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S&P 500 波動度的預測 – 考慮狀態轉換與指數風險中立偏態及 VIX 期貨之資訊內涵

The Information Content of S&P 500 Risk-neutral Skewness and VIX Futures for S&P 500 Volatility Forecasting: Markov Switching Approach

研究生：黃郁傑

Yu-Jie Huang

指導教授：陳威光 博士

林靖庭 博士

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謝詞

時光如炬、歲月如梭，碩士兩年的學生生涯隨著論文的完成即將劃下句點，回首這兩年，滿滿的感謝與感動。首先，要感謝我的指導教授陳威光老師與林靖庭老師，每當論文遇到瓶頸時總是給予寶貴的建議及方向。再者，我要感謝碩士班的同學們，碩士這兩年，在課業上給予我許多幫助，在生活中有大家的陪伴，創造許多美好的回憶。除此之外，我也要感謝中山財管所的同學們，無論在課業或生活方面，幫助我許多。

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黃郁傑　謹致於
國立政治大學金融研究所
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摘要

本研究探討 VIX 期貨價格所隱含的資訊對於 S&P 500 指數波動度預測的解釋力。過去許多文獻主要運用線性預測模型探討歷史波動度、隱含波動度和風險中立偏態對於波動度預測的資訊內涵。然而過去研究顯示，波動度具有長期記憶與非線性的特性，因此本文主要研究非線性預測模型對於波動度預測的有效性。本篇論文特別著重在不同市場狀態下(高波動與低波動)的實現波動度及隱含波動度異質自我迴歸模型(HAR-RV-IV model)。因此，本研究以考慮馬可夫狀態轉化下的異質自我迴歸模型(MRS-HAR model)進行實證分析。

本研究主要目的有以下三點: (1) 以 VIX 期貨價格所隱含的資訊提升 S&P 500 波動度預測的準確性。(2) 結合風險中立偏態與 VIX 期貨的資訊內涵，進一步提升 S&P 500 波動度預測的準確性。(3) 考慮狀態轉換後的波動度預測模型是否優於過去文獻的線性迴歸模型。

本研究實證結果發現: (1) 相對於過去的實現波動度及隱含波動度，VIX 期貨可以提供對於預測未來波動度的額外資訊。 (2) 與其他模型比較，加入風險中立偏態和 VIX 期貨萃取出的隱含波動度之波動度預測模型，只顯著提高預測未來一天波動度的準確性。 (3) 考慮狀態轉換後的波動度預測模型優於線性迴歸模型。

關鍵字: 波動度預測、實現波動度、風險中立偏態、VIX 期貨、馬可夫狀態轉換模型

Abstract

This paper explores whether the information implied from VIX futures prices has incremental explanatory power for future volatility in the S&P 500 index. Most of prior studies adopt linear forecasting models to investigate the usefulness of historical volatility, implied volatility and risk-neutral skewness for volatility forecasting. However, previous literatures find out the long-memory and nonlinear property in volatility. Therefore, this study focuses on the nonlinear forecasting models to examine the effectiveness for volatility forecasting. In particular, we concentrate on Heterogeneous Autoregressive model of Realized Volatility and Implied Volatility (HAR-RV-IV) under different market conditions (i.e., high and low volatility state).

This study has three main goals: First, to investigate whether the information extracted from VIX futures prices could improve the accuracy for future volatility forecasting. Second, combining the information content of risk-neutral skewness and VIX futures to enhance the predictive power for future volatility forecasting. Last, to explore whether the nonlinear models are superior to the linear models.

This study finds that VIX futures prices contain additional information for future volatility, relative to past realized volatilities and implied volatility. Out-of-sample analysis confirms that VIX futures improves significantly the accuracy for future volatility forecasting. However, the improvement in the accuracy of volatility forecasts is significant only at daily forecast horizon after incorporating the information of risk-neutral skewness and VIX futures prices into the volatility forecasting model. Last, the volatility forecasting models are superior after taking the regime-switching into account.

Keywords: Volatility forecasting, Realized volatility, Risk-neutral skewness, VIX futures, Markov regime-switching

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1. Introduction

Volatility is one of the most critical issues in asset pricing, investment and risk management, so measure and forecast the volatility accurately is important. Along with the high frequency data becomes widely available, we can estimate more accurate volatility measure. Realized Volatility (RV) is a method to measure volatility by square root of summing the intraday squared returns. It is a more accurate volatility measure than other measures, such as squared or absolute daily returns. In this study, we focus on the volatility of S&P 500 index, which is one of the most representative index in the United State. The index is composed of the top 500 large capitalization companies traded on the United State.

Some of the studies point out that the implied volatility and the risk-neutral skewness are effective at future volatility forecasting. However, the VIX index could not easily be traded, the CBOE launched the Volatility Index (VIX) futures on March 26, 2004 and VIX options on February 24, 2006. In this study, we concentrate on the informational role of VIX futures, since Szado (2009) has proven VIX futures to be a far more convenient hedging tool than S&P 500 option. Therefore, we examine whether the implied volatility extracted from VIX futures prices can be used to improve the predictive accuracy of future volatility in the S&P 500 index.

This study has three main goals: First, to investigate whether the implied volatility extracted from VIX futures exists incremental information content for future volatility forecasting. We define the implied volatility extracted from VIX futures prices as Residual of VIX, which is uncorrelated with VIX. Therefore, combining the VIX and Residual of VIX, we regard this kind of risk-neutral volatility as an adjusted risk-neutral volatility. We argue that this adjusted implied volatility measure can improve the predictive accuracy for future volatility forecasting, as compared to the implied volatility measure (VIX). Second, since many previous researchers have demonstrated that risk-neutral skewness extracted from S&P 500 options prices has significant effect on future volatility forecasting, we wonder if we combine the information of the risk-neutral skewness and the adjusted risk-neutral volatility, it can improve the predictive power for future volatility forecasting. Third, to explore whether the nonlinear forecasting models are superior to the linear forecasting models.

We adopt the implied volatility, the risk-neutral skewness and VIX futures to forecast future volatility. Byun and Kim (2013) derive a linear relationship between the physical variance and the high-order risk-neutral moments, such as the risk-neutral variance and the risk-neutral skewness.[1] If return innovation is normal distribution, the physical variance is identical to the risk-neutral variance. However, for non-normal return innovation, the implied volatility and risk-neutral skewness play the important roles on future volatility forecasting.

Over the past two decades, several volatility forecasting models based on various factors have been developed. One of the most famous volatility forecasting models is Heterogeneous Autoregressive model of Realized Volatility (HAR-RV model) of Corsi (2009), which utilizes a combination of volatilities measured over different time horizons to capture the long-memory characteristic of future volatility. However, Granger and Ding (1996) find out that the long-memory property in volatility tends to be non-constant over time and Longin (1997) provides evidence that the long-memory property in high volatility is less persistent, thus suggesting the presence of nonlinearities. Therefore, we use Markov regime-switching model to capture the nonlinear feature of realized volatility, which is different from the previous researches. With this framework, we run the

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[1] The proposition in Byun and Kim (2013) states the following equation: $\sigma_t^2 \approx \sigma_t^{*2} + \beta \times SKEW_t^* + \gamma \times KURT_t^*$ where σ_t^2 (σ_t^{*2}) is the conditional variance under the physical (risk-neutral) measure, and $SKEW_t^*$ is the conditional risk-neutral skewness, and $KURT_t^*$ is the conditional risk-neutral kurtosis.

multivariate regression with Markov regime-switching to analyze the in-sample performance and out-of-sample forecasting ability.

The rest part of this study is organized as follows. Section 2 is the literature reviews covers the VIX, VIX futures, risk-neutral skewness and Markov regime-switching model. Section 3 is the methodology including measuring the realized volatility, Residual of VIX, risk-neutral skewness and modeling Markov regime-switching model. Section 4 is the empirical analysis of the study. Final Section is the conclusion of the study.

2. Literature

In this Section, we introduce the studies of VIX, VIX futures and risk-neutral skewness about future volatility forecasting. Furthermore, owing to the nonlinear feature of volatility, some researches that adopt Markov regime-switching model to capture this characteristic is also included in this Section.

2.1 VIX for Future Volatility Forecasting

The CBOE Volatility Index (VIX), introduced by the Chicago Board Options Exchange (CBOE) in 1993, is designed to measure the expected volatility of the S&P 500 index over the next 30 calendar days. When introduced in 1993, the VIX was originally based on implied volatilities of eight S&P 100 at-the-money put and call options. Since 2003, the VIX has been calculated, based on a model-free formula, by a wide range of S&P 500 out-of-the-money call and put option prices. This change is to reflect a more accurate view of market volatility. Some earlier studies find that implied volatility has predictive power for future volatility. For example, Latane and Rendleman (1976) demonstrate that the weighted average of Black and Scholes (B-S) call option implied volatilities is typically a better predictor of future volatility than volatility based on the historical return data. Fleming (1998) indicates that VIX has dominated predictive power for future volatility compared to historical volatility. Poon and Granger (2003) reviewed studies related to volatility forecasting and conclude that VIX is the best predictor for future volatility, although it may be a bias one. Moreover, Jiang and Tian (2005) suggest that their model-free volatility, which does not depend on option pricing model, subsumes all the information contained in B-S implied volatility and historical volatility and is a more efficient forecast for future volatility.

2.2 VIX Futures for Future Volatility Forecasting

VIX futures were listed by the CBOE in March 26, 2004. They are exchange-traded futures contracts on volatility, and may be used to trade and hedge volatility. Since the VIX is untradable, a number of different studies involving forecast accuracy have been applied to VIX futures markets. Konstantinidi and Skiadopoulus (2011) demonstrate that VIX futures are predictable by their historical patterns, however the coefficients are too small to attain abnormal trading profits. Chung et al. (2011) investigate the informational role of S&P 500 index option and VIX option on the prediction of return, volatility and density in the S&P 500 index. They find that the information content implied from these two option market is not identical and all the predictions significantly improved by the information recover from VIX option. They apply the put-call parity to recover the information from VIX option, named implied VIX, which is similar to VIX futures. Shu and Zhang (2012) apply traditional linear Engle-Granger cointegration test and find that VIX futures prices have predictive ability on the underlying VIX. Furthermore, Frijns et al. (2013) document that VIX futures dominance VIX when the index returns is negative and the value of VIX is high. This finding suggests that on those days investors use VIX futures to hedge their positions rather than trading in the S&P 500 options.

Regarding to tests of Expectation Hypothesis, Nossman and Wilhelmsson (2009) test the expectation hypothesis, whether the VIX futures price is an unbiased estimator of the changes in the VIX index. They find that if the futures price is not adjusted by a risk premium, the expectation hypothesis is rejected. They report that risk premium adjusted futures prices predict the direction of one-day ahead VIX index correctly in 73 percent of the times.

2.3 Risk-Neutral Skewness for Future Volatility Forecasting

The curve of S&P 500 implied volatility, also known as the smile or "skew", has been one of the most studied features of S&P 500 option prices. Therefore, the Chicago Board Exchange (CBOE) introduced the CBOE Skew Index (SKEW) to measure the slope of the implied volatility curve that increases as the curve tends to steepen. This indicator can also measure the tail risk of the S&P 500 returns. Bakshi et al. (2003) show that the risk-neutral skewness can be expressed on the basis of option prices. Dennis and Mayhew (2002) describe a negative relation between the risk-neutral skewness and systematic risk, beta. They suggest that market risk is important in pricing individual stock options and indicate that market risk is reflected in the risk-neutral skewness extracted from the option prices. Similarly, Doran et al. (2007) report the evidence that risk-neutral skewness has strong predictive power in short-term crash/spike of the stock market. They find that large jump premium in the short term is the best explanation of significant negative skew for short maturity options. More recently, Byun and Kim (2013) investigate that risk-neutral skewness has incremental information content for future volatility in the S&P 500 index. Particularly, they concentrate on Heterogeneous Autoregressive model of Realized Volatility and Implied Volatility (HAR-RV-IV). They find that risk-neutral skewness significantly improve the accuracy of volatility forecasting at only daily and weekly horizons.

2.4 Markov Regime-Switching Model

Hamilton (1989) pioneered in the use of a Markov process to model the regime-changes, also termed "Markov switching model". Since the introduction of Markov switching models to econometrics, there are considerable studies on using GARCH type with regime-switching models to capture the volatility dynamics of financial time series, in part because they give rise to a believable interpretation of nonlinearities associated with time-varying. Gray (1996) develops a generalized regime-switching (GRS) model using the conditional expectation of the past variance and the model can be regarded as the first MRS-GARCH. Marcucci (2005) compares different standard GARCH models and Markov regime-switching GARCH (MRS-GARCH) in terms of their ability to forecast the US stock market volatility at horizons that range from one day to one month. The empirical analysis demonstrates that MRS-GARCH models outperform all standard GARCH models in forecasting volatility at horizons shorter than one week. However, at forecast horizons longer than one week, the asymmetric GARCH models are superior.

Relative to nonlinearities of realized volatility (RV), many previous researches evidence that RV exhibits high persistence or long-memory. Heterogeneous Autoregressive (HAR) model and Autoregressive Fractionally Integrated Moving Average (ARFIMA) models are generally used to capture this feature. Corsi (2009) utilizes past daily, weekly and monthly realized volatility to capture the long-memory property of realized volatility. Raggi and Bordignon (2012) adopt MRS-ARFIMA to capture long-memory and nonlinearities characteristic simultaneously. The out-sample results of volatility forecasting at several forecast horizons reveal that introducing these nonlinearities produces superior forecasts.

3. Methodology

In Section 3.1 and 3.2, we introduce how to measure the realized volatility via intraday returns and the method to extract the implied volatility, risk-neutral skewness from VIX futures prices and S&P 500 option prices. In Section 3.3 and 3.4, we will specify the volatility forecasting model with Markov regime-switching and the methods to compare out-of-sample forecasting performance, respectively.

3.1 Measuring Realized Volatility

Consider a standard Brownian motion with jump model for the logarithmic asset price at time t:

$$dp_t = \mu_t dt + \sigma_t dW_t + \kappa_t dq_t \tag{1}$$

where μ_t is the mean of the instantaneous change in the value of p_t, σ_t is the variance of the instantaneous change in the value of p_t, W_t is a Standard Brownian motion, κ_t is the jump size, and q_t is the counting process which is normalized, that is $dq_t = 1$, whenever there is a jump at time t, and $dq_t = 0$, otherwise. The continuously compounded intraday return over the trading day t is denoted by

$$r_{t,i} \equiv p_{t,i} - p_{t,t-1} \tag{2}$$

where $i = 1,\ 2,\ \cdots,\ N$

The daily realized volatility for the time t is defined by the sum of the square root of the daily realized variance.

$$RV_{t-1,t} = \sqrt{\sum_{i=1}^{N} r_{t,i}^2} \tag{3}$$

where $r_{t,i}$ is the $i-th$ intraday return.

For the comparison of other forecast horizons, let's denote the multi-period realized volatility as

$$RV_{t-h,t} = \sqrt{(RV_{t-1,t}^2 + RV_{t-2,t-1}^2 + \cdots + RV_{t-h,t-h+1}^2)/h} \tag{4}$$

where $h = 1, 2, 3, \cdots$ We take $h = 5$ and $h = 22$ as weekly and monthly realized volatility, respectively.

3.2 Measuring Risk-Neutral Skewness and Residual of VIX

SKEW is defined as $SKEW = 100 - 10 \times S$, where $S = E[(\frac{R-\mu}{\sigma})^3]$, R is 30-days log return of S&P 500, μ is the expected return and σ is the standard deviation.

S can be easily recognized as the risk-neutral version of a coefficient of statistical skewness.

S can be expanded as the following function of prices P_1, P_2 and P_3:

$$S = \frac{E[R^3] - 3E[R]E[R^2] + 2E[R]^3}{(E[R^2] - E^2[R])^{3/2}} = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}} \tag{5}$$

where P_1, P_2, and P_3 is the price of the power payoffs R, R^2, and R^3, which derived from the option prices. Detail of risk-neutral skewness is in Appendix A.

We defined the implied volatility extracted from VIX futures prices as Residual of VIX which is the residuals of running the regression of VIX futures on VIX. It can be formulated as:

$$VIX\_F_t = \alpha + \beta VIX_t + \varepsilon_t \tag{6}$$

where Residual of VIX $(reVIX_t) \equiv \varepsilon_t$.

3.3 Regime Switching Model for Volatility Forecasting

In this Subsection, we introduce the concept of regime-switching model and the parameter estimates with EM algorithm. Then, we will set up the model specification based on Heterogeneous Autoregressive model of Realized Volatility (HAR-RV).

3.3.1 Regime-Switching Model

The major characteristic of regime-switching model is that it allows some or all parameters to switch across the different regimes/states by a Markov process, which is governed by an unobserved state variable S_t. The logic behind this kind of model is having a mixture of distributions, from which the model gets the current value, according to a more possible (unobserved) state that could determine such observation. The state variable is assumed to follow first-order Markov chain with transition probability:

$$\Pr(S_t = j \mid S_{t-1} = i) = P_{ij} \tag{7}$$

where P_{ij} indicates the probability from state i at time $t-1$ to state j at time t.

The transition matrix P is:

$$P = \begin{pmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{pmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \tag{8}$$

We consider the following regression model with Markov regime-switching:

$$\begin{aligned} & y_t = x_t \beta^{S_t} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_{S_t}^2) \\ & S_t = \begin{cases} 1, & \text{low volatility state} \\ 2, & \text{high volatility state} \end{cases} \\ & \Pr(S_t = j \mid S_{t-1} = i) = P_{ij} \end{aligned} \tag{9}$$

where y_t is a 1×1 matrix, x_t is a $1 \times k$ matrix, $\beta_t^{S_t}$ is a $k \times 1$ matrix.

Let $\mathcal{Y}_t \equiv [y_{1,} y_2, \cdots, y_T]^{'}$ denote the observations obtained up to time T and $\mathcal{S}_t \equiv [y_{1,} y_2, \cdots, y_T]^{'}$ denote unobserved state variable up to time T. We estimate the parameter vector $\theta = [\beta^1, \beta^2, \sigma_1^2, \sigma_2^2, P_{11}, P_{22}]^{'}$, and get the parameter estimates by the following conditional log likelihood function with EM algorithm:[2]

$$\hat{\theta} = \arg\max_{\theta} \prod_{t=1}^{T} \mathrm{f}(y_t \mid \mathcal{Y}_T; \theta) \tag{10}$$

where $\mathrm{f}(y_t \mid \mathcal{Y}_T; \theta) = \sum_{j=1}^{2} \mathrm{p}(y_t \mid S_t = j, \mathcal{Y}_{t-1}; \theta) \times \Pr(S_t = j \mid \mathcal{Y}_{t-1}; \theta)$

The parameter estimates is as follows:

$$\beta^j = \left(\sum_{t=1}^{T} x_t x_t^{'} \times \mathrm{p}(S_t = j \mid \mathcal{Y}_T; \theta) \right)^{-1} \left(\sum_{t=1}^{T} x_t y_t \times \mathrm{p}(S_t = j \mid \mathcal{Y}_T; \theta) \right), \quad j = 1,2 \tag{11}$$

$$\sigma_j^2 = \frac{\sum_t (y_t - x_t^{'} \beta^j)^2 \times \mathrm{p}(S_t = j \mid \mathcal{Y}_T; \theta)}{\sum_t \mathrm{p}(S_t = j \mid \mathcal{Y}_T; \theta)}, \quad j = 1,2 \tag{12}$$

$$P_{jj} = \frac{\sum_t \mathrm{p}(S_t = j, S_{t-1} = j \mid \mathcal{Y}_T; \theta)}{\sum_t \mathrm{p}(S_{t-1} = j \mid \mathcal{Y}_T; \theta)}, \quad j = 1,2 \tag{13}$$

3.3.2 Volatility Forecasting Model

Corsi (2009) proposes HAR-RV model which utilizes linear regression of past daily, weekly and monthly realized volatility to capture the long-memory characteristic of future volatility. The implied volatility which extracted from option prices contains additional information for future volatility has suggested a new model. Busch et al. (2011) show that the volatility forecasting can be improved by taking the implied volatility into account and this model is simply regarded as HAR-RV-IV model. Recently, Byun and Kim (2013) investigate that risk-neutral skewness has incremental information content for future volatility and evolve a HAR-RV-IV-SK model.

[2] For detail, see Kim and Nelson (1999) chapter 4.

In this study, we set up the previous models with regime-switching and suggest a new model, HAR-RV-IV-IM, to improve the predictive accuracy of HAR-RV-IV model for future volatility. The five models are as follows:

HAR-RV model with regime-switching (MRS-HAR-RV):

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \varepsilon_{t,t+h}, \quad \varepsilon_{t,t+h} \sim \mathrm{N}(0, \sigma_{S_t}^2) \tag{14}$$

HAR-RV-IV model with regime-switching (MRS-HAR-RV-IV):

$$\begin{aligned} RV_{t,t+h} &= \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \varepsilon_{t,t+h}, \\ \varepsilon_{t,t+h} &\sim \mathrm{N}(0, \sigma_{S_t}^2) \end{aligned} \tag{15}$$

HAR-RV-IV-SK model with regime-switching (MRS-HAR-RV-IV-SK):

$$\begin{aligned} RV_{t,t+h} &= \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{SK}^{S_t} SKEW_t + \varepsilon_{t,t+h}, \\ \varepsilon_{t,t+h} &\sim \mathrm{N}(0, \sigma_{S_t}^2) \end{aligned} \tag{16}$$

HAR-RV-IV-IM model with regime-switching (MRS-HAR-RV-IV-IM):

$$\begin{aligned} RV_{t,t+h} &= \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{IM}^{S_t} reVIX_t + \varepsilon_{t,t+h}, \\ \varepsilon_{t,t+h} &\sim \mathrm{N}(0, \sigma_{S_t}^2) \\ \text{where, } reVIX_t &= VIX\_F_t - (\alpha + \beta VIX_t) \end{aligned} \tag{17}$$

HAR-RV-IV-IM-SK model with regime-switching (MRS-HAR-RV-IV-IM-SK):

$$\begin{aligned} RV_{t,t+h} &= \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{IM}^{S_t} reVIX_t \\ &\quad + \beta_{SK}^{S_t} SKEW_t + \varepsilon_{t,t+h}, \quad \varepsilon_{t,t+h} \sim \mathrm{N}(0, \sigma_{S_t}^2) \\ \text{where, } reVIX_t &= VIX\_F_t - (\alpha + \beta VIX_t) \end{aligned} \tag{18}$$

3.4 Out-of-sample Comparisons

In this Subsection, we illustrate the procedure to get the forecast values under regime-switching model. Then, we adopt the Diebold-Mariano test (DM) of Diebold and Mariano (1995), and the Weighted Likelihood Ratio test (WLR) of Amisano and Giacomini (2007) to compare the models in the out-of-sample analysis. DM test is a point forecast, based on the two competing loss functions. However, WLR test is a density forecast, which is an estimate of the probability distribution of a random variable, conditional on the realization of a variable at some future time.

3.4.1 Forecasting Procedures

In regime-switching model, there are three kinds of probabilities. First, the smoothed probability, $\Pr(S_t = j \mid \mathcal{Y}_T)$, utilizes all the information to infer the probability of each regime at time t. Second, the filtered probability, $\Pr(S_t = j \mid \mathcal{Y}_t)$, utilizes the information available up to time t to infer the probability of each regime at time t. Last, the prediction probability, $\Pr(S_t = j \mid \mathcal{Y}_{t-1})$, utilizes the information available up to time $t-1$ to infer the probability of each regime at time t.

Due to the unobservable state process S_t, we use filtered inference to infer the probability of each regime at each time $t+1$. The idea of filtered inference is that we obtain the prediction probability of each regime at time $t+1$ from matrix multiplication of filtered probability and transition matrix. Then, multiplying prediction probability of each regime by their conditional forecast, we could get the forecast value at time $t+1$. For example, the prediction probability in the regime 1 at time $t+1$ probably comes from two parts. First, in the regime 1 at time t and staying in the regime 1 at time $t+1$. Second, in the regime 2 at time t but changing to regime 1 at time $t+1$, that is $\Pr(S_{t+1} = 1 \mid \mathcal{Y}_t) = \Pr(S_t = 1 \mid \mathcal{Y}_t) \times P_{11} + \Pr(S_t = 2 \mid \mathcal{Y}_t) \times P_{21}$. Then, we can get the predicted probability of realized volatility in the regime 1 at time $t+1$ by multiplying the prediction probability in the regime 1 at time $t+1$ by the predicted realized volatility in the regime 1 at time $t+1$, that is, $\Pr(S_{t+1} = 1 \mid \mathcal{Y}_t) \times \widehat{RV}^{1}_{t+1}$. In the same manner, we could get the prediction probability and the predicted probability of realized volatility in the regime 2 at time $t+1$. Finally, we can obtain the predicted realized volatility at time $t+1$ by summing these two predicted probability of realized volatility. The procedures are in the following:

Step 1: The prediction probability of each regime at time $t+1$.

$$\Pr(S_{t+1} \mid \mathcal{Y}_t) = \begin{pmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{pmatrix} \begin{pmatrix} \Pr(S_t = 1 \mid \mathcal{Y}_t) \\ \Pr(S_t = 2 \mid \mathcal{Y}_t) \end{pmatrix} \tag{19}$$

Step 2: The forecast value at time $t+1$.

$$\widehat{RV}_{t+1} = \begin{pmatrix} \Pr(S_{t+1} = 1 \mid \mathcal{Y}_t) \\ \Pr(S_{t+1} = 2 \mid \mathcal{Y}_t) \end{pmatrix}' \begin{pmatrix} \widehat{RV}^{1}_{t+1} \\ \widehat{RV}^{2}_{t+1} \end{pmatrix} \tag{20}$$

3.4.2 Diebold-Mariano Test

We use three loss functions to measure the out-of-sample predictive power of the models, namely mean square errors (MSE), mean absolute errors (MAE) and quasi-likelihood errors (QLIKE).[3]

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\widehat{RV}_{t,t+h} - RV_{t,t+h})^2 \tag{21}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\widehat{RV}_{t,t+h} - RV_{t,t+h}\right| \tag{22}$$

$$QLIKE = \frac{1}{N}\sum_{i=1}^{N}(\log(RV_{t,t+h}) + \frac{\widehat{RV}_{t,t+h}}{RV_{t,t+h}}) \tag{23}$$

where $\widehat{RV}_{t,t+h}$ is the forecast value estimated by the forecasting model, $RV_{t,t+h}$ is the true value of the future realized volatility and N is the length of evaluation period.

DM test is to test if the two competing models have the same loss function. The test statistic is the following:

$$\begin{aligned} \mathrm{DM} &= \frac{\bar{d}}{\sqrt{\frac{\hat{g}}{N}}} \sim \mathrm{N}(0,1) \text{ as } N \to \infty, \\ \hat{g} &= \hat{\gamma}(0) + 2\sum_{k=1}^{h-1}\hat{\gamma}(k) \end{aligned} \tag{24}$$

[3] The QLIKE criterion proposed by Bollerslev et al. (1994) is the loss implied by a Gaussian likelihood.

where,

$$\bar{d} = \frac{1}{N}\sum_{t=1}^{N} d_t$$

$d_t = (loss\ function_i - loss\ function_j)$ represents the loss differential.

h is the forecast horizon.

$\hat{\gamma}(k)$ is the autocovariance of the loss differential at lag k and is the consistent estimator of $\gamma(k) = Cov(d_t, d_{t-k})$.

In this study, we set the null and alternative hypothesis as follows:

H_0 : DM = 0, which represents no difference between the model.

H_1 : DM > 0, which represents that the model j has better predictive power.

3.4.3 Weighted Likelihood Ratio Test

WLR test is based on a given weighted function $w(\cdot)$ and two alternative conditional density forecasts f and g for $RV_{t,t+h}$. The test is thus based on

$$WLR_{t,t+h} = w(\widetilde{RV}_{t,t+h})(\log \hat{f}(RV_{t,t+h}) - \log \hat{g}(RV_{t,t+h})) \tag{25}$$

where $\widetilde{RV}_{t,t+h}$ is the standardized observation computed on the same sample on which the density forecasts are estimated, $\hat{f}$ and $\hat{g}$ are the density forecasts of model i and model j and finally $w(\cdot)$ is a weighted function that allows forecaster to put greater weight on specific regions of the distribution of the variable. Note that a positive difference means model i has better predictive power than model j. Following Amisano and Giacomini (2007), a test for equal performance of h-steps-ahead density forecasts f and g can be formulated as a test of the hypothesis system.

H_0 : $E[\mathrm{WLR}_{t,t+h}] = 0$

H_1 : $E[\mathrm{WLR}_{t,t+h}] \neq 0$

The statistic is

$$t_h = \frac{\overline{WLR}_{t,t+h}}{\hat{\sigma}_{t,t+h} \big/ \sqrt{N}} \tag{26}$$

where,

$$\overline{WLR}_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N} WLR_{t,t+h}$$

$$\hat{\sigma}_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N} WLR_{t,t+h}^{2}$$

Note that the density forecast is $\hat{f} = \sum_{j=1}^{2} \phi_{(x_t'\hat{\beta}_t^j,\, \hat{\sigma}_j^2)} \times \Pr(S_{t+1} = j \mid \mathcal{Y}_t)$

where $\phi_{(\mu,\sigma^2)}$ is the probability density function of a normal with mean μ and variance σ^2 and $\hat{\beta}_t^j$, $\hat{\sigma}_j^2$ are MLE estimates at time t based on the information available up to time t.

4. Empirical Analysis

We analyze the ability of the forecasting models with Markov switching approach to forecast future volatility of S&P 500 index based on five-minute intraday return. In Section 4.1, we describe the data of S&P 500 realized volatility, VIX, SKEW and Residual of VIX, and the filtering rule of VIX futures to ensure the confidence in empirical analysis. We also illustrate the economic implication of Residual of VIX in this Section. In Section 4.2 and Section 4.3, we evaluate the predictive power of the VIX futures and risk-neutral skewness from the empirical results of the in- and out-of-sample, respectively. Last, in Section 4.4, we will compare the differences of predictive accuracy between the MRS-HAR models and the corresponding HAR models.

4.1 Data

In this study, data can be simply divided into three parts. First, the index, including the S&P 500 intraday index and the implied volatility index measure (VIX), are provided by Chicago Mercantile Exchange (CME) and Chicago Board of Options Exchange (CBOE), respectively. For the S&P 500 intraday index, the period is from 08:30 a.m. to 03:00 p.m. To construct the realized volatility, we divide the intraday data of S&P 500 index at five-minute frequency into 78 intra-daily return groups.[4] Second, SKEW, the risk-neutral skewness measure, is provided by Chicago Board of Options Exchange (CBOE). Last, VIX futures is obtained from Chicago Board of Options Exchange (CBOE). The data frequency is daily for VIX, SKEW and VIX futures, and we use the daily settlement prices for VIX futures. All of our data covers the period from 3 January 2006 to 31 October 2012, which consists of 1704 daily observations.

[4] We use five-minute sampling frequency in order to keep the balance of accuracy and avoid the microstructure problems.

We filter the VIX futures by the following rules to strengthen the reliability of the empirical results. First, trading volume less than five contracts are excluded. Second, we only consider the near-term contract and when time to maturity less than nine calendar days, move to the next-term contract.

Table 1 presents the summary statistics of the S&P 500 index daily realized volatility, VIX, SKEW and Residual of VIX. The average values of daily realized volatility and VIX are 1.03% and 1.20%, and the standard deviation are 0.76% and 0.57%, respectively. The skewness and excess kurtosis of daily realized volatility are 2.90 and 13.29, respectively. However, the skewness and excess kurtosis of VIX are 1.90 and 4.66, respectively. The daily realized volatility is more positive skewness than VIX and both daily realized volatility and VIX are leptokurtic. The minimum and maximum of daily realized volatility are 0.23% and 8.68%, and the minimum and maximum of VIX are 0.52% and 4.23%, respectively. For SKEW and Residual of VIX, the average value are 119.59 and 0.00, and standard deviation are 5.06 and 1.88, respectively. The skewness of SKEW and Residual of VIX are 0.29 and -1.85, and the excess kurtosis of SKEW and Residual of VIX are 0.21 and 12.11, respectively. The minimum and maximum of SKEW are 106.43 and 142.02, and the minimum and maximum of Residual of VIX are -16.67 and 6.45, respectively. The Ljung-Box test statistic of daily realized volatility, VIX, SKEW and Residual of VIX are all high and the corresponding p-value is zero at 0.01 significance level, which means the time series of these four variables are high serial correlation. Lastly, Augmented Dickey-Fuller test statistic of these four variables are all significant which means they are stationary process.[5]

Figure 1 exhibits three time series plots during the whole sample period, from 3 January 2006 to 31 October 2012. First is the time series of daily realized volatility and

[5] The ADF test statistic of weekly and monthly realized volatility are -3.6062 and -3.8857, respectively. These variables are all significant at 5% which implies they are stationary process.

VIX; Second is the time series of the difference between daily realized volatility and VIX (RV-VIX) and SKEW; Last is the time series of RV-VIX and Residual of VIX. From the first plot, we can observe that both volatility track each other considerable closely. From the second and third plot, we could observe that SKEW and Residual of VIX move to the opposite direction against RV-VIX simultaneously when RV-VIX has large movement.[6]

Figure 2 displays the economic implication of Residual of VIX during the whole sample period, from 3 January 2006 to 31 October 2012. First is the time series of daily VIX and VIX futures; Second is the time series of RV-VIX and Residual of VIX; Last is the time series of RV-VIX and Spread.[7] From the first plot, we could observe that the values of VIX futures generally below the values of VIX, especially during the Financial Crisis.[8] This may imply that the investors actively take positions in the VIX futures to hedge volatility rather than trading in the S&P 500 index options when the market undergoes severe volatile. Interestingly, we can observe that the path of Residual of VIX and Spread are analogous from the second and third plots.[9] This may suggest that the economic implication of Residual of VIX is similar to Spread. However, the Residual of VIX is uncorrelated to VIX while Spread is highly correlated to VIX. Therefore, the Residual of VIX, relative to VIX, contains incremental information for future volatility forecasting.

[6] Here, we roughly regard the Financial Crisis as large movement or the periods of high volatility state. During the whole sample period, the correlation between the difference of daily realized volatility and VIX and SKEW (*corr*(*RV-VIX*, *SKEW*)) is -0.1454. The correlation between the difference of daily realized volatility and VIX and Residual of VIX (*corr*(*RV-VIX*, *reVIX*)) is -0.4514. During the Financial Crisis, *corr*(*RV-VIX*, *SKEW*) is -0.2394 and *corr*(*RV-VIX*, *reVIX*) is -0.4789.

[7] Spread is defined as the differences between VIX futures and VIX.

[8] The proportion of VIX futures below the VIX is 0.2411 during the whole sample period, and the proportion is 0.3780 during the Financial Crisis.

[9] During the whole sample period, *corr*(*RV-VIX*, *Spread*) is -0.4847 and *corr*(*reVIX*, *Spread*) is 0.8076. During the Financial Crisis, *corr*(*RV-VIX*, *Spread*) is -0.5080 and *corr*(*reVIX*, *Spread*) is 0.8859.

Table 1: Summary Statistics of Daily Realized Volatility, VIX, SKEW and Residual of VIX

The table reports the summary statistics of the S&P 500 index daily realized volatility, VIX, SKEW and Residual of VIX from 3 January 2006 to 31 October 2012. Note: (1) The VIX here is daily VIX which is calculated by VIX divided by square root of 365. (2) LB test for lag=10 represents Ljung-Box test statistic for ten lags serial correlation. (3) ADF test represents Augmented Dickey-Fuller test statistic for testing the stationary process. (4) ** and *** denote Significant at 5% and 1%, respectively.

Statistics	Realized Volatility	VIX	SKEW	Residual of VIX
Mean	1.03%	1.20%	119.59	0.00
Std. dev.	0.76%	0.57%	5.06	1.88
Skewness	2.90	1.90	0.29	-1.85
Excess Kurtosis	13.29	4.66	0.21	12.11
Min	0.23%	0.52%	106.43	-16.67
Max	8.68%	4.23%	142.02	6.45
LB test for lag=10	8877.91***	15032.88***	7769.18***	7639.25***
ADF test	-3.9586**	-3.5955**	-5.3084***	-5.8037***

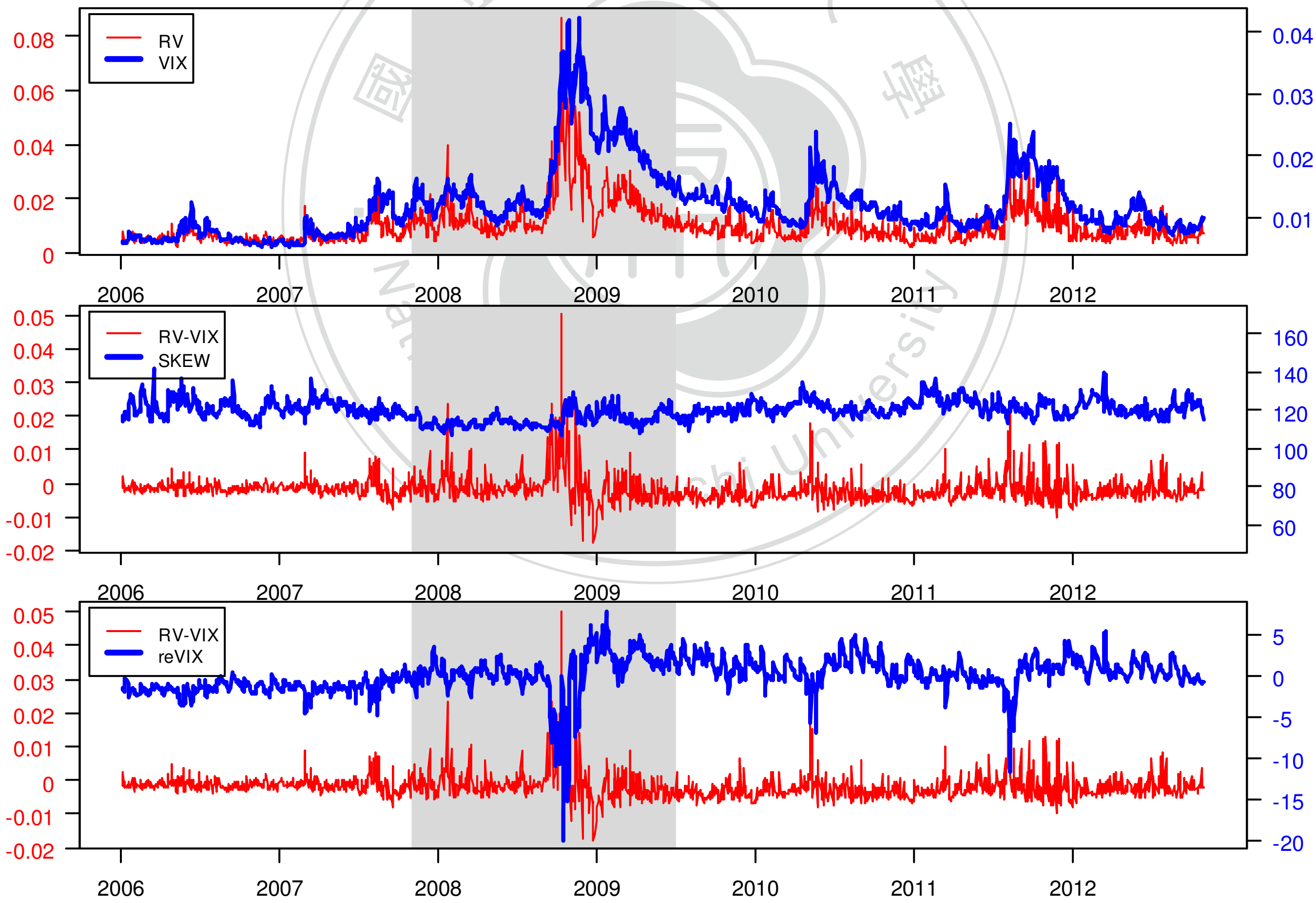

Figure 1: Time series of daily realized volatility, VIX, SKEW and Residual of VIX.

Figure 1 exhibits three time series plots from 3 January 2006 to 31 October 2012. First is the time series of daily realized volatility (left scale) and VIX (right scale); Second is the time series of the difference between daily realized volatility and VIX (RV-VIX) (left scale) and SKEW (right scale); Third is the time series of RV-VIX (left scale) and Residual of VIX (right scale). Gray area denotes the Financial Crisis from November 2007 to June 2009. (Here, we roughly regard the Financial Crisis as the periods of high volatility state.)

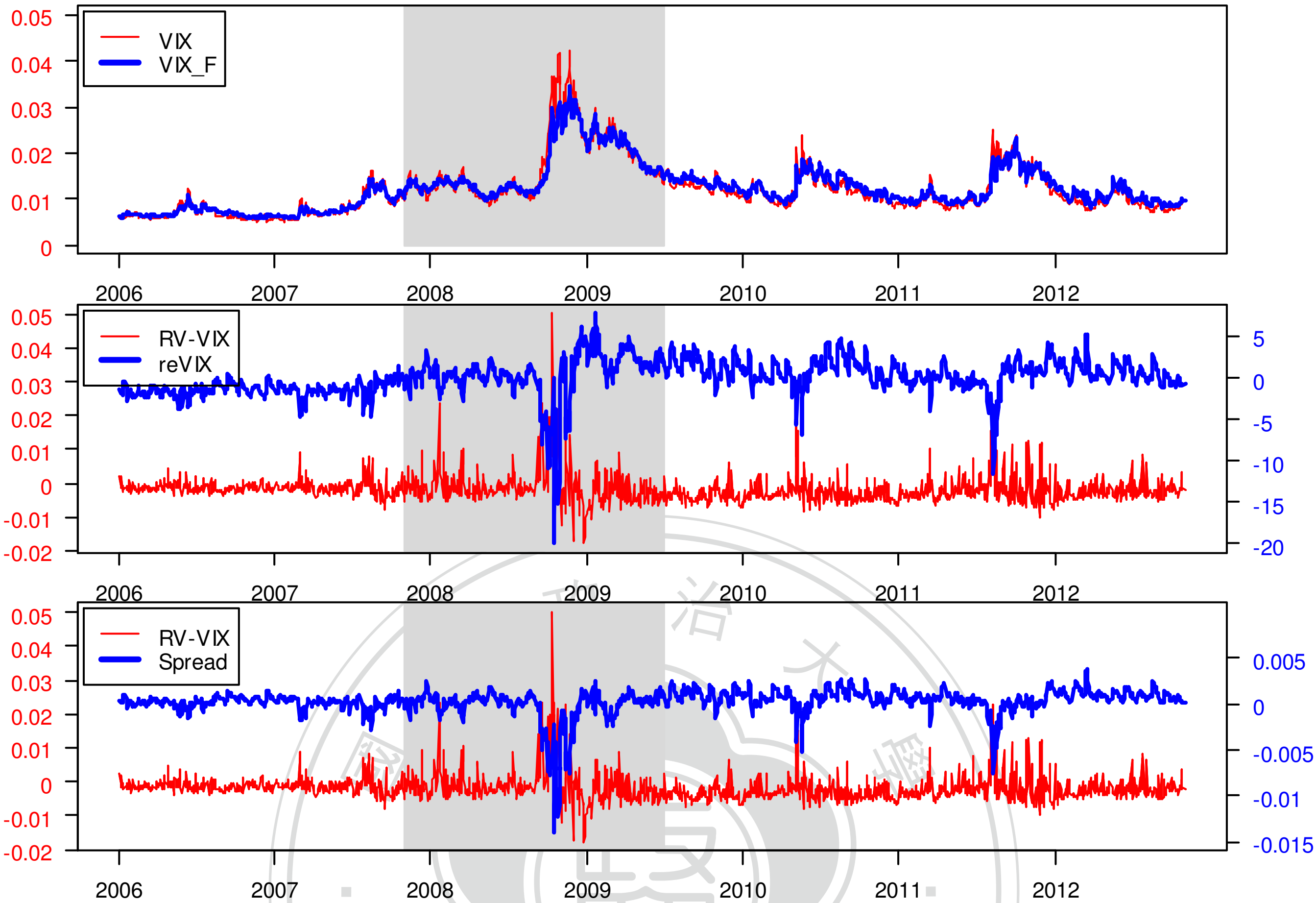

Figure 2: Time series of daily VIX, VIX futures, Residual of VIX and Spread.

Figure 2 exhibits three time series plots from 3 January 2006 to 31 October 2012. First is the time series of daily VIX and VIX futures; Second is the time series of the difference between daily realized volatility and VIX (RV-VIX) (left scale) and Residual of VIX (right scale); Third is the time series of RV-VIX (left scale) and Spread (right scale). Gray area denotes the Financial Crisis from November 2007 to June 2009 and Spread denotes the difference of VIX futures and VIX. (Here, we roughly regard the Financial Crisis as the periods of high volatility state.)

4.2 In-sample Performance

To investigate if there exists the information content of Residual of VIX and risk-neutral skewness for future volatility forecasting, the following regression models with Markov regime-switching are adopted:

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \varepsilon_{t,t+h} \quad \text{(M0)}$$

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \varepsilon_{t,t+h} \quad \text{(M1)}$$

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{SK}^{S_t} SKEW_t + \varepsilon_{t,t+h} \quad \text{(M2)}$$

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{IM}^{S_t} reVIX_t + \varepsilon_{t,t+h} \quad \text{(M3)}$$

$$RV_{t,t+h} = \beta_0^{S_t} + \beta_D^{S_t} RV_{t-1,t} + \beta_W^{S_t} RV_{t-5,t} + \beta_M^{S_t} RV_{t-22,t} + \beta_{IV}^{S_t} VIX_t + \beta_{IM}^{S_t} reVIX_t + \beta_{SK}^{S_t} SKEW_t + \varepsilon_{t,t+h} \quad \text{(M4)}$$

Table 2 presents the coefficient estimates from (M0), (M1), (M2), (M3) and (M4).[10] The estimates of β_{SK} confirm the effectiveness of risk-neutral skewness for different time horizons. In the low volatility state, the size and the significance of β_{SK} decrease with the forecast horizon, and the coefficient is negative and highly significant. However, in the high volatility state, the size and the significance of β_{SK} increase with the forecast horizon, and the coefficient is negative and highly significant except the daily forecast horizon. The coefficient of β_{SK} is larger in the high volatility state than low volatility state for all forecast horizons. These results are consistent with Byun and Kim (2013), which suggest that the risk-neutral skewness has more explanatory power in the short-term regression than the long-term regression and is more important, especially in the high volatility state.

The estimates of β_{IM} verify the effectiveness of Residual of VIX in different time horizons. In the low volatility state, the size β_{IM} decreases with the forecast horizon, but the coefficient is significant for all forecast horizons. On the other hand, in the high volatility state, the size and the significance of β_{IM} increase with the forecast horizon, and the coefficient is negative and high significant for all forecast horizons. In addition, the estimates of β_{IM} is significant in the high volatility state at monthly regression, as compared with the estimates of β_{IV}. These results imply that VIX futures somewhat has information content for future volatility forecasting, especially in the high volatility state.

Regarding the average log likelihood, the MRS-HAR-RV-IV-IM model outperforms the MRS-HAR-RV-IV model, stressing the relevance of Residual of VIX as a predictor.

[10] To clearly report the coefficient estimates of each regression, the realized volatility is scaled by 100 times the square root of 252 of the original realized volatility and VIX is scaled by the square root of 252/365 of the original VIX.

Besides, the MRS-HAR-RV-IV-IM-SK model outperforms the others. Last, in the each model, both of the probability of staying in the regime 1 and regime 2 increase with the forecast horizon, however, the magnitude is larger in the regime 2 than regime 1. This implies that it is not easy to change regimes, especially in the monthly time horizon and high volatility state. On the other hand, the expected duration of regime 1 and regime 2 generally decrease from Model 0 to Model 4 under different forecast horizon.

Figure 3 plots daily realized volatility and estimated smoothed probability of regime 2 (i.e., $\Pr(S_t = 2 \mid \mathcal{Y}_T)$) from 3 January 2006 to 31 October 2012. All the models provide similar regime estimates, detecting the high volatility from November 2007 to June 2009. Apart from the MRS-HAR-RV model, the other models detect the high volatility on 27 April 2006.

Table 2: In-sample Performance Result for Future Volatility of MRS-HAR Models

The table presents the estimation result of risk-neutral skewness and Residual of VIX for future volatility. The sample period is from 3 January 2006 to 31 October 2012, for a total 1704 daily observations. The specification for M0: $RV_{t,t+h} = \beta_0^{St} + \beta_D^{St}RV_{t-1,t} + \beta_W^{St}RV_{t-5,t} + \beta_M^{St}RV_{t-22,t} + \varepsilon_{t,t+h}$; M1: $RV_{t,t+h} = \beta_0^{St} + \beta_D^{St}RV_{t-1,t} + \beta_W^{St}RV_{t-5,t} + \beta_M^{St}RV_{t-22,t} + \beta_{IV}^{St}VIX_t + \varepsilon_{t,t+h}$; M2: $RV_{t,t+h} = \beta_0^{St} + \beta_D^{St}RV_{t-1,t} + \beta_W^{St}RV_{t-5,t} + \beta_M^{St}RV_{t-22,t} + \beta_{IV}^{St}VIX_t + \beta_{SK}^{St}SKEW_t + \varepsilon_{t,t+h}$; M3: $RV_{t,t+h} = \beta_0^{St} + \beta_D^{St}RV_{t-1,t} + \beta_W^{St}RV_{t-5,t} + \beta_M^{St}RV_{t-22,t} + \beta_{IV}^{St}VIX_t + \beta_{IM}^{St}reVIX_t + \varepsilon_{t,t+h}$; M4: $RV_{t,t+h} = \beta_0^{St} + \beta_D^{St}RV_{t-1,t} + \beta_W^{St}RV_{t-5,t} + \beta_M^{St}RV_{t-22,t} + \beta_{IV}^{St}VIX_t + \beta_{IM}^{St}reVIX_t + \beta_{SK}^{St}SKEW_t + \varepsilon_{t,t+h}$. $reVIX_t$ represents the Residual of VIX which is the residuals obtained from regressing VIX futures on VIX. For each model, the left column is the estimated coefficient in the regime 1 and the right column is in the regime 2. P_{ii} indicates the probability of staying in regime I and all the P_{ii} are significant at 1%. Duration indicates the expected duration of each regime which is calculated by 1/ (1-P_{ii}) and the unit is days. Last, Log likelihood for each model is reported in last row. Note that the parentheses is t-statistics and *, ** and *** denote Significant at 10%, 5% and 1%, respectively.

Panel A : Daily forecast horizon

Model	M0		M1		M2		M3		M4	
β_0^{St}	2.032***	6.299***	-0.018	0.382	11.744***	14.902	-1.350***	2.035	9.578***	28.411*
	(9.151)	(4.557)	(-0.156)	(0.114)	(4.859)	(0.993)	(-4.582)	(1.109)	(4.392)	(1.898)
β_D^{St}	0.238***	0.348***	0.124***	0.199**	0.118***	0.192**	0.093***	0.131	0.089***	0.132
	(6.873)	(5.021)	(4.666)	(2.654)	(3.919)	(2.351)	(3.577)	(1.533)	(3.582)	(1.501)
β_W^{St}	0.297***	0.304***	0.216***	0.335***	0.196***	0.359***	0.126***	0.209	0.115***	0.201
	(6.637)	(2.743)	(6.870)	(2.879)	(4.710)	(2.615)	(2.974)	(1.407)	(3.289)	(1.425)
β_M^{St}	0.231***	0.195**	-0.007	-0.373***	-0.022	-0.382***	0.021	-0.274**	0.002	-0.262**
	(7.932)	(2.101)	(-0.067)	(-7.272)	(-0.628)	(-3.177)	(0.606)	(-2.245)	(0.147)	(-2.179)
β_{IV}^{St}			0.481***	0.908***	0.496***	0.889***	0.636***	0.949***	0.653***	0.935***
			(8.256)	(5.992)	(12.124)	(5.644)	(14.451)	(5.880)	(17.211)	(5.892)
β_{IM}^{St}							-0.583***	-1.283***	-0.599***	-1.331***
							(-8.088)	(-4.906)	(-8.122)	(-5.102)
β_{SK}^{St}					-0.095***	-0.121			-0.089***	-0.221*
					(-4.908)	(-0.958)			(-5.075)	(-1.770)
σ_{St}^2	8.822	132.815	8.023	116.770	7.679	113.852	7.414	106.544	7.203	104.455
	(26.642)	(15.568)	(26.801)	(15.426)	(26.776)	(15.669)	(26.941)	(15.580)	(26.981)	(15.731)
P_{ii}	0.947	0.814	0.942	0.783	0.937	0.773	0.927	0.721	0.922	0.703
Duration	19.03	5.37	17.19	4.61	15.97	4.41	13.61	3.58	12.74	3.36
Log. Lik.	-5016.090		-4924.658		-4911.425		-4884.618		-4869.969	

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Panel B : Weekly forecast horizon										
Model	M0		M1		M2		M3		M4	
β_0^{St}	2.752***	8.526***	0.611***	3.775***	4.369**	46.323***	0.424*	4.348***	4.200**	49.990***
	(17.089)	(7.439)	(3.232)	(2.794)	(2.485)	(4.054)	(1.916)	(3.046)	(2.382)	(4.706)
β_D^{St}	0.087***	0.304***	0.023	0.215***	0.026	0.201***	0.020	0.144**	0.020	0.127**
	(4.188)	(5.276)	(1.369)	(3.648)	(1.540)	(3.511)	(1.161)	(2.530)	(1.179)	(2.298)
β_W^{St}	0.275***	0.315***	0.182***	0.398***	0.175***	0.346***	0.164***	0.232***	0.162***	0.183**
	(9.721)	(3.195)	(7.708)	(4.505)	(7.275)	(3.922)	(6.800)	(2.652)	(6.733)	(2.129)
β_M^{St}	0.361***	0.195**	0.146***	-0.200**	0.138***	-0.142	0.154***	-0.103	0.147***	-0.047
	(15.740)	(2.533)	(6.081)	(-2.040)	(5.621)	(-1.462)	(6.395)	(-1.120)	(6.010)	(-0.504)
β_{IV}^{St}			0.444***	0.575***	0.449***	0.568***	0.466***	0.676***	0.467***	0.666***
			(16.290)	(4.471)	(16.227)	(4.555)	(15.703)	(5.441)	(15.715)	(5.535)
β_{IM}^{St}							-0.104*	-1.127***	-0.098*	-1.143***
							(-1.932)	(-5.337)	(-1.915)	(-5.857)
β_{SK}^{St}					-0.031**	-0.358***			-0.031**	-0.384***
					(-2.150)	(-3.753)			(-2.151)	(-4.352)
σ_{St}^2	4.866	86.753	4.027	78.218	3.979	75.229	3.976	70.870	3.850	66.960
	(26.228)	(14.398)	(26.281)	(14.473)	(26.235)	(14.605)	(26.254)	(14.626)	(26.137)	(14.900)
P_{ii}	0.976	0.909	0.970	0.887	0.970	0.890	0.969	0.886	0.968	0.889
Duration	40.82	10.96	33.11	8.88	33.25	9.11	32.14	8.80	31.30	8.99
Log. Lik.	-4454.256		-4336.161		-4326.934		-4320.424		-4308.799	

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Panel C : Monthly forecast horizon										
Model	M0		M1		M2		M3		M4	
β_0^{St}	3.882***	13.940***	2.035***	12.057***	4.009***	56.356***	1.318***	13.732***	4.472***	59.772***
	(25.271)	(16.005)	(12.400)	(10.198)	(4.775)	(6.237)	(5.201)	(10.943)	(2.756)	(6.930)
β_D^{St}	0.064***	0.147**	-0.001	0.212***	0.000	0.186***	0.000	-0.055	0.012	0.106*
	(4.512)	(2.504)	(-0.142)	(3.596)	(-0.011)	(3.244)	(0.005)	(-0.921)	(0.833)	(1.848)
β_W^{St}	0.214***	0.437***	0.165***	0.354***	0.162***	0.334***	0.204***	0.480***	0.185***	0.200***
	(9.326)	(5.149)	(7.404)	(4.363)	(7.696)	(4.326)	(9.747)	(8.049)	(8.213)	(2.605)
β_M^{St}	0.355***	0.083	0.076***	0.221**	0.076***	0.242***	0.151***	0.106	0.061***	0.287***
	(16.541)	(1.331)	(3.204)	(2.539)	(3.353)	(2.830)	(5.824)	(1.273)	(2.655)	(3.493)
β_{IV}^{St}			0.440***	-0.107	0.434***	-0.089	0.396***	0.163	0.397***	0.051
			(19.209)	(-0.913)	(18.757)	(-0.772)	(14.971)	(1.259)	(15.795)	(0.421)
β_{IM}^{St}							-0.229***	-2.066***	-0.163***	-1.025***
							(-5.140)	(-8.284)	(-3.315)	(-5.684)
β_{SK}^{St}					-0.016**	-0.373***			-0.017	-0.399***
					(-2.393)	(-4.975)			(-1.295)	(-5.567)
σ_{St}^2	4.529	75.738	2.832	78.162	2.833	73.566	4.417	60.170	2.809	68.567
	(25.498)	(14.585)	(24.722)	(16.177)	(24.704)	(16.219)	(25.799)	(14.040)	(24.654)	(16.189)
P_{ii}	0.988	0.962	0.983	0.959	0.983	0.959	0.987	0.954	0.984	0.961
Duration	83.95	26.49	58.47	24.10	58.55	24.29	76.89	21.86	61.27	25.53
Log. Lik.	-4330.345		-4204.835		-4192.623		-4241.768		-4170.533	

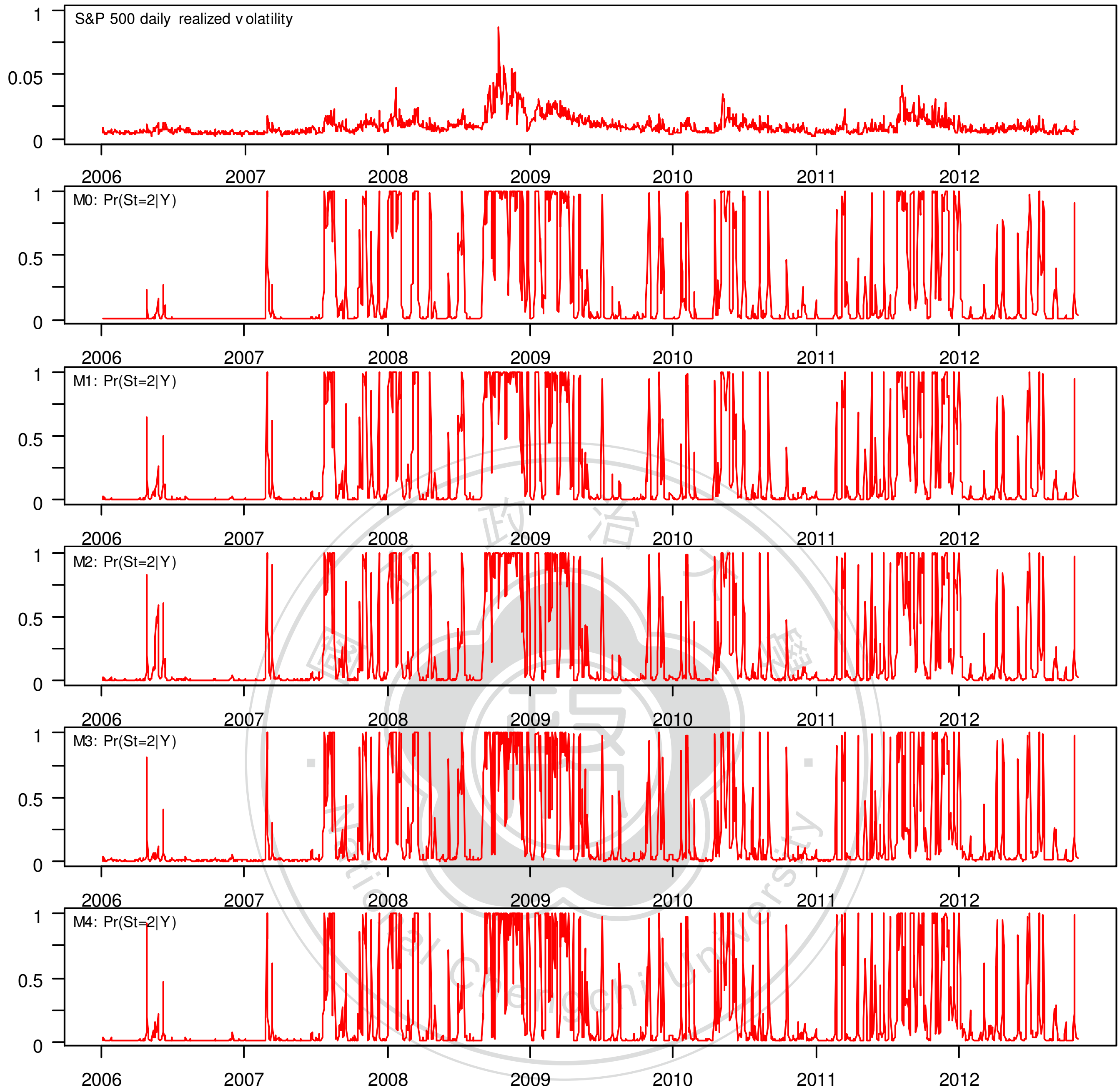

Figure 3: Daily realized volatility and estimated smoothed probability of $\Pr(S_t = 2|\mathcal{Y}_T)$.

Figure 3 plots the daily realized volatility and estimated smoothed probability of $\Pr(S_t = 2|\mathcal{Y}_T)$ from 3 January 2006 to 31 October 2012. The first panel is the S&P 500 daily realized volatility and from the second to the last panels are $\widehat{\Pr}(S_t = 2|\mathcal{Y}_T)$ for all Markov switching model discussed in Section 3.

4.3 Out-of-sample Forecasting Performance

In Section 4.2, we report the results of the in-sample analysis with the time-varying forecasting performance of the information content of VIX futures and risk-neutral skewness based on the level of market volatile. In this Section, we use a re-estimate procedure to estimate the coefficient of the forecasting model. After that, we would use the loss functions to evaluate the accuracy of the forecasts.

We obtain the out-of-sample forecasts of future volatility from the estimates with an increasing window scheme. First of all, we use the first $m = 1200$ observations to initialize the models. Then, we re-estimate the forecasting model at each day t conditional on all the observations available up to day $t-1$.[11] That is, the first h-steps-ahead forecasts is based on $(y_1, y_2, ..., y_m)$, the second h-steps-ahead forecasts is based on $(y_1, y_2, ..., y_{m+1})$, until the last one, which is based on $(y_1, y_2, ..., y_{T-h})$. We could obtain a sequence of $N_h = T - m - h + 1$ forecasts. In our study, the full sample contains $T = 1704$ observations, and the forecast horizon $h = 1,\ 5,\ 22$ for daily, weekly and monthly forecast, respectively, leading to a sequence of $N_h = 504,\ 500$ and 483 forecast values.[12]

Table 3 reports some classic loss functions of forecasting accuracy, namely RMSE[13], MAE and QLIKE. It is evident that the MRS-HAR-RV-IV-IM model outperforms the MRS-HAR-RV-IV model from daily to monthly forecast horizon. In this case, VIX futures seems to provide incremental information to forecast future volatility. Similarly, the MRS-HAR-RV-SK model outperforms the MRS-HAR-RV-IV model at all forecast horizons except the RMSE criteria at daily horizon. Furthermore, the MRS-HAR-RV-IV-IM-SK model provides good forecasts at all forecast horizons. However, at monthly

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[11] We also specify the Residual of VIX with an increasing window scheme.

[12] Since we use the first 1200 daily observations to estimate the parameter of the model, the sample period of out-of-sample analysis is from 13 October 2010 to 31 October 2012.

[13] To clearly report the results, we adopt RMSE instead of MSE, and RMSE is the square root of MSE.

forecast horizon, the MRS-HAR-RV model has the best forecasts. Last, we observe that the MRS-HAR-RV-IV-IM model outperforms the MRS-HAR-RV-IV-SK model in RMSE and MAE criteria at all forecast horizons, while in QLIKE criteria, the MRS-HAR-RV-IV-IM model inferiors to the MRS-HAR-RV-IV-SK model.

Table 3: RMSE, MAE and QLIKE for MRS-HAR Models

The table reports the 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecast errors of different models during the whole sample period from 3 January 2006 to 31 October 2012. Three loss functions are adopted, namely RMSE, MAE and QLIKE. Note that RMSE is the square root of MSE.

	Daily			Weekly			Monthly		
	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE
Model 0	6.267	4.428	3.688	4.390	2.884	3.663	3.670	2.591	3.667
Model 1	5.835	4.138	3.677	4.141	2.694	3.674	4.291	2.733	3.728
Model 2	5.846	4.086	3.651	4.080	2.677	3.654	4.216	2.681	3.705
Model 3	5.736	4.080	3.662	4.070	2.644	3.669	4.164	2.674	3.711
Model 4	5.741	4.022	3.630	3.999	2.634	3.648	4.084	2.629	3.706

In addition, we adopt the Diebold-Mariano test of Diebold and Mariano (1995), and the Weighted Likelihood Ratio test of Amisano and Giacomini (2007) to compare the predictive power between the models. The Diebold-Mariano test has the null hypothesis of no difference between two competing models and reports the t-statistics to show the significance of predictive accuracy. If the t-statistic of two competing model is positive, the forecast error of the former forecasting model is smaller than that of the latter one. Also in the same manner, If the t-statistic of two competing model is negative, the forecast error of the former forecasting model is larger than that of the latter one. The Weighted Likelihood Ratio test has the null hypothesis of no difference in the expectation of two weighted loss function and presents p-value to show the significance of predictive accuracy. A positive value of the test indicates that the former forecasting model provides a superior predictive accuracy with respect to the later forecasting model.

Table 4 reports the result for the out-of-sample forecasts of future volatility. Compared to the MRS-HAR-RV-IV model, the Diebold-Mariano test shows that the

MRS-HAR-RV-IV-IM model has significant improvement for all forecast horizons, except the MAE and QLIKE criteria at the weekly forecast horizon. In addition, the Diebold-Mariano test also shows that the improvement of the MRS-HAR-RV-IV-IM model in QLIKE criteria for the daily forecast horizon is significant at 1%, and in MAE and QLIKE criteria for the monthly forecast horizon is significant at 5%. Furthermore, the Diebold-Mariano test shows that the MRS-HAR-RV-IV-IM-SK model has significant improvement at daily and weekly forecast horizons with respect to the other models, except the MRS-HAR-RV-IV-SK model at weekly forecast horizon. Interestingly, the Diebold-Mariano test shows that the MRS-HAR-RV-IV-IM-SK model in QLIKE criteria for the daily forecast horizon is significant at 1%. Similar to the results of Byun and Kim (2013), the Diebold-Mariano test shows the improvement in the MRS-HAR-RV-IV-SK model is significant at all forecast horizons, as compared to the MRS-HAR-RV and MRS-HAR-RV-IV model, except the MRS-HAR-RV model at monthly forecast horizon. Finally, compared to the MRS-HAR-RV model, the Diebold-Mariano test shows that all the models (M1-M4) do not have significant improvement at the monthly forecast horizon.

The Weighted Likelihood Ratio test reveals that the MRS-HAR-RV-IV-IM model has superior predictive accuracy than the MRS-HAR-RV-IV model at the daily forecast horizon. Besides, the Weighted Likelihood Ratio test provides equivalent performance of the MRS-HAR-RV-IV model and MRS-HAR-RV-IV-IM model at the weekly and monthly forecast horizon. Regarding to the MRS-HAR-RV-IV-IM-SK model, at the daily forecast horizon, the Weighted Likelihood Ratio test reveals that the MRS-HAR-RV-IV-IM-SK model outperforms the MRS-HAR-RV and MRS-HAR-RV-IV models. Furthermore, the MRS-HAR-RV model is outperformed by the MRS-HAR-RV-IV-IM-SK model at the weekly forecast horizon. However, at monthly forecast horizon, the MRS-HAR-RV-IV-IM-SK model has equivalent performance with other models.

Based on the results of the Diebold-Mariano test and the Weighted Likelihood Ratio test, we could confirm the significant improvement in the effectiveness of the volatility forecasting model after incorporating the information content of VIX futures. In addition, we could also confirm the significant improvement in the effectiveness of the daily volatility forecasting model after incorporating the information content of VIX futures and risk-neutral skewness. Last, analogous to the results of Byun and Kim (2013), the MRS-HAR-RV-IV-SK model outperforms the MRS-HAR-RV model and the MRS-HAR-RV-IV model.

Table 4: Out-of-sample Forecasting Performance of MRS-HAR Models

The table shows the comparison of out-of-sample forecasting performance for 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecasts of different models during the whole sample period from 3 January 2006 to 31 October 2012. In panel A, the t-statistic of the Diebold-Mariano test based on different loss function is reported. In panel B, for each forecast horizon, the first column is the t-statistic of the Weighted Likelihood Ratio test and the second column is the corresponded p-value are reported. Note that *, ** and *** denote Significant at 10%, 5% and 1%, respectively.

Panel A: Diebold-Mariano test

	Daily			Weekly			Monthly		
	MSE	MAE	QLIKE	MSE	MAE	QLIKE	MSE	MAE	QLIKE
M1 vs. M0	3.944***	4.883***	2.063**	2.006**	1.931**	-0.782	-1.388	-0.515	-1.823
M2 vs. M0	3.530***	5.312***	6.793***	2.503***	2.209**	0.720	-1.282	-0.415	-1.298
M3 vs. M0	3.577**	4.254***	3.582***	1.646*	2.238**	-0.443	-1.116	-0.301	-1.251
M4 vs. M0	3.355***	4.625***	7.728***	2.514***	2.424***	1.244	-1.192	-0.178	-1.353
M2 vs. M1	-0.353	2.177**	11.287***	1.034	0.413	3.699***	0.412	0.507	1.798**
M3 vs. M1	1.427*	1.394*	4.351***	1.445*	0.843	0.976	1.389*	2.244**	1.880**
M4 vs. M1	1.308*	2.296**	10.108***	1.453*	1.012	3.170***	1.889**	1.109	1.442*
M3 vs. M2	1.329*	0.136	-3.113	0.055	0.460	-2.693	0.193	0.063	-0.307
M4 vs. M2	1.434*	1.465*	5.977***	0.776	0.948	1.249	1.161	1.346*	-0.285
M4 vs. M3	-0.133	2.080**	12.227***	0.663	0.219	5.042***	0.455	0.446	0.228

Panel B: Weighted Likelihood Ratio test

	Daily		Weekly		Monthly	
	WLR	P-value	WLR	P-value	WLR	P-value
M1 vs. M0	3.652	<0.000	2.585	0.010	1.209	0.227
M2 vs. M0	4.583	<0.000	3.126	0.002	0.982	0.326
M3 vs. M0	3.889	<0.000	2.180	0.029	0.109	0.913
M4 vs. M0	3.870	<0.000	2.210	0.027	1.122	0.262
M2 vs. M1	2.150	0.032	1.843	0.065	-0.528	0.598
M3 vs. M1	2.320	0.020	-0.942	0.346	-2.581	0.010
M4 vs. M1	2.149	0.032	-0.480	0.632	-0.339	0.735
M3 vs. M2	0.537	0.592	-1.920	0.055	-1.707	0.088
M4 vs. M2	1.344	0.179	-1.913	0.056	0.195	0.846
M4 vs. M3	0.973	0.331	0.250	0.802	1.746	0.081

4.4 Comparison between MRS-HAR Models and HAR Models

In this Subsection, we compare the MRS-HAR models with HAR models. In order to investigate the improvement of forecasting accuracy after considering Markov regime-switching approach, the following HAR models are adopted:

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \varepsilon_{t,t+h} \tag{m0}$$

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_{IV} VIX_t + \varepsilon_{t,t+h} \tag{m1}$$

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_{IV} VIX_t + \beta_{SK} SKEW_t + \varepsilon_{t,t+h} \tag{m2}$$

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_{IV} VIX_t + \beta_{IM} reVIX_t + \varepsilon_{t,t+h} \tag{m3}$$

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_{IV} VIX_t + \beta_{IM} reVIX_t + \beta_{SK} SKEW_t + \varepsilon_{t,t+h} \tag{m4}$$

Table 5 reports the differences of some classic loss functions of forecasting accuracy between MRS-HAR and HAR models.[14] The negative value implies the MRS-HAR model has less forecast error than the corresponding HAR model. It is evident that the MRS-HAR models generally outperform the HAR models at weekly and monthly horizons which implies the improvement of forecasting accuracy after taking the regime-switching into consideration. However, at the daily horizon, the forecast error of the HAR models are typically smaller than the MRS-HAR models. One of the possible reasons is the instability of parameter estimates. Dacco and Satchell (1999) mention that if the change between regimes is frequently, it may lead to the regime misclassification for out-of-sample forecasting.

[14] For the detail of loss functions of forecasting accuracy of HAR models, see Appendix B.

Table 5: Comparison of RMSE, MAE and QLIKE for MRS-HAR Models and HAR Models

The table reports the differences of 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecast errors of different models during the whole sample period from 3 January 2006 to 31 October 2012. Three loss functions are adopted, namely RMSE, MAE and QLIKE. Note that RMSE is the square root of MSE. The negative value implies the MRS-HAR model has less forecast error than the corresponding HAR model.

	Daily			Weekly			Monthly		
	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE
M0-m0	0.134	0.189	0.033	-0.430	-0.277	-0.010	-1.303	-0.546	-0.079
M1-m1	-0.015	0.071	0.020	-0.522	-0.415	-0.002	-0.653	-0.415	-0.020
M2-m2	0.008	0.069	0.017	-0.560	-0.398	0.005	-0.817	-0.537	-0.010
M3-m1	-0.114	0.013	0.006	-0.592	-0.465	-0.007	-0.779	-0.474	-0.037
M3-m3	-0.072	0.068	0.034	-0.548	-0.473	0.013	-0.642	-0.366	-0.021
M4-m4	-0.076	0.063	0.040	-0.624	-0.478	0.031	-0.852	-0.545	0.016

Table 6 reports the result for the out-of-sample forecasts of future volatility about the comparison between the MRS-HAR models and the HAR models.[15] The Diebold-Mariano test shows the significant improvement for all MRS-HAR models at the weekly horizon, except the QLIKE criteria. Furthermore, at the monthly horizon, the Diebold-Mariano test shows the improvement of MRS-HAR-RV model in MSE and MAE criteria are significant at 10%, and in QLIKE criteria is significant at 1%. Besides, the Diebold-Mariano test also shows the improvement of MRS-HAR-RV-IV-SK model in MAE criteria is significant at 10%, and MRS-HAR-RV-IV-IM-SK model in MAE criteria is significant at 5%, as compared to the corresponding HAR model. However, at the daily horizon, the MRS-HAR models do not significantly outperform the corresponding HAR models.[16]

The Weighted Likelihood Ratio test revels the MRS-HAR models have superior predictive accuracy than the corresponding HAR models at all forecast horizons. In addition, the Weighted Likelihood Ratio test also provides the improvement of the MRS-HAR models are significant at 1% in all forecast horizons, except the MRS-HAR-RV-IV-IM-SK models in monthly forecast horizon is significant at 5%.

[15] For the detail of out-of-sample forecasting performance of HAR models, see Appendix C.

[16] As we mentioned, this could be caused by the instability of parameter estimates.

Based on the results of the Diebold-Mariano test and the Weighted Likelihood Ratio test, we could confirm the significant improvement in the effectiveness of the volatility forecasting model after taking the regime-switching into consideration.

Table 6: Comparison of Out-of-sample Forecasting Performance for MRS-HAR Models and HAR Models

The table shows the comparison of out-of-sample forecasting performance for 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecasts of MRS-HAR models and corresponding HAR models during the whole sample period from 3 January 2006 to 31 October 2012. In panel A, the t-statistic of the Diebold-Mariano test based on different loss function is reported. In panel B, for each forecast horizon, the first column is the t-statistic of the Weighted Likelihood Ratio test and the second column is the corresponded p-value are reported. Note that *, ** and *** denote Significant at 10%, 5% and 1%, respectively.

Panel A: Diebold-Mariano test

	Daily			Weekly			Monthly		
	MSE	MAE	QLIKE	MSE	MAE	QLIKE	MSE	MAE	QLIKE
M0 vs. m0	-1.895	-3.868	-7.538	1.546*	2.124**	0.671	1.365*	1.534*	2.748***
M1 vs. m1	0.246	-1.547	-4.668	1.976**	3.346***	0.100	0.824	1.114	0.506
M2 vs. m2	-0.137	-1.483	-4.097	2.305**	3.816***	-0.356	1.165	1.629*	0.282
M3 vs. m1	1.261	-0.229	-1.187	2.637***	3.495***	0.534	0.884	1.176	0.882
M3 vs. m3	0.967	-1.300	-7.299	1.858**	3.966***	-0.876	0.764	0.981	0.582
M4 vs. m4	1.072	-1.242	-8.693	2.436***	4.381***	-2.079	1.128	1.789**	-0.532

Panel B: Weighted Likelihood Ratio test

	Daily		Weekly		Monthly	
	WLR	P-value	WLR	P-value	WLR	P-value
M0 vs. m0	10.127	<0.000	9.850	<0.000	13.291	<0.000
M1 vs. m1	11.202	<0.000	9.653	<0.000	11.531	<0.000
M2 vs. m2	11.296	<0.000	9.823	<0.000	11.460	<0.000
M3 vs. m1	11.282	<0.000	9.700	<0.000	11.386	<0.000
M3 vs. m3	10.259	<0.000	9.105	<0.000	11.268	<0.000
M4 vs. m4	4.859	<0.000	3.388	0.001	2.148	0.032

5. Conclusion and Suggestions

Within this study, we demonstrate the important roles of VIX futures and risk-neutral skewness for future volatility forecasting during high and low market volatility state. It is well documented in the literature that the implied volatility extracted from VIX futures and the risk-neutral skewness have the predictive ability regarding future volatility.

Using S&P 500 index, S&P 500 index option prices and VIX futures prices, this study finds two important results. First, in the in-sample analysis, the VIX futures has a significant effect on forecasting future volatility, especially in the short-term forecast horizon (i.e., daily and weekly horizons). More specifically, the VIX futures has more significant effect on the high volatile market in the weekly and monthly regressions, while the VIX futures has more significant effect on the low volatile market in the daily regression. Second, in the out-of-sample analysis, the VIX futures improves significantly forecasting ability in the daily, weekly, and monthly regressions.

In addition, similar to the findings of Byun and Kim (2013), the risk-neutral skewness has more significant effect on the high volatile market in the monthly regression, while risk-neutral skewness has more significant effect on the low volatile market in the daily and weekly regressions. We also find the volatility forecasting model that take the information of risk-neutral skewness and VIX futures into account improves the forecasting ability in the in-sample analysis. However, for the out-of-sample analysis, the volatility forecasting model is valid only in the daily regression.

For the comparison between MRS-HAR models and HAR models, we find that the MRS-HAR models outperform the corresponding HAR models in the weekly and monthly regressions. However, the MRS-HAR models are outperformed by HAR models in the daily regression. As we mentioned in Section 4.4, it is probably caused by the parameter estimates instability.

Finally, this study proposes some suggestions for further research. First, based on the proposition of Byun and Kim (2013), it is possible that the VIX of VIX (VVIX) derived from VIX option market prices exists the information content for future volatility forecasting. Second, regarding to the parameter estimates instability of Markov regime-switching models, Book and Pick (2014) suggest the optimal weights for Markov regime-switching models. Last, relative to the unobserved state of Markov regime-switching models, threshold models, which are alternative nonlinear models, use a threshold variable determining the regimes based on observable past level of volatility. These extensions are left for future research.

Appendix

Appendix A

The derivation of S from CBOE website is as follows:

$$S = \frac{E[R^3] - 3E[R]E[R^2] + 2E[R]^3}{(E[R^2] - E^2[R])^{3/2}} = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}}$$

(1) $P_1 = \mu = E[R_T] = e^{rT}(-\sum_i \frac{1}{K_i^2} \mathrm{Q}_{K_i}\, \Delta_{K_i}) + \varepsilon_1$

(2) $P_2 = E[R_T^2] = e^{rT}(\sum_i \frac{2}{K_i^2}(1 - \ln(\frac{K_i}{F_0}))\mathrm{Q}_{K_i}\, \Delta_{K_i}) + \varepsilon_2$

(3) $P_3 = E[R_T^3] = e^{rT}(\sum_i \frac{3}{K_i^2}\{2\ln(\frac{K_i}{F_0}) - \ln^2(\frac{K_i}{F_0})\}Q_{K_i}\Delta_{K_i}) + \varepsilon_3$

(4) $\varepsilon_1 = -(1 + \ln(\frac{F_0}{K_0}) - \frac{F_0}{K_0})$

(5) $\varepsilon_2 = 2\ln(\frac{K_0}{F_0})(\frac{F_0}{K_0} - 1) + \frac{1}{2}\ln^2(\frac{K_0}{F_0})$

(6) $\varepsilon_3 = 3\ln^2(\frac{K_0}{F_0})(\frac{1}{3}\ln(\frac{K_0}{F_0}) - 1 + \frac{F_0}{K_0})$

where,

F_0 : Forward index price derived from index option prices.

K_0 : First strike below F_0.

K_i : Strike price of i^{th} out-of-the money option.

ΔK_i : Half the difference between the strike on either side of K_i :

$$\Delta K_i = \frac{1}{2}(K_{i+1} - K_{i-1})$$

Q_{K_i} : The midpoint of the bid-ask spread for each option with strike K_i.

r : Risk-free interest rate to maturity.

T : The time to maturity expressed as a fraction of a year.

ε_i : Adjustment terms.

Appendix B

Table 7: RMSE, MAE and QLIKE for HAR Models

The table reports the 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecast errors of different HAR models during the whole sample period from 3 January 2006 to 31 October 2012. Three loss functions are adopted, namely RMSE, MAE and QLIKE. Note that RMSE is the square root of MSE.

	Daily			Weekly			Monthly		
	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE	RMSE	MAE	QLIKE
model 0	6.134	4.239	3.655	4.819	3.160	3.673	4.973	3.137	3.746
model 1	5.850	4.067	3.657	4.662	3.109	3.676	4.943	3.148	3.748
model 2	5.837	4.018	3.634	4.641	3.075	3.649	5.033	3.218	3.715
model 3	5.808	4.012	3.628	4.618	3.117	3.656	4.806	3.040	3.732
model 4	5.817	3.958	3.590	4.623	3.112	3.617	4.937	3.174	3.690

Appendix C

Table 8: Out-of-sample Forecasting Performance of MRS-HAR Models

The table shows the comparison of out-of-sample forecasting performance for 1-day- (daily), 5-days- (weekly), and 22-days-ahead (monthly) forecasts of different HAR models during the whole sample period from 3 January 2006 to 31 October 2012. In panel A, the t-statistic of the Diebold-Mariano test based on different loss function is reported. In panel B, for each forecast horizon, the first column is the t-statistic of the Weighted Likelihood Ratio test and the second column is the corresponded p-value are reported. Note that *, ** and *** denote Significant at 10%, 5% and 1%, respectively.

Panel A: Diebold-Mariano test

	Daily			Weekly			Monthly		
	MSE	MAE	QLIKE	MSE	MAE	QLIKE	MSE	MAE	QLIKE
m1 vs. m0	3.075***	3.053***	-0.396	1.978**	0.779	-0.412	0.949	-0.285	-0.363
m2 vs. m0	3.072***	3.835***	4.107***	2.014***	1.199	2.879***	-0.871	-0.896	2.827***
m3 vs. m0	2.211**	2.577***	3.299***	1.381*	0.442	1.347*	1.671	0.917	1.353*
m4 vs. m0	2.029**	2.978***	7.593***	1.217	0.432	4.094***	0.330	-0.258	3.833***
m2 vs. m1	0.707	2.595**	11.642***	0.692	0.898	4.762***	-1.310	-0.769	2.496***
m3 vs. m1	0.547	1.046	6.030***	0.436	-0.121	2.606***	1.714**	1.260	1.888**
m4 vs. m1	0.391	1.714**	10.954***	0.334	-0.028	5.096***	0.067	-0.193	3.658***
m3 vs. m2	0.363	0.108	1.167	0.214	-0.592	-0.746	1.856**	1.483*	-0.909
m4 vs. m2	0.249	1.031	8.457***	0.160	-0.529	3.733***	1.096	0.469	2.644***
m4 vs. m3	-0.280	1.742**	12.174***	-0.109	0.099	4.940***	-1.771	-1.429	2.654***

Panel B: Weighted Likelihood Ratio test

	Daily		Weekly		Monthly	
	WLR	P-value	WLR	P-value	WLR	P-value
m1 vs. m0	2.969	0.003	6.090	<0.000	3.546	<0.000
m2 vs. m0	3.577	<0.000	6.871	<0.000	1.288	0.198
m3 vs. m0	4.861	<0.000	6.180	<0.000	5.598	<0.000
m4 vs. m0	5.162	<0.000	5.924	<0.000	3.454	0.001
m2 vs. m1	3.694	<0.000	2.177	0.029	-0.345	0.730
m3 vs. m1	6.904	<0.000	5.062	<0.000	6.093	<0.000
m4 vs. m1	6.804	<0.000	4.081	<0.000	2.926	0.003
m3 vs. m2	5.926	<0.000	3.433	0.001	4.596	<0.000
m4 vs. m2	6.572	<0.000	3.940	<0.000	4.366	<0.000
m4 vs. m3	2.741	0.006	0.877	0.375	-0.904	0.366

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