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SIMILARITY AND PREFERENTIAL CHOICE

A Comparison of Two Behavioral Ratio Models

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This paper is to present empirical results which conclude that on the aggregate level, Restle's model predicts better than Luce's model the probability of choosing one alternative over the other in a two-choice task.

I. INTRODUCTION

This study deals with an interesting issue associated with the marketing applications of nonmetric multidimensional scaling. It concerns the relationship of similarity judgments and consumer preferential choice.

There are generally two types of models relating the similarity judgments to preferential choices for a set of alternatives, i.e., Ideal Point Models (IPM) and Behavior Ratio Models (BRM). IPM emphasizes deterministic relationship between similarity and preferential choice, whereas BRM focuses on the stochastic relation. A number of marketing studies deal with the deterministic relations between similarity and preferential choice. Few studies center on the possible stochastic relationship, though two competing models of a stochastic type, i.e., Luce's and Restle's, have been in existence for a number of years.[1]

A report by Bass introduces strong empircial evidence which suggests that individual consumer "brand choice behavior is substantially stochastic".[2] The crux of this study is to compare the predictive power of two behavior ratio models --- Restle's and Luce's. Two variants of Restle's model are compared with two variants of Luce's model. The empirical results of this study show that Restle's model has better predictive power than Luce's so far as consumer preference is concerned.

II. BEHAVIOR RATIO MODELS

The behavior-ratio theory was developed by Luce in consideration of the effects on choice of different attributes of alternatives. According to Luce's model:[3]

(1)

$$P(a,b) = \frac{v(a)}{v(a) + v(b)}$$

where:

P(a,b) = probability that a is chosen from the offered set (a,b)

v (a) = measure of total valence for a.

Each choice alternative is viewed as possessing various attributes or elements. Luce's model simply adds up the valences of each alternative producing a total valence for each and then forms the behavior ratio. The model as Restle and Greeno point out leads to some surprising results in overlapping choice situations because it fails to separate the differential from the common elements.[4]

Restle has shown that "overlap" or the degree of similarity between alternatives can be given precise theoretical meaning in the context of BRM. He defines the distance between two alternatives a and b as:

(2) $$d(a,b) = v(a-b) + v(b-a)$$

where:

$$v(a-b) = v(a) - v(a \cap b) \text{ (set difference)}.$$

The function d in equation (2) satisfies the properties of a metric. Furthermore, by direct substitution Luce's model may be rewritten:

(3)
$$P(a,b) = \frac{v(a-b) + v(a \cap b)}{d(a,b) + 2v(a \cap b)}$$

which has limit $P(a,b) \rightarrow .5$ as $d(a,b) \rightarrow 0$. The form in equation (3) demonstrates the direct relationship of similarity and preference postulated by the Luce model. The model requires that the decision-maker be nearly indifferent between two very similar alternatives.

Restle eliminates all dependence between similarity and stochastic preference in his BRM. According to Restle only the differential elements of the alternatives enter into a given choice! Symbollically:

(4)
$$P(a,b) = \frac{v(a-b)}{v(a-b) + v(b-a)}$$

Equations (1) and (4) are the same if a and b are disjoint for then (a-b) = a and (b-a) = b. In an overlapping situation, however, the two models lead to different results.

This difference may be explained by example.

Example 1: Commodity Bundles

Economists studying utility and choice tend to deal with commodity bundles as alternatives. Some reports in marketing have also examined choices involving such alternatives. Suppose

$a \equiv$ (shirt, \$20)

$b \equiv$ (pants, \$20), and

v(shirt) = 2

v (pants) = 3

v (\$20) = 5

For this illustration the total measure for an alternative will be derived by a simple additive rule, i.e.,

$v(a) = v(\text{shirt}) + v(\$20) = 7$. Of course the actual compositional rule used by a decision-maker may be more complex. Then in Luce's model

$$P(a,b) = 7/(7+8) = .467$$

and in Restle's model

$$P(a,b) = \frac{(7-5)}{(7-5)+(8-5)} = .400$$

Note that the valence of the twenty dollars plays no part in the choice according to Restle since it is common to both bundles.

Example 2: Simple Stimuli

Most marketing research in consumer preference conceptualizes the alternatives as what might be called "simple stimuli." For example, suppose

$a \equiv (C_a, F_a)$ $\qquad$ $b \equiv (C_b, F_b)$

are soft drinks with two perceived attributes, calorie (C) and flavor (F).

Let,

$v(C_a) = 8$ $\qquad$ $v(F_a) = 6$

$v(C_b) = 4$ $\qquad$ $v(F_b) = 7$

The total valences are $v(a) = v(C_a) + v(F_a) = 14$,

$v(b) = v(C_b) + v(F_b) = 11$

and the valence of the overlap $v(a \cap b) = v(F_a) + v(C_b) = 10$.

The Luce and Restle models predict respectively :

$$P(a,b) = .560 ; \qquad P(a,b) = .800$$

Ⅲ. EXPERIMENTAL DESIGN

A laboratory experiment was carried out to evaluate the relative and absolute effectiveness of Luce's and Restle's models to predict consumer preference. The subjects for the experiment were sixty students (32 males and 28 females) randomly selected from the total student population of a U. S. university. They were led to believe that they were judging "real" brands of soft drink and that the experimentor was interested in differences in perception between students and housewives and among different sex and age groups.

The stimulus set includes six "imaginary" brands of soft drink. These imaginary brands were presented to each subject graphically in two dimensions (calorie and cola flavor) on computer print-out as shown in Figure 1.

To insure that subjects would interpret the direction and magnitude of the attributes in a consistent manner, data was generated at two sessions. At the first session each subject was asked to rate their ideal soft drink on calorie content and cola-noncola flavor. At the next session (at least two days later) the subjects were told that the attribute scores (Figure 1) were relative values derived from a comparison of each soft drink with their ideal. The subject circled the preferred brand. While the ideal brand could presumably completely satisfy the individual's desires on each attributs, as shown in Figure 1, a particular brand might achieve only a fraction of this satisfaction. The relative scale values of the six imaginary brands and the ideal brand are given in Table 1. These numerical scores however were not shown to the subjects. Though the relative scale values were identical for all subjects, each subject was led to believe that the values were unique to his situation.

GRAPHIC PRESENTATION OF RELATION OF TWO BRANDS (A AND B) TO IDEAL BRAND ON CALORIE AND FLAVOR

SUBJECT NO: 101

COMPARISON OF YOUR HYPOTHETICAL IDEAL BRAND AND THE REAL BRANDS UNDER STUDY

IDEAL BRAND

CALORIE ++
FLAVOR ==

BRAND A

CALORIE ++++++++++++++++++++++++++++++
FLAVOR ==========

BRAND B

CALORIE ++++++++++++++++++++++++++
FLAVOR ==========

WHICH BRAND DO YOU PREFER? A OR B
PLEASE CIRCLE THE BRAND OF YOUR CHOICE.

Table 1

RELATIVE SCALE VALUES OF SIX IMAGINARY BRANDS AND THE IDEAL BRAND

Brand	Calorie	Flavor
Ideal	1.000	1.000
A	.850	.300
B	.700	.450
C	.500	.900
D	.100	.925
E	.600	.500
F	.300	.750

Ⅳ. ANALYSIS OF RESULTS

Model variations

Two compositional rules were used to compute the total valence for each choice alternative. The first used the city-block metric to measure the distance from the origin to the scale value on the attributes. The second used the Euclidean rule to measure this distance. For example, the total valence for brand A would be:

(1) city-block:

$v(A) = .85 + .30 = 1.15$

(2) Euclidean:

$v(A) = [(.85)^2 + (.30)^2]^{1/2} = .90$

Results

The results of the empirical tests are summarized in Table 2. The results show that Restle's model using the city-block metric fits the data quite well. The obtained χ^2 (chi-square) value is 10.247, which is substantially smaller than the χ^2 value needed to reject the model at .05 level (23.685). The observed χ^2 values for Euclidean version of Restle's model and both versions of Luce's model are larger than the critical χ^2 value at .05 level.

The observed χ^2 for Restle's model are smaller than those for Luce's model whenever the same compositional rule is employed, indicating Restle's domination over Luce's in a comparative sense. As to the predictive power of the two compositional rules, the city-block is more powerful than the Euclidean in both models.

Examining the results on a pair-by-pair basis shows that for the city-block version 11 out of 15 choices favor Restle's model and only three favor Luce's. The predicted probabilities for these three choices favoring Luce's model are close in both models and near one-half. Some of the choices favoring Restle's model are quite dramatic, especially the pairs CD and CF. For example, subjects should be nearly indifferent between brands C and F according to Luce's model, while Restle's predicts strong preference for C. The data show that 59 out of 60 subjects prefer C to F, suggesting that subjects ignored the similarity between the two brands and based their preference jedgments on the fact that brand C dominates F on both attributes.

Table 2

OBSERVED VS. PREDICTED RELATIVE FREQUENCIES
FOR THE LUCE AND RESTLE BRM

Pair	Observed		City Block			Euclidean		
	P(i,j)	Frequency	Luce	Restle	(L or R)[a]	Luce	Restle	(L or R)
AB	.433	26	.50000	.50000	-	.51997	.66450	L
AC	.293	17	.45098	.36842	R	.46681	.41620	R
AD	.567	34	.52874	.54545	R	.49208	.48791	L
AE	.483	29	.51111	.55556	L	.53577	.67660	L
AF	.567	34	.52273	.55000	R	.52739	.55439	R
BC	.367	22	.45098	.30769	R	.44699	.30886	R
BD	.617	37	.52874	.55814	R	.47214	.44158	L
BE	.500	30	.51111	.66667	L	.51585	.72591	L
BF	.583	35	.52273	.57143	R	.50744	.52185	R
CD	.800	48	.57732	.94118	R	.52530	.83307	R
CE	.717	43	.56000	.80000	R	.56863	.81351	R
CF	.983	59	.57143	1.00000	R	.56036	1.00000	R
DE	.433	26	.48235	.45946	R	.54364	.60798	L
DF	.350	21	.49398	.46667	R	.53527	.77261	L
EF	.467	28	.51163	.54545	L	.49158	.46835	R
OBSERVED χ^2			35.659	10.247		45.939	38.596	
$P(\chi^2 \geq$ OBSERVED $\chi^2)$			$\doteq$ .001	$\doteq$ .750		$<$ .001	$<$ .001	

[a] A L or R is used to signify that the absolute error | predicted-observed | is a minimum for the Luce or Restle model respectively.

V. SUMMARY AND CONCLUSION

This study is aimed at exploring the relationship between degree of similarity and preference probabilities. Two stochastic models -- Luce's and Restle's -- were compared. The results of a laboratory experiment support the following three conclusions:

1. The city-block compositional rule has better predictive power than the Euclidean for use in behavior ratio models.
2. On the aggregate level, when city-block is used, Restle's model can predict consumer preference probabilities with reasonable accuracy.
3. Restle's model predicts preference probabilities on the aggregate level better than Luce's model.

The comparison of Restle's and Luce's models seems to suggest the dominance of differential elements over common elements in a given choice. It would be fruitful for the management to distinguish the differential elements from the common elements and to strengthen as much as possible the differential subset of their brands.

Footnotes:

*The author would like to thank Professor David J. Curry of the University of Iowa, U.S.A., for his assistance in the research for this study.

1. These two models of stochastic type have been applied to choice problems previously. For example, see Frank Restle and James Greeno, Introduction to Mathematical Psychology, Reading, Mass. : Addison-Wesley, 1970, pp. 221-229.
2. Frank M. Bass, "The Theory of Stochastic Preference and Brand Switching," Journal of Marketing Research, 11 (Feb. 1974), p.2.
3. R. Duncan Luce, Individual Choice Behavior, New York: John Wiley and Sons, 1959.
4. See footnote 1.