\sigma})/2 \geqslant \mu_1 \geqslant (\overline{\mu_2} + \overline{\mu_\sigma})/2 > \mu_2$, then investor 1 does not trade and investor 2 sells, and $x_1 = 0, x_2 = (\mu_2 - (\overline{\mu_2} + \overline{\mu_\sigma})/2)/(\gamma\sigma_2^2) < 0$.

Case 6. If $(\overline{\mu_1} + \overline{\mu_\sigma})/2 > (\overline{\mu_2} + \overline{\mu_\sigma})/2 > \mu_1 > \mu_2$, then both investors sell, and $x_i = (\mu_i - (\overline{\mu_2} + \overline{\mu_\sigma})/2)/(\gamma\sigma_i^2) < 0, i = 1, 2$.

The inventory of the market maker equals the negative investors' demand, which gives the result of the inventory in Proposition 3. □

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