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On the Optimization Methods for Fully Fuzzy Regression Models

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ABSTRACT

The goal of this paper is to construct a multiple fuzzy regression model by fuzzy parameters estimation using the fuzzy samples. We propose an optimization model that provides fuzzy coefficients to minimize the distance between the fuzzy regressands and the fuzzy regressors. It concerned with imprecise measurement of observed variables, linear programming estimation and non-parametric methods. This is different from the assumptions as well as the estimation techniques of the classical analysis. Empirical results demonstrate that our new approach is efficient and more realistic than the traditional regression analysis did.

Keywords: Fuzzy regression; Fuzzy parameter; H-cut; Methods of least square; Triangular membership function

1. Introduction

Regression analysis has been a very popular method with many successful applications. And the problem of parameter estimation in the linear regression models has been an important research topic for statisticians. Conventional study on the regression analysis is based on the conception that the observed data are random with certain measurement errors or noise. However, in the empirical study those assumptions may hardly meet the realization. Since there are many observations experience linguistic or vague data inside the classical type.

For example, the official record of exchange rate for Japanese Yen to US dollar in January 1999 is 118.4. However, this exchange rate do only account for the last exchange data, it can not exactly display the variation of exchange rate (Japanese Yen to US dollar) during January 1999. Under such situation, it may have a great chance of misleading if we try to apply this inaccurate data to fit a regression model.

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Tanaka et. al. [4] proposed the study in linear regression analysis using fuzzy set theory. They consider the linear interval regression model as

$$Y = A_0 + A_1x_1 + A_2x_2 + ... + A_px_p, \tag{1}$$

where parameters A_i are triangular fuzzy numbers and the explained variables $x_1, x_2, ..., x_p$ are real value numbers. Therefore, the estimated value Y is also a fuzzy number. Tanaka et. al. designed a useful technique to solve the estimation problem by transforming the optimization problem of estimation into a linear programming scheme. However, their method is a little complicated. Isshibuchi and Tanaka [2] presented an interval regression analysis base on the back-propagation neural networks. Their method is to obtain a nonlinear interval regression model by identifying the upper bound and the lower bound of the data interval. Recently, Yang and Ko [5] proposed a cluster-wise fuzzy in two approaches: the two stage weighted fuzzy regression and the one stage generalized fuzzy regression. The two stage procedure extends the results of Jajuga [3] and Diamond [1]. The one stage is created by embedding fuzzy clustering into the fuzzy regression model fitting at each step of procedures.

The regression analysis dealing with fuzzy data is usually called fuzzy regression analysis. While a linear interval regression model with fuzzy parameters is called fuzzy regression model. One advantage of using fuzzy regression analysis is that it can process the fuzzy sample data such as: $(\mathbf{x}_i, Y_i)$, where Y_i is a fuzzy number and $\mathbf{x}_i$ is the vector of the explained variables, in a way which is closer to the reality. However, the estimation of fuzzy regression coefficients about fuzzy regression models has not been studied very much. That is, when the parameters A_i in the equation (1), exhibits a linguistics form, such as A_i contains five linguistic values (vary low, low, medium, high, very high). In order to get an appropriate model to exhibit the real case, we had better apply the concept of fuzzy theory as well as the membership functions for these fuzzy sample data.

In this paper we propose a new approach to fuzzy regression models by using fuzzy number and method of least square. It is connected with imprecise measurement of observed variables, fuzzy least square estimation and non-parametric methods. This is different from the assumptions as well as the estimation techniques of the classical analysis. A generalized least square method with nonparametric statistics estimating the regression coefficients is derived. Empirical results demonstrate that our new approach is efficient and more realistic than the traditional regression analysis did.

2. Preliminary

Definition 1 A fuzzy set of X is a mapping $\tilde{u} : X \to [0, 1]$.
Definition 2 Let $\tilde{u}$ be a fuzzy set of R. The α-level set of $\tilde{u}$, denoted $[\tilde{u}]_\alpha$,

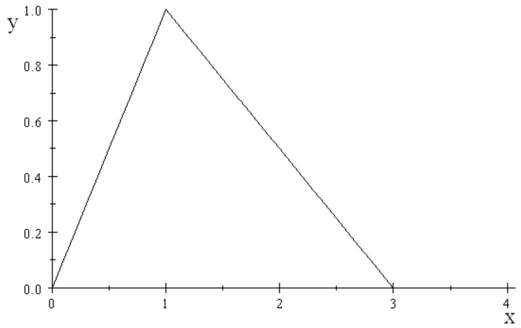

Figure 1. The graph of the fuzzy set $\tilde{u}(x)$

$0 \leq \alpha \leq 1$, is

$$[\tilde{u}]_\alpha = \{x \in R | \tilde{u}(x) \geq \alpha\}.$$

For $\alpha = 0$ the support of $\tilde{u}$ us defined as

$$[\tilde{u}]_0 = \overline{\{x \in R | \tilde{u}(x) \geq 0\}}.$$

Remark Let $\tilde{u}$ be a triangle fuzzy set and $0 \leq \alpha \leq 1$. The α-level set of $\tilde{u}$ can be represented as

$$[\tilde{u}]_\alpha = (u, (1-\alpha)\underline{u}, (1-\alpha)\overline{u}),$$

where u is the center of $\tilde{u}$ and $(1-\alpha)$ or $(1-\alpha)\overline{u}$ are the length of the left or right spread from the center. Without any confusion, the triangle fuzzy set $\tilde{u}$ can now be denoted as $\tilde{u} = (u, \underline{u}, \overline{u})$.

Example 1 Let $\tilde{u}$ be a triangle fuzzy set with

$$\tilde{u}(x) = \begin{cases} 0, & \text{if } x < 0, \\ x, & \text{if } 0 \leq x < 1, \\ \frac{3-x}{2}, & \text{if } 1 \leq x < 3, \\ 0, & \text{if } 3 \leq x. \end{cases}$$

Then the α-level set of $\tilde{u}$ can be represented as

$$[\tilde{u}]_\alpha = (u, (1-\alpha), 2(1-\alpha)).$$

Here, we say that $[\tilde{u}]_\alpha = (u, \underline{u}_\alpha, \overline{u}_\alpha)$ is positive if $u - \underline{u}_\alpha > 0$.

It is well-known that H-difference for fuzzy sets was initially introduced by Hukuhara as follows.

Definition 3 Let $\tilde{u}$, $\tilde{v}$ be two fuzzy sets. If there exists a fuzzy set $\tilde{w}$ such that $\tilde{u} = \tilde{v} + \tilde{w}$, then $\tilde{w}$ is called the H-difference of $\tilde{u}$ and $\tilde{v}$ and it is denoted by $\tilde{u}_{\overline{H}}\tilde{v}$.

For any two fuzzy numbers, we define the following operations.

Definition 4 If $[\tilde{a}]_\alpha = (a, \underline{a}_\alpha, \overline{a}_\alpha)$ and $[\tilde{b}]_\alpha = (b, \underline{b}_\alpha, \overline{b}_\alpha)$ are two fuzzy α-level sets, then two operations, say addition and multiplication, are defined as

(1) **Addition**

$$(a, \underline{a}_\alpha, \overline{a}_\alpha) \oplus (b, \underline{b}_\alpha, \overline{b}_\alpha) = (a + b, \underline{a}_\alpha + \underline{b}_\alpha, \overline{a}_\alpha + \overline{b}_\alpha).$$

(2) **Multiplication**
If $a > 0$ and $b > 0$, then

$$(a, \underline{a}_\alpha, \overline{a}_\alpha) \otimes (b, \underline{b}_\alpha, \overline{b}_\alpha) = (ab, b\underline{a}_\alpha + a\underline{b}_\alpha, b\overline{a}_\alpha + a\overline{b}_\alpha).$$

If $a < 0$ and $b > 0$, then

$$(a, \underline{a}_\alpha, \overline{a}_\alpha) \otimes (b, \underline{b}_\alpha, \overline{b}_\alpha) = (ab, b\underline{a}_\alpha - a\underline{b}_\alpha, b\overline{a}_\alpha - a\overline{b}_\alpha).$$

If $a < 0$ and $b < 0$, then

$$(a, \underline{a}_\alpha, \overline{a}_\alpha) \otimes (b, \underline{b}_\alpha, \overline{b}_\alpha) = (ab, -b\underline{a}_\alpha - a\underline{b}_\alpha, -b\overline{a}_\alpha - a\overline{b}_\alpha).$$

So far, many papers are proposed to ranking the fuzzy sets. In this paper, two triangle fuzzy sets are ranked by the following procedure.

Definition 5 Let $\tilde{u} = (u, \underline{u}, \overline{u})$ and $\tilde{v} = (v, \underline{v}, \overline{v})$ be two triangle fuzzy sets. We say $\tilde{u} \leq \tilde{v}$ if they satisfy the procedure

$$\begin{array}{rl} \text{If} & u < v, \\ \text{Elseif} & u - \underline{u} < v - \underline{v}, (or\ \underline{v} < \underline{u}) \\ \text{Else} & u + \underline{u} < v + \underline{v}, (or\ v < u). \end{array}$$

On the other hand, the fuzzy matrix is defined as follows.

Definition 6 A matrix $\tilde{X} = (\tilde{x}_{ij})$ is called a fuzzy matrix if all element in $\tilde{A}$ are fuzzy sets.

If $\tilde{x}_{ij}$ are all triangle fuzzy sets, then the fuzzy matrix can be represented as

$$[\tilde{X}]_\alpha = ([\tilde{x}_{ij}]_\alpha) = ((x_{ij}, (1-\alpha)\underline{x}_{ij}, (1-\alpha)\overline{x}_{ij})) \text{ or } X = ((x_{ij}, \underline{x}_{ij}, \overline{x}_{ij})).$$

Following the notation of Dehghan and Hashemi, the matrix $\tilde{X}$ can be represented as $[\tilde{X}]_\alpha = (X, M_\alpha, N_\alpha)$ since $\tilde{X} = (\tilde{x}_{ij})$ and $\tilde{x}_{ij} = ((x_{ij}, (1-\alpha)\underline{x}_{ij}, (1-\alpha)\overline{x}_{ij}))$. There crisp matrices $X = (x_{ij})$, $M = (\underline{x}_{ij}^\alpha)$ and $N = (\overline{x}_{ij}^\alpha)$ are called the center matrix and the right and the left spread matrices, respectively.

Now, we consider the fully fuzzy linear systems (FFLS) as the form:

$$\begin{array}{rcl}
(\tilde{x}_{11} \otimes \tilde{a}_1) \oplus (\tilde{x}_{12} \otimes \tilde{a}_2) \oplus ... \oplus \tilde{x}_{1n} \otimes \tilde{a}_n & = & \tilde{y}_1, \\
(\tilde{x}_{21} \otimes \tilde{a}_1) \oplus (\tilde{x}_{22} \otimes \tilde{a}_2) \oplus ... \oplus \tilde{x}_{2n} \otimes \tilde{a}_n & = & \tilde{y}_2, \\
& \vdots & \\
(\tilde{x}_{m1} \otimes \tilde{a}_1) \oplus (\tilde{x}_{m2} \otimes \tilde{a}_2) \oplus ... \oplus \tilde{x}_{mn} \otimes \tilde{a}_n & = & \tilde{y}_m,
\end{array}$$

where $\tilde{x}_{ij}$, $\tilde{a}_i$ and $\tilde{y}_j$ are all fuzzy numbers. The matrix form of this linear system is represented as

$$\tilde{X} \otimes \tilde{a} = \tilde{Y},$$

where $\tilde{X} = (X, M_\alpha, N_\alpha)$, is an $m \times n$ fuzzy matrix, $\tilde{a} = (a, \underline{a}_\alpha, \overline{a}_\alpha)$ and $\tilde{Y} = (y, \underline{y}_\alpha, \overline{y}_\alpha)$ are $n \times 1$ and $m \times 1$ fuzzy matrices. As $m = n$, the solution of nonnative FFLS have been proposed by Dehghan and Hashemi.

3. Fuzzy regression models

The equation

$$Y_i = a_1 X_{i1} + a_2 X_{i2} + ... + a_n X_{in} + \varepsilon_i, \quad i = 1, 2, ..., m$$

is a multiple linear model, where Y_i is the regressand and $X_{i1}, X_{i2}, ..., X_{in}$ are regressors. A disturbance terms ε_i is added to capture the influence of everything else on Y_i and $X_{i1}, X_{i2}, ..., X_{in}$. In general there are n parameters to be determined, $a_1, a_2, ..., a_n$. In order to estimate the parameters it is often useful to use the matrix notation

$$Y = aX + \varepsilon,$$

where Y is a column vector that includes the observed values of $Y_1, Y_2, ..., Y_m$, ε includes the unobserved stochastic components $\varepsilon_1, \varepsilon_2, ..., \varepsilon_m$ and the matrix X the observed values of the regressors:

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & & X_{2n} \\ \vdots & & \ddots & \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix}$$

.

To determine the parameters, one possible method is to find the solution of the minimization model

$$\min ||Y - aX||.$$

In the following, we assume that the regressors and the regressand are all triangle fuzzy sets. Let $\tilde{X} = (X, M_\alpha, N_\alpha)$, be an $m \times n$ fuzzy matrix, $\tilde{a} = (a, \underline{a}_\alpha, \overline{a}_\alpha)$ and $\tilde{Y} = (y, \underline{y}_\alpha, \overline{y}_\alpha)$ are $n \times 1$ and $m \times 1$ fuzzy matrices. We shall propose an optimization model that provides a solution $\tilde{a} = (a, \underline{a}_\alpha, \overline{a}_\alpha)$ to minimize the difference between $\tilde{X} \otimes \tilde{a}$ and $\tilde{Y}$, that is

$$\min ||\tilde{X} \otimes \tilde{a} - \tilde{Y}||_1,$$

where the fuzzy multiplication, H-difference and the ranking procedure are given in the previous section. Since $(\tilde{X} \otimes \tilde{a})_i = \sum_{k=1}^{n} \tilde{x}_{ik} \otimes \tilde{a}_k$,

$$||\tilde{X} \otimes \tilde{a} - \tilde{Y}||_1 = \sum_{i=1}^{m} |\sum_{k=1}^{n} (\tilde{x}_{ik} \otimes \tilde{a}_k) - \tilde{y}_i|.$$

Since

$$\begin{aligned} &\tilde{X} \otimes \tilde{a} \\ &= (X, M_\alpha, N_\alpha) \otimes (a, \underline{a}_\alpha, \overline{a}_\alpha) \\ &= (Xa, X\underline{a}_\alpha + M_\alpha a, X\overline{a}_\alpha + N_\alpha a) \end{aligned}$$

an optimization model can be formulated as follows.

$$\begin{array}{ll} \min & \beta h_1 + \gamma h_2 + \delta h_3 \\ s.t. & Xa - y = d_1^+ - d_1^- \\ & X\underline{a}_\alpha + M_\alpha a - \underline{y}_\alpha = d_2^+ - d_2^- \\ & X\overline{a}_\alpha + N_\alpha a - \overline{y}_\alpha = d_3^+ - d_3^- \\ & h_1 = \sum_{i=1}^{m} d_{1i}^+ + d_{1i}^- \\ & h_2 = \sum_{i=1}^{m} d_{2i}^+ + d_{2i}^- \\ & h_3 = \sum_{i=1}^{m} d_{3i}^+ + d_{3i}^- \\ & d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^- \geq 0 \quad i = 1, 2, ..., m \end{array}$$

where β, γ and δ are given weighted coefficients.

Example 2 Let us consider the triangle fuzzy data shown in Table 1. A first order linear regression is performed by modeling the data by a linear equation

$$\tilde{Y}_i = \tilde{a} \otimes \tilde{X}_i + \varepsilon_i, \quad i = 1, 2, 3, 4, 5.$$

Table 1. Triangle fuzzy data.

	$\tilde{X}$	$\tilde{Y}$
A	(0.6, 0.06, 0.06)	(0.6, 0.04, 0.09)
B	(0.8, 0.1, 0.1)	(0.65, 0.06, 0.06)
C	(0.4, 0.03, 0.08)	(0.6, 0.08, 0.03)
D	(0.3, 0.05, 0.05)	(0.7, 0.05, 0.05)
E	(0.9, 0.09, 0.04)	(0.85, 0.06, 0.06)

The optimization model is formulated as

$$
\begin{aligned}
\min \quad & \beta h_1 + \gamma h_2 + \delta h_3 \\
s.t. \quad & \begin{bmatrix} 0.6 \\ 0.8 \\ 0.4 \\ 0.3 \\ 0.9 \end{bmatrix} a - \begin{bmatrix} 0.6 \\ 0.8 \\ 0.4 \\ 0.3 \\ 0.9 \end{bmatrix} = d_1^+ - d_1^- \\
& \begin{bmatrix} 0.6 \\ 0.8 \\ 0.4 \\ 0.3 \\ 0.9 \end{bmatrix} \underline{a}_\alpha - \begin{bmatrix} 0.06 \\ 0.1 \\ 0.03 \\ 0.05 \\ 0.09 \end{bmatrix} a - \begin{bmatrix} 0.04 \\ 0.06 \\ 0.08 \\ 0.05 \\ 0.06 \end{bmatrix} = d_2^+ - d_2^- \\
& \begin{bmatrix} 0.6 \\ 0.8 \\ 0.4 \\ 0.3 \\ 0.9 \end{bmatrix} \overline{a}_\alpha - \begin{bmatrix} 0.06 \\ 0.1 \\ 0.03 \\ 0.05 \\ 0.09 \end{bmatrix} a - \begin{bmatrix} 0.09 \\ 0.06 \\ 0.03 \\ 0.05 \\ 0.06 \end{bmatrix} = d_3^+ - d_3^- \\
& h_1 = \sum_{i=1}^{5} d_{1i}^+ + d_{1i}^- \\
& h_1 = \sum_{i=1}^{5} d_{2i}^+ + d_{2i}^- \\
& h_1 = \sum_{i=1}^{5} d_{3i}^+ + d_{3i}^- \\
& d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^- \geq 0 \quad i = 1, 2, ..., 5
\end{aligned}
$$

When the weighted coefficients β, γ and δ are given as 0.5, 0.3 and 0.2, respectively.

Table 2. Personal income, level of education and marriageable age for 12 females.

Number	Personal Income	Years of Education	Marriageable Age
1	1.5∼2.5	6∼9 (high school)	26∼30
2	2.6∼3.5	12∼16 (college)	26∼30
3	1.5∼2.5	6∼9 (high school)	31∼35
4	2.6∼3.5	12∼16(college)	31∼35
5	1.5∼2.5	9∼12 (high school)	31∼35
6	2.6∼3.5	12∼16(college)	31∼35
7	1.5∼2.5	6∼9 middel high school	31∼35
8	2.6∼3.5	12∼16(college)	26∼30
9	2.6∼3.5	12∼16(college)	31∼35
10	2.6∼3.5	12∼16(college)	31∼35
11	3.6∼4.5	12∼16(college)	36∼40
12	3.6∼4.5	12∼16(college)	36∼40

The coefficient $\tilde{a}$ is (1.227273, 0.107692, 0.100000); i.e.

$$\tilde{Y}_i = (1.227273, 0.107692, 0.100000) \otimes \tilde{X}_i + \varepsilon_i.$$

4. Empirical studies

In this research, we attempt to figure out whether there is a relationship between marriageable age and some special personal characteristics, such as personal income and level of education. These data are obtained through random sampling of 24 marriageable people at Taipei area, including 12 females and 12 males. The variables include: personal income, level of education, and marriageable age. Here, we define the dependent variable Y as marriageable age, while two independent variables X_1, X_2 are defined as personal income (monthly) and level of education, respectively. These data are shown respectively in Table 2 and Table 3.

The ranges of marriageable age are respectively denoted as: 18∼20 years old; 21∼25 years old; 26∼30 years old; 31∼35 years old; 36∼40 years old; 41 years old and older.

The ranges of personal income are respectively denoted as: NT$15,000∼25,000; NT$26,000∼35,000; NT$36,000∼45,000; NT$46,000∼55,000; NT$56,000∼65,000; NT$66,000 and more.

The level of education is defined as the number of school years. So, we denote respectively as follows:

Table 3. Personal income, level of education and marriageable age for 12 males.

Number	Personal Income	Years of Education	Marriageable Age
1	1.5 2.5	6∼9 middel high school	35∼40
2	2.6∼3.5	12∼16(college)	26∼30
3	3.6∼4.5	6∼9 middel high school	35∼40
4	3.6∼4.5	12∼16(college)	≥ 41
5	2.6∼3.5	9∼12 (high school)	35∼40
6	3.6∼4.5	12∼16(college)	35∼40
7	3.6∼4.5	6∼9(college)	31∼35
8	2.6∼3.5	12∼16(college)	26∼30
9	2.6∼3.5	12∼16(college)	35∼40
10	3.6∼4.5	12∼16(college)	31∼35
11	$\geq$6.6	12∼16(college)	36∼40
12	4.6∼5.5	16∼18 (master)	31∼35

0∼6 years of schooling is denoted as the level of elementary school.

6∼9 years of schooling is denoted as the level of junior high school.

9∼12 years of schooling is denoted as the level of senior high school, including vocational school.

12∼16 years of schooling is denoted as college graduate with a bachelor's degree.

16∼18 years of schooling is denoted as holding a master's degree.

18 and more years of schooling is denoted as holding a doctor's degree.

Let personal income and level of education be two triangle fuzzy sets, and are denoted as $\tilde{u} = (u, \underline{u}, \overline{u})$ and $\tilde{v} = (v, \underline{v}, \overline{v})$, respectively. Let marriageable age be a triangle fuzzy set with $\tilde{w} = (w, \underline{w}, \overline{w})$. The equation for the regression line of $\tilde{u} = (u, \underline{u}, \overline{u})$ and $\tilde{v} = (v, \underline{v}, \overline{v})$ on $\tilde{w} = (w, \underline{w}, \overline{w})$ is given by

$$\tilde{w} = (5, 1, 0) \otimes \tilde{u} \oplus (1.214, 0, 0.071) \oplus \tilde{v}$$

for the female, and is

$$\tilde{w} = (6.239, 0, 0.278) \otimes \tilde{u} \oplus (0.806, 0, 0.155) \oplus \tilde{v}$$

for the male. Here, the infinite symbol is replaced with a large number since age and income are always finite in the real-world.

5. Conclusion

In this research, we proposed a new method for parameters estimation of linear regression models integrated with statistical theory and the concept of fuzzy logic. The presented procedure to find the α-level of fuzzy parameter for a set of regression data is carefully discussed. Experimental results show that the proposed method of estimated fuzzy parameters is efficient and practical in explanation of the real data with the significant α-level. The result in this research presented a feasible application and new promising area for constructing regression models.

The conventional method for parameters estimation of linear regression rested on concept of the linear programming. If we use fuzzy statistical concept to analyze the relationships among the endogenous variables Y and exogenous variables X, we will get a better explanation for those variables. Especially, from the statistical point of view, using the least square method to estimate the boundaries of fuzzy variable exhibits more appropriateness and its computation is also more efficient than the traditional linear programming method. Though the least square method can be infected by certain outliers, we placed the nonparametric technique to reduce the influence of those data and make the estimator more robustic.

Finally, linguistic value estimation by the use of regression data is very complicated, involving entities with many features and parts which interact with each other and their environment in intricate ways. The proposed method is also suitable in dealing with historical data, which are linguistic values. The method of fuzzy parameter estimation doesn't require precise knowledge about the structure in the data and can take full advantage of the model-free approach. However there still remains many problems for future studies, such as:

(1) In the traditional regression models construction, the estimation based on the least square is BLUE(Best Linear Unbiased Estimate). Is our estimator BLUE? How to define the BLUE from the fuzzy statistics point of view?

(2) How to identify the fuzzy endogenous and exogenous variables as well as detect the intervention among them?

(3) How to extend the estimated parameter methods to the seasonal regression models?

(4) How much precision is required in constructing the fuzzy parameters under the significant α level?

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