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英 文 摘 要： This paper develops a two-stage estimation of a dynamic panel quantile regression (DPQR) model with individual fixed effects. The regressors in the model include a lagged endogenous dependent variable and other explanatory variables, which are correlated with
the fixed effects. The estimation uses the fitted value of the endogenous variable from the first stage, and applies a penalized quantile regression method for panel data in the second stage. The Monte Carlo simulation shows that the proposed DPQR estimation
effectively reduces the dynamic bias and performs better than other estimators in finite samples. The proposed approach is easy to implement and effective in several practical applications.

英文關鍵詞： Quantile regression, dynamic panel data, penalized method

1 Introduction

Quantile regression (QR) for a panel data model has received wide attention in theoretical and empirical studies. Its advantages are that the QR reveals heterogeneity effects of regressors on the dependent variable, and the panel data can control for unobserved individual heterogeneity by including the individual effect. One issue associated with the model is that demeaning or differencing techniques for the incidental parameter problem cannot be used in the conditional quantile function. To solve the incidental parameter problem of QR for panel data, Koenker (2004) first proposes the penalized approach to estimate the QR for the fixed effect panel data model where the penalty of the estimation serves to shrink a vector of individual effects toward a common value. Lamarche (2010) further discusses the degree of this shrinkage and shows that a suitable selected tuning parameter can reduce the variability of the estimator. Recent papers on the QR for a panel data model include Geraci and Bottai (2007), Abrevaya and Dahl (2008), Wang and Fygenson (2009), Gamper-Rabindran et al. (2010), Canay (2011), and Kato et al. (2012).

The dynamic relationship of the panel data model is of great interest in empirical applications. The correlation between the lagged dependent variable and the fixed effect produces a dynamic bias in the estimation. Anderson and Hsiao (1982), Holtz-Eakin et al. (1988), and Arellano and Bond (1991) present that the two-stage least square estimation or the dynamic generalized method of moment (DGMM) in the first-differencing model can be used to eliminate the dynamic bias and produce consistent estimators. Such a first-differencing procedure is not feasible when applying the QR to a dynamic panel data model, and the model should be estimated directly. It follows that the dynamic bias arises when applying QR to a dynamic panel data model. Galvao and Montes-Rojas (2010) use the instrumental variable quantile regression (IVQR) method of Chernozhukov and Hansen (2005, 2006, 2008) to estimate the penalized QR for a dynamic panel data. Galvao (2011) also considers the IVQR method for the dynamic panel model, without using the penalized method. Other papers related to the QR for a panel data model with endogenous variables are Arias et al. (2001), and Harding and Lamarche (2009).

This paper adopts a "fitted value" approach to eliminate the dynamic bias and develops a two-stage estimation procedure for the dynamic panel quantile regression (DPQR) model. The first stage consists of estimating a fitted value for the lagged endogenous dependent variable. Under the assumption of independence between the instrumental variable and the disturbance term of the DPQR model, the dynamic bias can be eliminated by replacing the endogenous variable with its fitted value and adding a constant term in the regression. Moreover, the fixed effect in this paper is a pure location shift and does not depend on the quantiles. Specifying a dummy variable that identifies individuals for the fixed effect is not available in this setting. The fixed effects should be estimated directly, and this paper applies the penalized QR of Koenker (2004) to improve the estimation of common model parameters by controlling the variability introduced by the fixed effects. Therefore, the second step is to replace the endogenous variable in the DPQR model by its fitted value and to run a penalized QR for the panel data model to obtain the estimators.

The fitted value approach for the fixed effect DPQR model, by simply running two-stage regressions, is appealing in that it is generally applicable and easy to implement. The proposed estimator is extremely simple to compute and can be implemented in standard econometrics packages. The estimators introduced in this paper offers some computation advantages and should be viewed as a complement to those in Galvao and Montes-Rojas (2010). This paper shows that the proposed DPQR estimator is asymptotically normal with zero mean when both N and T are large. In addition, we compare the bias and root mean squared error (RMSE) of several estimators for the DPQR model under different scenarios. Using the Monte Carlo simulations, in a finite sample the dynamic bias is effectively reduced under the two-stage estimation. Comparing with the estimators of Koenker (2004), Galvao and Montes-Rojas (2010), and Galvao (2011), the proposed estimator performs better than the other estimators regarding the bias and RMSE. Thus, our estimator competes efficiently with those methodologies applying to the DPQR model.

The remainder of this paper is organized as follows. Section 2 introduces the econometrics method, proposes the DPQR model with a two-stage estimating procedure, and provides the large sample properties of the proposed estimator. Section 3 shows a Monte Carlo simulation.

2 Dynamic Panel Quantile Regression

2.1 Fitted Value Approach

Consider a dynamic panel data model with individual fixed effects:

$$y_{it} = \alpha y_{it-1} + x'_{it}\beta + \eta_i + u_{it}, \quad \forall i = 1, \cdots, N, t = 1, \cdots, T, \tag{1}$$

where y_{it} is a real-valued dependent variable, y_{it-1} is the lagged dependent variable, x_{it} is a $(d_X \times 1)$ vector of real-valued, continuously distributed, exogenous explanatory variables, η_i is the parameter that represents the individual fixed effects, α and β are unknown parameters, and u_{it} is the error term. The fixed effects η_i in (1) capture some source of variability, or "unobserved heterogeneity," that is not adequately controlled by other regressors in the model. When N is a large number, the estimation of $N+k+1$ parameters is complicated and suffers from an incidental parameter problem. In addition, by construction, y_{it-1} is a function of the unobserved individual effect η_i and is correlated with the error term. An endogeneity problem thus arises in the dynamic panel data model. To eliminate the dynamic bias, Anderson and Hsiao (1982) suggest a two-stage least squares estimation by using further lags of the dependent variable as instruments for first-differenced lag dependent variable. Moreover, Holtz-Eakin et al. (1988) and Arellano and Bond (1991) suggest using the DGMM estimator, which is based on moment equations constructed from the first-differenced error term and lags of regressors.

To capture the heterogeneous covariate effects of the dependent variable, this paper applies the QR for the dynamic panel data with fixed effects. The DPQR model is able to capture the dynamic relationship of variables of interest, control for unobserved individual heterogeneity with η_i, and reveal heterogeneity effects of regressors on the dependent variable. Since the first-differencing procedure is not feasible in the conditional quantile function, the QR for dynamic panel data model should be estimated directly, but the correlation between the lagged dependent variable and the fixed effect in (1) produces a dynamic bias in the estimation. Several studies propose to solve this endogeneity problem in the DPQR model. For example, Arias et al. (2001), following the control function approach, suggest a two-stage estimation. Harding and Lamarche (2009), Galvao and Montes-Rojas (2010), and Galvao (2011) introduce the IVQR method for panel data model.

This paper proposes the fitted value approach to eliminate the dynamic bias in the DPQR model, and develops a two-stage estimation procedure. First, let the endogenous lagged dependent variable can be divided into two parts: one is a function of exogenous and instrument variables $\hat{y}_{it-1}$, and the other is the residual between y_{it-1} and $\hat{y}_{it-1}$. Let z_{it} be a $(d_Z \times 1)$ vector of the instrumental variable, then we have

$$y_{it-1} = \hat{y}_{it-1} + v_{it}, \tag{2}$$

where $\hat{y}_{it-1} := \hat{y}_{it-1}(x_{it}, z_{it})$ is a function of instruments, and $v_{it} = y_{it} - \hat{y}_{it-1}$. In this setting, v_{it} is a real-valued unobserved random variable and is independent of both x_{it} and z_{it}. Here, further lags of the first-differenced dependent variable, $\Delta y_{it-j}, j = 1, 2, \cdots, T-1$, can be used as instrumental variables, since the fixed effect is eliminated by construction. For example, $\Delta y_{it-1}, \Delta y_{it-2}$ can be valid instruments for y_{it-1}. In addition, the exogenous variable x_{it} is allowed to be incorporated into the first stage.

To identify the estimation procedure used, we need the following assumptions. First, we need the consistency of the fitted value: $\hat{y}_{it-1}$ should be a consistent estimator for y_{it-1}. Second, we need an assumption for the independence between u_{it} and z_{it} as well as v_{it} and η_i to identify the model.

Assumption 1. *$Wp \to 1$, the function $\hat{y}_{it-1}(x, z) \xrightarrow{p} y_{it-1}$ uniformly in (x, z)*

Assumption 2. *u_{it} is independent of z_{it}, and v_{it} is independent of η_i.*

Replacing y_{it-1} in (1) by the function (2) yields:

$$\begin{aligned} y_{it} &= \alpha(\hat{y}_{it-1} + v_{it}) + x'_{it}\beta + \eta_i + u_{it} \\ &= \alpha\hat{y}_{it-1} + x'_{it}\beta + \eta_i + (\alpha v_{it} + u_{it}). \end{aligned}$$

By construction of model (1), u_{it} is the above model is independent of the exogenous explanatory variable x_{it} and the fixed effect η_i. Also, v_{it} is independent of x_{it} and z_{it} in model (2). With the independence assumption (Assumption 2), we obtain that the conditional quantile function of $Q_{\alpha v_{it}+u_{it}}(\tau|x_{it}, z_{it}, \eta_i)$ equals an unconditional quantile function $Q_{\alpha v_{it}+u_{it}}(\tau)$. Let $Q_{\alpha v_{it}+u_{it}}(\tau)$ be $c(\tau)$. Therefore, the τ-th conditional quantile function for the DPQR model is:

$$Q_{y_{it}}(\tau|x_{it}, z_{it}, \eta_i) = \alpha(\tau)\hat{y}_{it-1} + x'_{it}\beta(\tau) + \eta_i + c(\tau). \tag{3}$$

where $Q_{y_{it}}(\tau|x_{it}, z_{it}, \eta_i)$ is the τ-th conditional quantile function of the dependent variable, and $\alpha(\tau)$ and $\beta(\tau)$ are parameters at the τ-th quantile. In this paper the fixed effect η_i is a pure location shift effect and does not depend on the quantile τ. The penalized QR approach of Koenker (2004) can then be used to obtain consistent estimators of $\alpha(\tau)$ and $\beta(\tau)$ in (3), where $c(\tau)$ is viewed as the coefficient of a constant term. This suggests that the parameters of the DPQR model can be estimated by a two-stage procedure. The first stage of the two-stage procedure is to construct a regression of y_{it-1} on z_{it} and x_{it} and obtain the fitted value $\hat{y}_{it-1}$. In the second step of the two-stage procedure, the fitted value $\hat{y}_{it-1}$ is inserted in place of the endogenous dependent variable y_{it-1}, and the penalized QR approach for the panel data model is used for (3). Therefore, the two-stage estimation corrects for endogeneity of the DPQR model by replacing y_{it-1} by $\hat{y}_{it-1}$ and can be viewed as a variant of the fitted value approach. One may note that Galvao and Montes-Rojas (2010) and Galvao (2011) also study the estimation and inference for the DPQR model. They consider using the IVQR method for the endogenous problem, whereas this paper uses a two-stage fitted value approach for the endogenous problem and can be viewed as a complement to their papers.

2.2 Estimation and Asymptotics

The estimation procedure consists of two-stages wherein the first stage estimates the fitted value of y_{it-1}. A practical formulation for $\hat{y}_{it-1}$ is to use the least squares projection of y_{it-1} on x_{it} and z_{it} and possibly their powers. Efficiency can be improved by choosing $\hat{y}_{it-1}$ appropriately. In the second stage, we suggest using the penalized QR of Koenker (2004) to improve the properties of the estimation of (3). The penalized QR method for the conditional quantile function (3) is to estimate (3) for several quantiles simultaneously with a penalty term, as characterized by:

$$\min_{\alpha,\beta,c} \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{t=1}^{T} \omega_k \rho_{\tau_k}(y_{it} - x_{it}'\beta(\tau_k) - \alpha(\tau_k)\hat{y}_{it-1} - c(\tau_k) - \eta_i) - \lambda \sum_{i=1}^{N} |\eta_i|, \quad (4)$$

where ω_k is the weight for the τ_k-quantile, $k = 1, \cdots, K$, $\rho_\tau(u) = u \cdot (\tau - \mathbf{1}(u < 0))$ is the check function as in Koenker and Bassett (1978), with $\mathbf{1}(\cdot)$ an indicator function, and $\lambda \sum_{i=1}^{N} |\eta_i|$ is an ℓ_1 penalty term with tuning parameter λ. The weights ω_k control the relative influence of the q quantiles, $\{\tau_1, \cdots, \tau_K\}$, on the estimation of the η_i parameters. The penalty term in (4) is designed to shrink the variability of a resulting estimator with $\lambda \geq 0$, and the tuning parameter λ controls the degree of the shrinkage. As Tibshirani (1996), Donoho et al. (1998), and Lamarche (2010) point out, when N is large relative to T, the ℓ_1 shrinkage is advantageous in controlling the variability introduced by the large number of estimated η_i parameters, since the fixed effects are estimated directly in the penalized method. Note that since $c(\tau)$ can be viewed as the parameter of intercept, we therefore have q, τ-specific, estimates of the intercept in the estimation.

In addition to Assumptions 1 and 2, to obtain the asymptotic properties of the proposed estimator, we need more assumptions as follows.

Assumption 3. The variables y_{it} are independent across individuals, stationary with conditional distribution F_{it}, and continuous conditional densities f_{it} are uniformly bounded away from 0 and ∞, for $i = 1, \cdots, N$, and $t = 1, \cdots, T$.

Assumption 4. Let $\tilde{x}_{it} = [1, \hat{y}_{it-1}, x'_{it}]'$ and $\tilde{\beta}(\tau) = [c(\tau), \alpha(\tau), \beta(\tau)]$ be a $(d_X+2)\times 1$ vector of parameters. There exist positive definite matrices $J_0(\tau)$ and $S(\tau)$ such that:

$$J_0(\tau) = \lim_{N\to\infty, T\to\infty} \frac{1}{NT} \begin{bmatrix} \omega_1 X' M_D(\tau_1)' \Phi(\tau_1) M_D(\tau_1) X & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_K X' M_D(\tau_K)' \Phi(\tau_K) M_D(\tau_K) X \end{bmatrix},$$

$$S(\tau) = \lim_{N\to\infty, T\to\infty} \frac{1}{NT} \begin{bmatrix} \Omega_{11} X' M_D(\tau_1)' M_D(\tau_1) X & \cdots & \Omega_{1K} X' M_D(\tau_1)' M_D(\tau_K) X \\ \vdots & \ddots & \vdots \\ \Omega_{K1} X' M_D(\tau_K)' M_D(\tau_1) X & \cdots & \Omega_{KK} X' M_D(\tau_K)' M_D(\tau_K) X \end{bmatrix},$$

where $X = [\tilde{x}_{it}]$, $M_D(\tau_k) = I - P_D(\tau_k)$, $P_D(\tau_k) = D(D'\Phi(\tau_k)D)^{-1}D'\Phi(\tau_k)$, $\Phi(\tau_k) = \text{diag}(f_{y_{it}}(\tilde{x}'_{it}\tilde{\beta}(\tau) + \eta_i))$ and $\Omega_{jl} = \omega_j(\tau_j \wedge \tau_l - \tau_j\tau_l)\omega_l$, with $D = [d_{it}]$ as an incidence matrix of dummy variable, and d_{it} as a dummy variable that identifies the N distinct individuals in the sample.

Assumption 5. $\max_{it} ||x_{it}|| = O(\sqrt{NT})$, and $\max_{it} ||z_{it}|| = O(\sqrt{NT})$.

Assumption 6. $\lambda_T/\sqrt{T} \to \lambda_0$.

Assumption 3 is standard in the QR literature. Assumption 4 uses the definition of the positive definite matrices for the central limit theorem to obtain the asymptotic normality. Assumption 5 imposes bounds on the variable x_{it} and $\tilde{x}_{it}$.

This assumption ensures the finite dimensional convergence of the objective function. In Assumption 6, the shrinkage of the fixed effects toward a common value can decrease the variability caused by the presence of unobserved individual heterogeneity. Similar assumptions can also be found in Lamarche (2010), Galvao and Montes-Rojas (2010), and Galvao (2011). Note that Kato et al. (2012) also discuss the asymptotics for panel data QR model, and use asymptotics that do not invoke the Bahadur representation in Koenker (2004). However, Kato et al. (2012) should impose substantially stronger conditions on the rate at which T is supposed to grow to infinity relative to N, and we rely on results from Koenker (2004), Galvao and Montes-Rojas (2010), and Galvao (2011). Under the assumptions above, we can establish the asymptotic normality of the proposed estimator.

Theorem 1. *Suppose that* Assumptions 1-6 *hold. When* $N \to \infty$, $T \to \infty$, *and* $N^a/T \to 0$, *for some* $a > 0$, *then* $\sqrt{NT}(\hat{\tilde{\beta}}(\tau) - \tilde{\beta}(\tau))$ *converges to a normal distribution with mean zero and covariance matrix* $J_0(\tau)^{-1}S(\tau)J_0(\tau)^{-1}$.

An asymptotically valid standard error of the estimator for the second step requires a correction for the first step estimation, which makes the variance function of the estimator very complicated. Thus, the bootstrap method is considered to calculate the standard error of the proposed estimator. Let $y_{i\cdot} = \{y_{i1}, \cdots, y_{iT}\}$, $x_{i\cdot} = \{x_{i1}, \cdots, x_{iT}\}$, and $z_{i\cdot} = \{z_{i1}, \cdots, z_{iT}\}$ be $T \times 1$ vectors that contain T observations. For the bootstrapping procedure, first, a random sample $\{y^*_{i\cdot}, x^*_{i\cdot}, z^*_{i\cdot}\}$ is randomly drawn from the data $\{y_{i\cdot}, x_{i\cdot}, z_{i\cdot}, i = 1, \cdots, N\}$ to create a resampled data set $\{y^*_{i\cdot}, x^*_{i\cdot}, z^*_{i\cdot}, i = 1, \cdots, N\}$. The DPQR estimates, $\hat{\tilde{\beta}}^*(\tau)$, can be computed from the resampled data. Second, the above procedure is repeated B times to obtain DPQR estimates, $\hat{\tilde{\beta}}^*_1(\tau), \hat{\tilde{\beta}}^*_2(\tau), \cdots, \hat{\tilde{\beta}}^*_B(\tau)$. Third and finally, a consistent variance estimator is obtained by:

$$\widehat{\mathrm{Var}}(\hat{\tilde{\beta}}(\tau)) = \frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\tilde{\beta}}^*_b(\tau) - \bar{\hat{\tilde{\beta}}}^*_b(\tau)\right)\left(\hat{\tilde{\beta}}^*_b(\tau) - \bar{\hat{\tilde{\beta}}}^*_b(\tau)\right)',$$

with $\bar{\hat{\tilde{\beta}}}^*_b(\tau) = B^{-1}\sum_{b=1}^{B}\hat{\tilde{\beta}}^*_b(\tau)$.

3 Monte Carlo Simulations

This section studies the Monte Carlo study to investigate the small sample properties of estimators for the DPQR model. We compare the bias and RMSE of the following estimators: (1) the two-stage estimator for the DPQR model in this paper (DPQR); (2) the penalized QR for panel data estimator in Koenker (2004) (PQR); (3) the penalized QR for panel data estimator using the IVQR method in Galvao and Montes-Rojas (2010) (PQR-IVQR); and (4) the fixed effect QR estimator of Galvao (2011) (FEQR). The latter two estimators use the IVQR method to reduce the dynamic bias. Three DPQR models are considered in this section: (A) the pure location shift model,

$$y_{it} = \eta_i + \alpha y_{it-1} + \beta x_{it} + u_{it};$$

(B) the location-scale shift model I,

$$y_{it} = \eta_i + \alpha y_{it-1} + \beta x_{it} + (\gamma_0 x_{it}) u_{it};$$

and (C) the location-scale shift model II,

$$y_{it} = \eta_i + \alpha y_{it-1} + \beta x_{it} + (1 + \gamma_1 x_{it}) u_{it}.$$

In all cases, we follow Galvao and Montes-Rojas (2010), set $y_{i,-50} = 0$ and generate y_{it} for $t = -49, -48, \cdots, T$, and discard the first 50 observations, using the observations $t = 0$ through T for estimation. The error term u_{it} follows the normal distribution $N(0, \sigma_u^2)$ with $\sigma_u^2 = 1, 3, 5$, the heavy-tail t-distribution with 3 degree of freedom (t_3 distribution), or the χ^2-distribution with 3 degree of freedom (χ_3^2 distribution).

The regressor x_{it} is generated according to $x_{it} = \mu_i + \xi_{it}$, where the fixed effect:

$$\mu_i = e_{1i} + \frac{1}{T} \sum_{t=1}^{T} x_{it}, \quad e_{1i} \sim N(0, \sigma_{e_1}^2),$$

and ξ_{it} follows the same distribution as u_{it}. The fixed effects, η_i are generated as:

$$\eta_i = e_{2i} + \frac{1}{T} \sum_{t=1}^{T} \epsilon_{it}, \quad e_{2i} \sim N(0, \sigma_{e_2}^2).$$

From the above specification of the fixed effect, there is correlation between the individual effects and the explanatory variables, ensuring that the random effects are inconsistent. In the simulation, $T = 10, N = 50$, and the number of replications is 2000. We also compare different sample sizes, with $T = \{5, 15, 25\}$, and $N = \{50, 100, 150\}$. In addition, the parameters $\alpha = \{0.3, 0.4, 0.5, 0.6, 0.7\}$, $\beta = 1$, and $\sigma_{e_1} = \sigma_{e_2} = 1$. For the location-scale shift models, we use $\gamma_0 = 0.5$ and $\gamma_1 = 0.1$.

The estimators are analyzed for three quantiles $\tau = (0.25, 0.5, 0.75)$. For the DPQR, PQR-IVQR and FEQR estimators, we consider instruments y_{it-2} and x_{it} for the lagged dependent variable. Tables 1 and 2 report the bias and RMSE results for estimates of the autoregressive parameter values for the location shift and the location-scale shift models, respectively. Both Tables 1 and 2 show that, under the normal error distribution, the autoregressive coefficient is biased upward for the PQR and PQR-IVQR estimations, and is slightly biased for the DPQR estimation. Tables 1 and 2 present that the DPQR estimator has lower bias and RMSE of $\hat{\alpha}$ than those of the PQR and PQR-IVQR estimators. Thus, the DPQR estimator performs best among the estimations for the three DPQR models. Tables 1 and 2 also shows that $\hat{\beta}$ is biased upward for all three estimators, except for some values of the DPQR estimators. Regarding the bias and the RMSE of β, in Table 1 and the upper panel of Table 2, the DPQR estimator also performs better than the other two estimators. In the lower panel of Table 2, regarding the bias and RMSE, the DPQR estimator performs better than the other two estimators for the lower to middle quartiles ($\tau = 0.25, 0.5$) and the PQR-IVQR estimator performs better than the other two estimators for the higher quartile ($\tau = 0.75$).

Tables 3 and 4 report the bias and RMSE results under the $N(0, 3)$, $N(0, 5)$, t_3, and χ^2_3 distributions, for the location and location-scale shift models, respectively. In Table 3, the autoregressive estimates of PQR and PQR-IVQR are biased upward, and those of DPQR are biased downward. Regarding the bias and RMSE, Table 3 shows that for the normal distributions $N(0, 3)$ and $N(0, 5)$, and t_3 distributions, the PQR estimator performs best among the three estimators; for the χ^2_3 distribution, the DPQR estimator performs best among the three estimators. Table 4 presents that the PQR estimator has the lowest bias and RMSE for different error distributions except the χ^2_3 distribution. The DPQR estimator performs better than the other two estimators for the χ^2_3 distribution for $\tau = 0.5, 0.75$. Moreover, regarding the bias and RMSE of $\hat{\beta}$, both Tables 3 and 4 show that, in all three DPQR models, the DPQR estimator of β has lower bias and RMSE than the other two estimators for all error distributions and across all quartiles.

Table 5 reports the bias and RMSE results with different panel sizes. The autoregressive parameter and β estimates are biased upward for all three estimators. Both the bias and RMSE are larger for small T and decrease as T increases, but the bias and RMSE do not depend on n. In addition, it is seen that the PQR estimator performs better than the other two estimators for the autoregressive parameter; the DPQR estimator performs better than the other two estimators for β in the location shift model. Similar results can be found in the location-scale shift models in the lower two panels of Table 5. In the final part of the simulation, we consider different values of tuning parameter λ. It is noted that the special case $\lambda = 0$ of the PQR-IVQR estimator is the FEQR estimator of Galvao (2011).

Table 6 shows the bias and RMSE results for different lambda values. For $\lambda = 0$, the PQR, DPQR, and FEQR estimators of α and β are biased downward; for $\lambda = 0.5$ and 1, the DPQR estimator of α is biased downward; for $\lambda = 0.5$, the DPQR estimator of β is biased downward. For other values of λ, the three estimators of α and β are biased upward. In the location shift model the FEQR estimator performs best among the three estimator when $\lambda = 0$, and for other values of λ the DPQR estimator has a lower bias and RMSE values. In the location-scale shift models the PQR estimator performs best for $\lambda = 0, 0.5$, while the DPQR estimator performs best for $\lambda = 1, 1.5, 2$ regarding the bias and RMSE of α and β. The results suggest that the DPQR estimator performs well in large values of λ for all three dynamic panel models.

4 Conclusions

This paper has proposed a two-stage estimator for the DPQR model. In this paper, the two-stage estimation method, adjusting the dynamic bias, depends crucially on the assumption of independence between the instrumental variable and the disturbance term of the DPQR model. Thus, it would be useful to extend this assumption. In addition, future research could also benefit by investigating the issue of efficiency of the estimation, and the selection of λ.

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Table 1: Bias and RMSE results for different autoregressive parameter values (The location shift model)

		$\tau = 0.25$					$\tau = 0.5$					$\tau = 0.75$				
	α	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7
PQR																
$\hat{\alpha}$	Bias	0.1120	0.1221	0.1293	0.1289	0.1198	0.1118	0.1219	0.1292	0.1292	0.1197	0.1116	0.1218	0.1291	0.1294	0.1197
	RMSE	0.1194	0.1281	0.1339	0.1318	0.1212	0.1190	0.1278	0.1336	0.1319	0.1211	0.1190	0.1280	0.1336	0.1322	0.1211
$\hat{\beta}$	Bias	0.0956	0.0923	0.0906	0.0814	0.0633	0.0958	0.0920	0.0904	0.0816	0.0633	0.0959	0.0915	0.0906	0.0817	0.0635
	RMSE	0.1112	0.1097	0.1078	0.0998	0.0864	0.1110	0.1086	0.1073	0.0993	0.0855	0.1116	0.1085	0.1082	0.1000	0.0864
DPQR																
$\hat{\alpha}$	Bias	-0.0378	-0.0487	-0.0312	0.0151	0.0683	-0.0369	-0.0484	-0.0308	0.0153	0.0684	-0.0369	-0.0488	-0.0295	0.0159	0.0681
	RMSE	0.0699	0.0766	0.0702	0.0628	0.0791	0.0681	0.0755	0.0688	0.0622	0.0791	0.0694	0.0770	0.0686	0.0624	0.0791
$\hat{\beta}$	Bias	0.0265	0.0170	0.0201	0.0256	0.0222	0.0250	0.0181	0.0188	0.0242	0.0228	0.0246	0.0164	0.0148	0.0240	0.0236
	RMSE	0.0711	0.0699	0.0750	0.0801	0.0818	0.0646	0.0657	0.0663	0.0700	0.0728	0.0708	0.0703	0.0723	0.0791	0.0809
PQR-IVQR																
$\hat{\alpha}$	Bias	0.2029	0.2199	0.2032	0.1721	0.1388	0.1812	0.2118	0.2021	0.1724	0.1388	0.2025	0.2201	0.2029	0.1727	0.1388
	RMSE	0.2248	0.2296	0.2076	0.1744	0.1401	0.2118	0.2246	0.2066	0.1745	0.1400	0.2243	0.2299	0.2071	0.1749	0.1401
$\hat{\beta}$	Bias	0.0722	0.0660	0.0642	0.0577	0.0454	0.0794	0.0674	0.0644	0.0576	0.0454	0.0728	0.0645	0.0643	0.0578	0.0455
	RMSE	0.0951	0.0902	0.0885	0.0828	0.0749	0.0993	0.0902	0.0879	0.0819	0.0738	0.0954	0.0891	0.0889	0.0829	0.0747

Table 2: Bias and RMSE results for different autoregressive parameter values (The location-scale shift models)

Model I		$\tau = 0.25$					$\tau = 0.5$					$\tau = 0.75$				
	α	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7
PQR																
$\hat{\alpha}$	Bias	0.0868	0.1002	0.1122	0.1206	0.1176	0.0867	0.1002	0.1121	0.1207	0.1177	0.0867	0.1002	0.1121	0.1208	0.1177
	RMSE	0.0917	0.1047	0.1161	0.1231	0.1188	0.0915	0.1046	0.1160	0.1231	0.1189	0.0915	0.1046	0.1160	0.1232	0.1190
$\hat{\beta}$	Bias	0.0554	0.0573	0.0562	0.0549	0.0483	0.0551	0.0575	0.0563	0.0555	0.0481	0.0549	0.0574	0.0561	0.0560	0.0481
	RMSE	0.0724	0.0738	0.0744	0.0715	0.0666	0.0700	0.0726	0.0724	0.0700	0.0639	0.0717	0.0743	0.0742	0.0721	0.0664
DPQR																
$\hat{\alpha}$	Bias	-0.0528	-0.0492	-0.0143	0.0442	0.0899	-0.0529	-0.0494	-0.0156	0.0429	0.0882	-0.0526	-0.0489	-0.0147	0.0447	0.0900
	RMSE	0.0711	0.0718	0.0610	0.0698	0.0953	0.0709	0.0717	0.0610	0.0690	0.0934	0.0711	0.0715	0.0603	0.0707	0.0952
$\hat{\beta}$	Bias	0.0014	-0.0016	0.0003	0.0090	0.0089	0.0012	-0.0040	-0.0013	0.0075	0.0078	0.0005	-0.0026	0.0003	0.0099	0.0082
	RMSE	0.0579	0.0607	0.0646	0.0659	0.0716	0.0482	0.0507	0.0537	0.0558	0.0590	0.0584	0.0617	0.0664	0.0673	0.0724
PQR-IVQR																
$\hat{\alpha}$	Bias	0.1438	0.1978	0.2053	0.1777	0.1430	0.1107	0.1796	0.2010	0.1777	0.1430	0.1439	0.1979	0.2056	0.1782	0.1431
	RMSE	0.1648	0.2122	0.2099	0.1792	0.1438	0.1433	0.2020	0.2073	0.1792	0.1438	0.1651	0.2122	0.2101	0.1797	0.1439
$\hat{\beta}$	Bias	0.0524	0.0503	0.0464	0.0441	0.0378	0.0598	0.0543	0.0470	0.0444	0.0375	0.0515	0.0507	0.0465	0.0448	0.0375
	RMSE	0.0723	0.0702	0.0684	0.0643	0.0599	0.0747	0.0715	0.0664	0.0621	0.0567	0.0711	0.0719	0.0682	0.0646	0.0596
Model II		$\tau = 0.25$					$\tau = 0.5$					$\tau = 0.75$				
	α	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7
PQR																
$\hat{\alpha}$	Bias	0.1121	0.1207	0.1266	0.1293	0.1183	0.1122	0.1208	0.1267	0.1294	0.1186	0.1125	0.1208	0.1268	0.1295	0.1189
	RMSE	0.1198	0.1268	0.1318	0.1323	0.1198	0.1196	0.1266	0.1316	0.1321	0.1199	0.1201	0.1268	0.1318	0.1323	0.1203
$\hat{\beta}$	Bias	0.0950	0.0924	0.0865	0.0787	0.0620	0.0959	0.0932	0.0867	0.0794	0.0627	0.0967	0.0940	0.0871	0.0802	0.0632
	RMSE	0.1101	0.1084	0.1036	0.0969	0.0848	0.1104	0.1089	0.1032	0.0969	0.0847	0.1117	0.1101	0.1039	0.0983	0.0857
DPQR																
$\hat{\alpha}$	Bias	-0.0403	-0.0509	-0.0339	0.0127	0.0642	-0.0382	-0.0486	-0.0321	0.0153	0.0666	-0.0362	-0.0476	-0.0294	0.0177	0.0693
	RMSE	0.0699	0.0775	0.0706	0.0609	0.0760	0.0680	0.0752	0.0691	0.0616	0.0777	0.0685	0.0755	0.0690	0.0632	0.0805
$\hat{\beta}$	Bias	-0.0301	-0.0347	-0.0323	-0.0249	-0.0239	0.0250	0.0178	0.0177	0.0234	0.0242	0.0796	0.0736	0.0660	0.0705	0.0686
	RMSE	0.0720	0.0773	0.0754	0.0780	0.0810	0.0640	0.0638	0.0648	0.0691	0.0730	0.1042	0.0996	0.0959	0.1019	0.1030
PQR-IVQR																
$\hat{\alpha}$	Bias	0.2034	0.2190	0.2018	0.1724	0.1387	0.1825	0.2124	0.2005	0.1723	0.1389	0.2035	0.2193	0.2023	0.1725	0.1393
	RMSE	0.2248	0.2297	0.2066	0.1747	0.1399	0.2124	0.2257	0.2056	0.1745	0.1400	0.2251	0.2299	0.2069	0.1747	0.1405
$\hat{\beta}$	Bias	0.0723	0.0665	0.0600	0.0561	0.0442	0.0803	0.0690	0.0605	0.0569	0.0448	0.0747	0.0680	0.0602	0.0580	0.0455
	RMSE	0.0938	0.0896	0.0842	0.0815	0.0742	0.0992	0.0906	0.0838	0.0808	0.0735	0.0957	0.0906	0.0843	0.0823	0.0748

Table 3: Bias and RMSE results for different error distributions (The location shift model)

		$\tau = 0.25$				$\tau = 0.5$				$\tau = 0.75$			
	u	$N(0,3)$	$N(0,5)$	t_3	χ_3^2	$N(0,3)$	$N(0,5)$	t_3	χ_3^2	$N(0,3)$	$N(0,5)$	t_3	χ_3^2
PQR													
$\hat{\alpha}$	Bias	0.0569	0.0207	0.0616	0.2343	0.0568	0.0206	0.0618	0.2796	0.0568	0.0207	0.0620	0.3429
	RMSE	0.0735	0.0519	0.0689	0.2346	0.0725	0.0505	0.0688	0.2800	0.0732	0.0520	0.0693	0.3436
$\hat{\beta}$	Bias	0.1380	0.1626	0.0581	0.1750	0.1386	0.1623	0.0581	0.1813	0.1390	0.1620	0.0581	0.1971
	RMSE	0.1692	0.2023	0.0721	0.1805	0.1688	0.1999	0.0719	0.1893	0.1703	0.2009	0.0722	0.2142
DPQR													
$\hat{\alpha}$	Bias	-0.1191	-0.1582	-0.1336	-0.1615	-0.1211	-0.1602	-0.1347	-0.1526	-0.1211	-0.1597	-0.1326	-0.1462
	RMSE	0.1389	0.1739	0.1423	0.1692	0.1391	0.1749	0.1426	0.1611	0.1403	0.1763	0.1412	0.1583
$\hat{\beta}$	Bias	0.0493	0.0731	-0.0165	-0.0161	0.0524	0.0739	-0.0178	-0.0215	0.0495	0.0753	-0.0188	-0.0217
	RMSE	0.1229	0.1617	0.0625	0.0532	0.1174	0.1501	0.0526	0.0595	0.1260	0.1639	0.0614	0.0830
PQR-IVQR													
$\hat{\alpha}$	Bias	0.1161	0.0548	0.1277	0.2642	0.1069	0.0472	0.1134	0.3126	0.1154	0.0526	0.1278	0.3821
	RMSE	0.1452	0.0978	0.1462	0.2644	0.1391	0.0919	0.1376	0.3130	0.1436	0.0968	0.1461	0.3828
$\hat{\beta}$	Bias	0.1169	0.1484	0.0583	0.0956	0.1217	0.1532	0.0620	0.0924	0.1184	0.1501	0.0582	0.0896
	RMSE	0.1553	0.1941	0.0730	0.0956	0.1569	0.1935	0.0754	0.1069	0.1567	0.1939	0.0729	0.1217

Table 4: Bias and RMSE results for different error distribution (The location-scale shift models)

Model I		$\tau = 0.25$				$\tau = 0.5$				$\tau = 0.75$			
	u	$N(0,3)$	$N(0,5)$	t_3	χ^2_3	$N(0,3)$	$N(0,5)$	t_3	χ^2_3	$N(0,3)$	$N(0,5)$	t_3	χ^2_3
PQR													
$\hat{\alpha}$	Bias	0.0752	0.0550	0.0418	0.0804	0.0752	0.0548	0.0417	0.0817	0.0752	0.0547	0.0416	0.0780
	RMSE	0.0811	0.0626	0.0474	0.0829	0.0809	0.0622	0.0472	0.0854	0.0812	0.0623	0.0472	0.0859
$\hat{\beta}$	Bias	0.0791	0.0928	0.0483	1.0687	0.0787	0.0923	0.0480	1.6140	0.0786	0.0917	0.0476	2.4501
	RMSE	0.1121	0.1352	0.0774	1.0771	0.1078	0.1294	0.0737	1.6235	0.1118	0.1345	0.0755	2.4638
DPQR													
$\hat{\alpha}$	Bias	-0.0897	-0.1188	-0.1390	-0.1909	-0.0925	-0.1203	-0.1412	-0.1787	-0.0916	-0.1183	-0.1391	-0.1762
	RMSE	0.1065	0.1332	0.1453	0.1995	0.1074	0.1326	0.1468	0.1888	0.1076	0.1323	0.1453	0.1916
$\hat{\beta}$	Bias	0.0027	0.0153	-0.0183	0.7142	0.0037	0.0129	-0.0178	1.1981	0.0051	0.0118	-0.0201	1.9371
	RMSE	0.0970	0.1257	0.0878	0.7377	0.0806	0.0995	0.0722	1.2204	0.0983	0.1238	0.0887	1.9650
PQR-IVQR													
$\hat{\alpha}$	Bias	0.1508	0.1161	0.0884	0.1194	0.1349	0.1013	0.0730	0.1208	0.1517	0.1154	0.0883	0.1164
	RMSE	0.1678	0.1356	0.1046	0.1223	0.1580	0.1269	0.0951	0.1258	0.1682	0.1352	0.1046	0.1266
$\hat{\beta}$	Bias	0.0691	0.0814	0.0510	0.8951	0.0735	0.0872	0.0553	1.4414	0.0680	0.0815	0.0497	2.2893
	RMSE	0.1075	0.1295	0.0804	0.9092	0.1049	0.1256	0.0800	1.4574	0.1070	0.1292	0.0787	2.3114

Model II		$\tau = 0.25$				$\tau = 0.5$				$\tau = 0.75$			
	u	$N(0,3)$	$N(0,5)$	t_3	χ^2_3	$N(0,3)$	$N(0,5)$	t_3	χ^2_3	$N(0,3)$	$N(0,5)$	t_3	χ^2_3
PQR													
$\hat{\alpha}$	Bias	0.0560	0.0200	0.0596	0.1828	0.0557	0.0202	0.0594	0.2206	0.0554	0.0202	0.0594	0.2733
	RMSE	0.0730	0.0525	0.0672	0.1836	0.0720	0.0511	0.0669	0.2216	0.0722	0.0519	0.0671	0.2751
$\hat{\beta}$	Bias	0.1348	0.1655	0.0558	0.4019	0.1359	0.1662	0.0562	0.5280	0.1368	0.1666	0.0567	0.7295
	RMSE	0.1653	0.2047	0.0696	0.4093	0.1652	0.2032	0.0697	0.5370	0.1669	0.2046	0.0706	0.7442
DPQR													
$\hat{\alpha}$	Bias	-0.1209	-0,1611	-0.1392	-0.1851	-0.1190	-0.1608	-0.1356	-0.1753	-0.1160	-0.1577	-0.1297	-0.1704
	RMSE	0.1394	0.1772	0.1470	0.1933	0.1367	0.1758	0.1430	0.1852	0.1358	0.1745	0.1384	0.1844
$\hat{\beta}$	Bias	-0.0510	-0.0531	-0.0710	0.1375	0.0482	0.0781	-0.0191	0.2254	0.1443	0.2050	0.0313	0.3739
	RMSE	0.1213	0.1551	0.0925	0.1619	0.1120	0.1516	0.0534	0.2478	0.1840	0.2477	0.0654	0.4029
PQR-IVQR													
$\hat{\alpha}$	Bias	0.1133	0.0522	0.1246	0.2228	0.1019	0.0478	0.1103	0.2657	0.1123	0.0520	0.1237	0.3261
	RMSE	0.1415	0,0985	0.1440	0.2235	0.1342	0.0937	0.1354	0.2667	0.1398	0.0975	0.1431	0.3280
$\hat{\beta}$	Bias	0.1135	0.1537	0.0565	0.2735	0.1207	0.1573	0.0612	0.3801	0.1182	0.1561	0.0583	0.5548
	RMSE	0.1510	0.1981	0.0710	0.2848	0.1534	0.1963	0.0741	0.3934	0.1532	0.1977	0.0723	0.5765

Table 5: Bias and RMSE results for different panel sizes ($\tau = 0.5$)

		$T=5$			$T=15$			$T=25$		
		$N=50$	$N=100$	$N=150$	$N=50$	$N=100$	$N=150$	$N=50$	$N=100$	$N=150$
					The location shift model					
PQR										
$\hat{\alpha}$	Bias	0.2134	0.2147	0.2157	0.0751	0.0749	0.0747	0.0400	0.0398	0.0400
	RMSE	0.2174	0.2165	0.2169	0.0822	0.0785	0.0771	0.0467	0.0432	0.0424
$\hat{\beta}$	Bias	0.1477	0.1472	0.1468	0.0587	0.0595	0.0595	0.0314	0.0324	0.0321
	RMSE	0.1713	0.1594	0.1552	0.0758	0.0680	0.0657	0.0482	0.0411	0.0383
DPQR										
$\hat{\alpha}$	Bias	0.1937	0.1978	0.1988	-0.1281	-0.1242	-0.1254	-0.1719	-0.1716	-0.1716
	RMSE	0.2029	0.2023	0.2015	0.1349	0.1279	0.1279	0.1745	0.1729	0.1726
$\hat{\beta}$	Bias	0.0566	0.0584	0.0577	-0.0075	-0.0055	-0.0054	-0.0240	-0.0221	-0.0229
	RMSE	0.1119	0.0899	0.0793	0.0490	0.0342	0.0275	0.0445	0.0345	0.0315
PQR-IVQR										
$\hat{\alpha}$	Bias	0.2947	0.2981	0.2998	0.1430	0.1697	0.1820	0.0881	0.1021	0.1089
	RMSE	0.2988	0.2998	0.3008	0.1635	0.1813	0.1891	0.1059	0.1122	0.1160
$\hat{\beta}$	Bias	0.0641	0.0654	0.0639	0.0558	0.0558	0.0548	0.0327	0.0336	0.0338
	RMSE	0.1141	0.0934	0.0842	0.0739	0.0651	0.0618	0.0491	0.0423	0.0398
					The location-scale shift model I					
PQR										
$\hat{\alpha}$	Bias	0.2284	0.2260	0.2274	0.0491	0.0476	0.0477	0.0220	0.0218	0.0216
	RMSE	0.2309	0.2272	0.2282	0.0521	0.0493	0.0487	0.0242	0.0228	0.0224
$\hat{\beta}$	Bias	0.1142	0.1185	0.1179	0.0281	0.0280	0.0272	0.0133	0.0126	0.0123
	RMSE	0.1345	0.1286	0.1244	0.0440	0.0363	0.0329	0.0277	0.0213	0.0187
DPQR										
$\hat{\alpha}$	Bias	0.2268	0.2274	0.2306	-0.1344	-0.1336	-0.1337	-0.1800	-0.1795	-0.1795
	RMSE	0.2323	0.2303	0.2324	0.1383	0.1356	0.1351	0.1813	0.1801	0.1800
$\hat{\beta}$	Bias	0.0175	0.0202	0.0185	-0.0170	-0.0164	-0.0165	-0.0224	-0.0222	-0.0227
	RMSE	0.0876	0.0631	0.0526	0.0431	0.0319	0.0279	0.0375	0.0305	0.0286
PQR-IVQR										
$\hat{\alpha}$	Bias	0.3125	0.3142	0.3160	0.0945	0.1166	0.1371	0.0625	0.0715	0.0726
	RMSE	0.3148	0.3151	0.3166	0.1127	0.1330	0.1505	0.0716	0.0766	0.0761
$\hat{\beta}$	Bias	0.0526	0.0558	0.0548	0.0335	0.0346	0.0344	0.0159	0.0156	0.0153
	RMSE	0.0918	0.0775	0.0690	0.0483	0.0426	0.0396	0.0298	0.0237	0.0212
					The location-scale shift model II					
PQR										
$\hat{\alpha}$	Bias	0.2129	0.2131	0.2137	0.0737	0.0732	0.0737	0.0389	0.0391	0.0387
	RMSE	0.2167	0.2151	0.2149	0.0806	0.0766	0.0761	0.0453	0.0426	0.0411
$\hat{\beta}$	Bias	0.1493	0.1469	0.1481	0.0589	0.0579	0.0583	0.0340	0.0326	0.0326
	RMSE	0.1714	0.1588	0.1563	0.0752	0.0665	0.0643	0.0492	0.0417	0.0384
DPQR										
$\hat{\alpha}$	Bias	0.1942	0.1946	0.1969	-0.1282	-0.1272	-0.1266	-0.1722	-0.1722	-0.1724
	RMSE	0.2037	0.1992	0.1999	0.1352	0.1306	0.1290	0.1749	0.1735	0.1733
$\hat{\beta}$	Bias	0.0576	0.0578	0.0572	-0.0065	-0.0074	-0.0077	-0.0216	-0.0217	-0.0227
	RMSE	0.1129	0.0885	0.0803	0.0488	0.0347	0.0291	0.0429	0.0343	0.0310
PQR-IVQR										
$\hat{\alpha}$	Bias	0.2948	0.2996	0.2993	0.1421	0.1654	0.1803	0.0881	0.1021	0.1063
	RMSE	0.2989	0.3012	0.3004	0.1618	0.1777	0.1878	0.1060	0.1120	0.1136
$\hat{\beta}$	Bias	0.0655	0.0637	0.0649	0.0560	0.0548	0.0551	0.0355	0.0344	0.0340
	RMSE	0.1140	0.0916	0.0842	0.0734	0.0643	0.0616	0.0504	0.0430	0.0399

Table 6: Bias and RMSE results for different tuning parameter values ($\tau = 0.5$)

λ		0	0.5	1	1.5	2
		The location shift model				
PQR						
$\hat{\alpha}$	Bias	-0.0755	0.0310	0.1234	0.1829	0.2173
	RMSE	0.0860	0.0507	0.1293	0.1863	0.2198
$\hat{\beta}$	Bias	-0.0098	0.0486	0.0943	0.1108	0.1078
	RMSE	0.0625	0.0757	0.1113	0.1255	0.1243
DPQR						
$\hat{\alpha}$	Bias	-0.3640	-0.2088	-0.0481	0.1031	0.1986
	RMSE	0.3666	0.2147	0.0781	0.1198	0.2049
$\hat{\beta}$	Bias	-0.0652	-0.0227	0.0189	0.0443	0.0385
	RMSE	0.0841	0.0647	0.0656	0.0782	0.0773
PQR-IVQR; FEQR						
$\hat{\alpha}$	Bias	-0.0719	0.0579	0.2127	0.2771	0.3003
	RMSE	0.0932	0.0926	0.2260	0.2808	0.3024
$\hat{\beta}$	Bias	-0.0172	0.0530	0.0687	0.0500	0.0307
	RMSE	0.0646	0.0786	0.0925	0.0805	0.0723
		The location-scale shift model I				
PQR						
$\hat{\alpha}$	Bias	-0.0234	0.0312	0.1003	0.1671	0.2110
	RMSE	0.0298	0.0368	0.1048	0.1707	0.2135
$\hat{\beta}$	Bias	-0.0046	0.0245	0.0568	0.0799	0.0851
	RMSE	0.0424	0.0482	0.0712	0.0919	0.0983
DPQR						
$\hat{\alpha}$	Bias	-0.3442	-0.1982	-0.0478	0.1113	0.2138
	RMSE	0.3467	0.2018	0.0706	0.1279	0.2194
$\hat{\beta}$	Bias	-0.0661	-0.0347	-0.0018	0.0199	0.0161
	RMSE	0.0818	0.0596	0.0498	0.0573	0.0597
PQR-IVQR; FEQR						
$\hat{\alpha}$	Bias	-0.0238	0.0540	0.1805	0.2787	0.3064
	RMSE	0.0393	0.0717	0.2024	0.2841	0.3079
$\hat{\beta}$	Bias	-0.0066	0.0312	0.0538	0.0420	0.0302
	RMSE	0.0430	0.0529	0.0702	0.0642	0.0604
		The location-scale shift model II				
PQR						
$\hat{\alpha}$	Bias	-0.0728	0.0291	0.1203	0.1820	0.2162
	RMSE	0.0836	0.0490	0.1265	0.1855	0.2187
$\hat{\beta}$	Bias	-0.0089	0.0491	0.0926	0.1085	0.1082
	RMSE	0.0596	0.0746	0.1089	0.1235	0.1229
DPQR						
$\hat{\alpha}$	Bias	-0.3643	-0.2114	-0.0510	0.1001	0.1964
	RMSE	0.3669	0.2172	0.0785	0.1178	0.2030
$\hat{\beta}$	Bias	-0.0647	-0.0230	0.0190	0.0433	0.0380
	RMSE	0.0832	0.0627	0.0635	0.0773	0.0747
PQR-IVQR; FEQR						
$\hat{\alpha}$	Bias	-0.0677	0.0558	0.2131	0.2770	0.3003
	RMSE	0.0914	0.0909	0.2271	0.2807	0.3023
$\hat{\beta}$	Bias	-0.0171	0.0534	0.0681	0.0506	0.0324
	RMSE	0.0623	0.0777	0.0901	0.0805	0.0719

國科會補助計畫衍生研發成果推廣資料表

日期:2013/10/31

<table>
<tr><td rowspan="3">國科會補助計畫</td><td>計畫名稱: 動態追蹤資料分量迴歸</td></tr>
<tr><td>計畫主持人: 林馨怡</td></tr>
<tr><td>計畫編號: 101-2410-H-004-011- 學門領域: 數理與數量方法</td></tr>
<tr><td colspan="2">無研發成果推廣資料</td></tr>
</table>

101 年度專題研究計畫研究成果彙整表

<table>
<tr><td colspan="3">計畫主持人：林馨怡</td><td colspan="5">計畫編號：101-2410-H-004-011-</td></tr>
<tr><td colspan="8">計畫名稱：動態追蹤資料分量迴歸</td></tr>
<tr><td colspan="3" rowspan="2">成果項目</td><td colspan="3">量化</td><td rowspan="2">單位</td><td rowspan="2">備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）</td></tr>
<tr><td>實際已達成數（被接受或已發表）</td><td>預期總達成數(含實際已達成數)</td><td>本計畫實際貢獻百分比</td></tr>
<tr><td rowspan="12">國內</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td></td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（本國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="12">國外</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
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<tr><td rowspan="4">參與計畫人力（外國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
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<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
</table>

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

   ■達成目標

   □未達成目標（請說明，以 100 字為限）

   □實驗失敗

   □因故實驗中斷

   □其他原因

   說明：

2. 研究成果在學術期刊發表或申請專利等情形：

   論文：□已發表 ■未發表之文稿 □撰寫中 □無

   專利：□已獲得 □申請中 ■無

   技轉：□已技轉 □洽談中 ■無

   其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）