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考量流動性風險下巨災債券與颶風衍生性商品之評價、實證與風險管理(第2年)

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中 文 摘 要 ： 本研究在縮減式模型的架構下提出了一個一般化的巨災債券評價模型，同時包含了巨災風險、違約風險以及利率風險。本研究模型具有相當的彈性，可廣泛地適用於各種支付函數與不同契約條款的巨災債券商品。本模型可以透過標的災害在特定地區所適用的損失幅度分配以及災害發生頻率來刻畫最重要的巨災風險。此外，信用風險與利率風險也具體地建構於本研究的評價模型之中。透過敏感度分析可以驗證巨災債券條款設定與巨災債券的風險對於債券價格的影響方向與程度，最後本研究以兼具全面性且合理變動的數值分析結果來佐證本評價模型的正確與穩健性。

中 文 關 鍵 詞 ： 巨災債券、縮減式模型、違約風險

英 文 摘 要 ： This paper proposes a general pricing formula based on a reduced-form model to evaluate catastrophe bonds (CAT bonds) with catastrophe risks, default risks, and interest rate risks. This model is flexible and can be widely used in CAT bond markets with a variety of payoff functions and CAT bond provisions. With regard to the most important catastrophe risks, we can choose the specific distributions for loss severity and the counting process of the frequency of catastrophe events according to the underlying perils and region. Moreover, the credit risks and interest rate risks are concretely modeled and incorporated into the pricing model. The scenario analysis demonstrates how the provisions and the risks of CAT bonds affect bond price, and the reasonable numerical results reveal the validity and robustness of our pricing model.

英 文 關 鍵 詞 ： CAT bond, reduced-form, default risk

1 Introduction

Catastrophes include exposure to losses from natural disaster events, such as earthquakes, tsunamis, storms, hurricanes, floods, droughts, and extreme temperatures that are infrequent, sudden, and unforeseen and cause critical casualties, severe destruction, and widespread property loss. The internationally recognized authority Property Claim Services (PCS) defines catastrophes as events that cause more than $25 million loss in insured property and affect a significant number of policyholders and insurers. The Centre for Research in the Epidemiology of Disaster (CRED) maintains an Emergency Events Database (EM-DAT) records the climatic events that have satisfied at least one of following disaster criteria: at least 10 people were killed; at least 100 people were affected; a state of emergency was declared or international assistance was requested.

The Intergovernmental Panel on Climate Change's (IPCC) fifth assessment report (2013) unequivocally documents the warming in the climate system and states that the changes in ocean surface temperature and greenhouse gases are very likely to cause many extreme climate events, for example, by increasing the number of heavy precipitation events and the frequency of heat waves. The frequency and severity of unanticipated catastrophic events have increased in recent years; in particular, weather disasters and natural hazards have increased economic loss and placed more people in harm's way (Cutter and Emrich, 2005; O'Brien et al., 2006; Chang et al., 2011; Nowak and Romaniuk, 2013; Smith and Katz, 2013). Figure 1 displays the statistical data provided by the EM-DAT Database of U.S. billion-dollar disaster events from 1980 to 2013. The figure reveals that the frequency of severe catastrophes has increased over the last several years.

[**Insert Figure 1 Here**]

Due to the threat of catastrophes, understanding how to transfer catastrophe risk has become a significant issue, and the availability of insurance-linked securities for transferring catastrophe risk has risen sharply as well. Although reinsurance is an efficient international instrument for diversifying risks among insurance companies, it is limited by insufficient market capacity, high early termination costs, the risk of illiquidity and asymmetric information caused by high-risk market participants (Cummins and Geman, 1994; Smith and Katz, 2013). Because of these limitations, insurers have turned their attention to the capital markets, whose sizable capital and high liquidity can better hedge against catastrophe risk. Hence, alternative capital market instruments for dispersing catastrophe risk, such as CAT bonds, catastrophe equity puts, and catastrophe derivatives, have been continuously evolving.

Issuance of catastrophe bonds transfers catastrophe risk to capital markets. A feature of CAT bonds is that the payoff conditions depend on whether the catastrophe actually occurs. The bondholders pay the principal in advance, and the bond issuer pays the fixed coupon to bondholders. The subsequent payments of coupons and the return of principal depend on the occurrence and the severity of the catastrophe. Cummins (2008) reviewed the market and development of CAT bonds and other risk-linked securities. Due to demand for transferring the risk of insurers and the attractive high yield for investors, the issuance volume of CAT bonds has been growing steadily. Moreover, CAT bonds are international investment instruments that serve as useful sources for diversifying portfolio risk because catastrophic events have low correlations with financial markets; thus, CAT bonds have gradually become the dominant instrument in catastrophe risk management.

CAT bond markets have witnessed tremendous growth in recent years. Figure 2 shows the trend of global CAT bond issuance volume from 1996 to 2013. We can observe that the issuance volume of CAT bonds has grown steadily as catastrophes have occurred more frequently. Therefore, the proper valuation of CAT bonds has become important.

[**Insert Figure 2 Here**]

One particular line of existing literature regarding valuation of CAT derivatives-linked derivatives focuses on modelling the frequency of catastrophes and applying different distributions for fitting the loss severity. These researchers use binomial process, negative binomial process, pure Poisson process (Cummis and Geman, 1995; Louberge et al., 1999; Lee and Yu, 2002; Cox et al., 2004; Jaimungal and Wang, 2006), doubly stochastic Poisson process (Dassios and Jang 2003; Lin et al., 2009; Wu and Chung, 2010; Ma and Ma, 2013), and Markov modulate Poisson process (Chang et al., 2011) to capture the occurrence of catastrophic events. Regarding the loss severity, the existing literature indicates that the lognormal, Burr, Pareto, and Gamma distributions are candidates for analyzing the loss data (Hogg and Klugman, 1984; Matthys et al., 2004; Jaimungal and Wang, 2006; Lescourret and Robert, 2006; Klugman et al., 2008). However, the extant empirical evidence indicates that the heavy-tailed distributions outperform the lighter-tailed distributions in terms of fitting the catastrophe loss severity.

Another line of research focuses on developing the sparse pricing framework used to evaluate catastrophic contracts and derivatives. Cox and Pedersen (2000) evaluated catastrophe risk bonds in an incomplete market setting and used the term structure of interest rates and the probability structure of catastrophe risk to develop the arbitrage-free valuation framework. Lee and Yu (2002) combined the stochastic interest rate model and loss process, which is assumed to be a lognormal distribution, to calculate the default-free, default-risky, and simultaneous default-risk and basis-risk CAT bond prices using the Monte Carlo method. Vaugirard (2003a) used the arbitrage pricing method to estimate CAT bond prices. They then adopted the stochastic interest rate and assumed the aggregate loss index as a jump-diffusion process. They used the Monte Carlo method to simulate other CAT bond prices in mid-range catastrophe cases and used sensitivity analysis to determine the effect of different parameter settings on the CAT bond prices. Vaugirard (2003b) simulated catastrophe bonds and weather-linked bonds with an arbitrage-free framework. The numerical simulations can be used to control the error of estimation of payoffs of insurance-linked securities, and they show that the term structure of CAT bond yield spreads is hump shaped. Hainaut and Devolder (2008) proposed a method of pricing a CAT bond with a stochastic seasonal effect and multiple coupons. To integrate the seasonal effect into the pricing model, the claim arrival process is modeled as a doubly stochastic Poisson process (DSPP) whose intensity is the sum of one deterministic seasonal function and one mean-reverting stochastic process, and the CAT bond is priced using the Fast Fourier Transform. Apart from the above-mentioned studies, Egami and Young (2008) proposed a structured CAT bond pricing method based on utility indifference. Unger (2010) developed a formulation and discretization strategy utilizing a numerical PDE approach to price components of a Bermudan-style callable CAT bond. Nowak and Romaniuk (2013) used Monte Carlo simulations to analyze the numerical properties of a general pricing formula that can be applied to CAT bonds with different payoff functions incorporating various interest rate models.

Jarrow (2010), noting that the existing literature for pricing CAT bonds contains models that are either too complex or based on equilibrium constructs, proposed a general methodology for valuing CAT bonds based on the reduced-form model used to price credit derivatives. Unlike the model proposed by Jarrow (2011) to evaluate CAT bonds, the dynamic reduced-form model we adopt can be applied to CAT bonds with different payoff functions and can explicitly evaluate the coupon corresponding to the expected residual principal. We further release the single disaster restrictions to allow for multiple disasters within the CAT bond contract period, and we incorporate default risk into the CAT bond price. Although most CAT bonds are issued by special purpose vehicles (SPV), Lee and Yu (2002) document the CAT bonds that issued by a traditional insurer may emerge the default risk. Wu and Chang (2010) note that the credit risks of financial institutions will become the counterparty risks of CAT instruments and result in significant risk pricing[1].

The remainder of this paper is organized as follows. The next section describes the structure and investigates the various risks of CAT bonds. Section 3 specifically models the CAT bond risks and introduces an efficiently reduced-form pricing model for CAT bonds. In Section 4, we provide a sensitivity analysis of the pricing formulas. Finally, we conclude the paper in Section 5.

2 Brief Description of a Catastrophe Bond

In addition to traditional reinsurance, CAT bonds are an additional instrument with which to hedge the underlying risk from capital markets. That is, CAT bonds are risk-linked securities, and sponsors can transfer the specific loss from disasters to the capital markets. Most CAT bonds are issued by an SPV, which is a legal entity established by the sponsor to insulate the bondholders and sponsor from credit risk, as the capital in the SPV is independent of the assets of the sponsor. The proceeds of SPV issued bonds can only be used on the payment of claims; they cannot be used by reinsurers on their financing problems even if the reinsurers are insolvent. In contrast, placement issue bonds do involve counterparty default risk because the proceeds collected from bondholders are not independent of the capital of the sponsor.

The CAT bond will track the loss of the ultimate beneficiary, a loss index such as PCS, or a specific index measured by reliable risk modeling agents. The bondholders will receive a high-yield coupon of LIBOR plus a spread (generally between 3 and 20%) on a predetermined schedule (usually three months) if the amount of losses from the disasters does not reach the attachment point, but they bear the underlying risk, which will cause

[1]The credit risk of CAT bonds will be discussed in detail in section 2.1 as we introduce the transaction structure of CAT bonds.

loss of principal as the losses exceed the attachment point. The principal linearly and proportionally decreases to zero, until the losses reach the exhaustion point, and the reduced principal is given to the ultimate beneficiary, who essentially helps the insurance company to pay claims or cover the losses that arise in the aftermath of the disaster. If the catastrophe events cause the loss of principal, the coupon will be reduced linearly and proportionally as well. However, CAT bonds are still a popular and useful instrument for fund managers due to their attractive high yields and the fact that there is hardly any correlation between CAT bonds and other financial securities or business cycles, and thus CAT bonds can be used to diversify the portfolio risk.

CAT bonds can be issued through private placements as well. The private placement of CAT bonds is not common due to the restrictions of legislative regulations in each country. However, private CAT bonds have become more widespread in recent years. In 2011, the service company Towers Watson Capital Markets (TWCM) successfully issued $11.95 million worth of private CAT bonds for an insurance company in Florida using an intensity-based trigger, and the company doubled the size of CAT bond issuances in 2012. After these issuances, more private CAT bonds have been issued. This case indicates important progress for middle-market insurance companies to access the CAT bond market.

2.1 Risk of CAT Bond

Before we analyze the types of CAT bonds, we must first evaluate the risks in CAT bonds so that we can incorporate those risks into the pricing formulas. The major risk to bondholders is the catastrophe risk, which includes loss occurrence and severity. A number of previous studies attempt to find the best fit model for loss data.

To model the catastrophe loss severity distribution, most of the prior studies use the heavy-tailed distribution such as the lognormal, Weibull, Pareto, and Burr distribution for modeling the loss severity. Levi and Partrat (1991) examined the losses caused by hurricane in the United State from 1954 to 1986. The Poisson and negative binomial distribution for catastrophe occurrence and exponential, Pareto, lognormal distribution for loss severity are tested and the empirical results confirmed by chi-square, Kolmogorov-Smirnov, nonparametric, and Anderson-Darling tests show that the Poisson lognormal model is the best fit. Burnecki et al. (2000) obtained a similar result by using the quarterly PCS index from 1950 to 1999 to find the best fit distribution, and found that the lognormal demonstrates a better fit than the Burr, Pareto, and Gamma distributions. Burnecki and Kukla (2003) obtained the same conclusion using PCS index from 1990 to 1999 as well. Milidonis and Grace (2008) used the historical catastrophe loss for the state of Florida from PCS to find the best frequency and severity distribution. The likelihood ratio test and Chi-square test support that the Binomial distribution has a better goodness of fit than the Poisson process and Negative Binomial distributions. The results of choosing the best model of loss severity distribution indicate that the Pareto and Burr distribution have better performance of fitting than the lognormal, exponential, and Weibull distributions. Braun (2011) proposed a two-stage contingent claims pricing approach to analyzing the catastrophe CAT swap. The occurrence of catastrophe is modeled as a doubly stochastic Poisson process with mean-reverting Ornstein-Uhlenbeck intensity. Braun used the historical loss data for hurricanes and earthquakes in the U.S. to find the appropriate loss distribution, and confirmed that heavy-tailed distributions, such as the Burr distribution, are more appropriate than the distributions with lighter tails.

Regarding the frequency of catastrophe events, Lin, Chang, and Powers (2009) proposed the doubly stochastic Poisson process (DSPP) instead of the pure Poisson process to describe the arrival process of catastrophic events. They also derived a general pricing formula for contingent capital and for pricing the catastrophe equity put using the data from the PCS loss index for the annual catastrophic events from 1950 to 2004. The results show that the arrival rate of catastrophe events is an important factor that significantly affects the price. Wu and Chung (2010) assumed the intensity process of catastrophe occurrence to be a DSPP with mean-reverting process in order to reflect climate phenomena such as the El Nino event. Ma and Ma (2013) apply the nonhomogeneous Poisson process with the trigonometric function to model the intensity of the frequency of catastrophic events and use the model to price catastrophe risk bonds because they argue the historical annual number of claims exhibit cyclic and periodic trends. From these studies, we know the DSPP is a flexible and generally used model and can be used to capture the characteristics of occurrence of disasters.

In addition to the catastrophic risk, the interest rate risk is also a crucial factor affecting the present value of a CAT bond. Most CAT bonds offer multi-year coverage, usually three to five years, and the coupon rate is linked with LIBOR so that the interest rate curve has direct influence on CAT bond price. Vaugirard (2003a, 2003b), Lin et al. (2009), and Chang et al. (2011) applied the spot interest rate using the Vasicek model (Vasicek, 1977) in their studies. Lee and Yu (2002), Wu and Chung (2009), and Ma and Ma (2013) document that CAT derivatives are interest rate sensitive instruments and apply the spot interest rate using the CIR model of Cox et al. (1985) in their studies. The CIR model captures the mean-reverting character of interest rates and avoids the negative interest rate that may appear in the Vasicek model. In short, from the above-mentioned studies, we know the stochastic interest rate model is necessary to reduce the interest rate risk.

The third risk in our model is the credit risk. As mentioned previously, most catastrophe bonds are usually issued by an SPV and have been regarded risk-free. In a typical catastrophe bond transaction, to insulate bondholders from interest rate movements and the mark-to-market exposure of collateral proceeds, CAT bonds are structured with total return swaps (TRS) to remove the investment risk and operation risk of the insurer. However, as the TRS counterparty goes into default status, the CAT bond will be downgraded and could even default if the counterparty cannot pay the full amount of scheduled interest. That is, the SPV issued bond isolates bondholders from the credit risk of the insurer, but it cannot protect bondholders from the SPV's credit risks resulting from TRS counterparty risk. The collapse of Lehman Brothers in 2008 is a famous example demonstrating the credit risk of CAT bonds. Lehman's bankruptcy directly affected four CAT bonds (Ajax Re, Carillon Ltd A-1, Newton Re 2008 A-1 and Willow Re B) that were Lehman's TRS counterparties, and these four bonds ended up defaulting one by one. Hence, we must consider the counterparty risk of TRS to obtain an accurate CAT bond price and reveal how credit risk affects the bond price in our pricing formula.

2.2 The Payoff of CAT Bond

The disaster losses will determine the amount of principal repayment. In the current state of the market, the most common CAT bond triggers are the indexed to industry loss and indemnity triggers. Both these trigger types have attachment point (a_L) and exhaustion point (a_H) that are specified in the contract. Here, we introduce the variable of CAT bond payment of the principal at maturity that we analyze in this study. The principal begins to reduce when the aggregate catastrophe losses exceed the attachment point, and no principal is repaid if the disaster losses exceed the exhaustion point. Let L be the catastrophe losses; the residual CAT bond principal at time t for per unit dollar is governed by:

$$W(t|L) = \begin{cases} 1 & , L < a_L, \\ 1 - \dfrac{L - a_L}{a_H - a_L} & , a_L < L < a_H, \\ 0 & , L > a_H. \end{cases} \qquad (1)$$

If the catastrophe losses fall below the attachment point, bondholders can receive the entire principal. When the catastrophe losses fall between attachment point and exhaustion point, bondholders can only receive a proportion of the principal. Finally, if the catastrophe losses exceeds the exhaustion point, bondholders cannot receive any portion of the principal.

3 The Pricing Model for Catastrophe Bonds

This section investigates the assumption of the interest rate, loss distribution and arrival rate of catastrophe events. Under the assumptions of a perfect market and no arbitrage opportunities, we evaluate the expected principal at each payment date and then derive the pricing formula of CAT bonds.

3.1 Modeling the CAT Bond Risks

The major uncertainties in this phase of modeling a CAT bond are the frequency and severity of natural disasters during the term of the contract. Following Lin et al. (2009) and Wu and Chung (2010), we assume the occurrence of catastrophe events following the DSPP and assume the process of the stochastic counting process $\lambda_c(t)$ to be:

$$\lambda_c(t) = \lambda_c(0) \exp\left[\left(\mu_\lambda - \frac{1}{2}\sigma_\lambda^2\right)t + \sigma_\lambda W_{\lambda,t}\right], \qquad (2)$$

where μ_λ and σ_λ is the instantaneous change rate and the volatilities of change rate of the arrival rate of catastrophic events, respectively. Then, we integrate the intensity to obtain the integrated density $\Lambda_t = \int_0^t \lambda_c(s)\, ds$. Thus, the probability of exactly n disasters in time t can be express by the integrated density:

$$\Pr\left[\Phi_t = n | \lambda_c(u), 0 \le u \le t\right] = \frac{e^{-\Lambda_t}\Lambda_t^n}{n!}, \qquad (3)$$

where Φ_t is the counting process of catastrophe events.

In addition to the frequency distribution, a distribution for the severity of the loss from natural disasters is also a crucial component for the pricing model. The claims of CAT bonds are triggered when catastrophe losses accumulate to a specific level before maturity. We assume the aggregate losses L is the sum of independent identically distributed random variables. Thus, given the number of catastrophe events $\Phi_t = n$ in a specified period, the distribution of aggregate losses can be expressed as $L = X_1 + \cdots + X_n$, where X_j represents the loss induced by the j^{th} disaster and follows the specific severity distribution investigated by previous studies, such as heavy tailed distributions. We further define the aggregate losses distribution, $f(L|\Phi_t)$, which can give the losses L a certain probability according to the distribution we choose, and the constant L can further determine the residual principal when pricing the CAT bond.

However, finding the aggregate losses distribution is not an easy task using any of the severity distributions. Some distributions, such as exponential distribution, Gamma distribution and Pareto distribution, have an explicit form for the sum of independent identically distributed random variables. According to the statistical theory, we know the sum of exponential random variables is an Erlang random variable and the sum of Gamma random variables is a Gamma random variable. Let $S_n = x_1 + \cdots + x_n$. If $x_i \overset{IID}{\sim} f_{X_i}^{Exp}(x;\lambda)$ for $x \geq 0$, then $S_n \sim f_{S_n}^{Erlang}(x; n, \lambda)$ which is a special case of the Gamma distribution, $f_{S_n}^{Erlang}(x; n, \lambda) = \frac{e^{-\lambda x}\lambda^n x^{n-1}}{(n-1)!}$ for $x > 0$. If $x_i \overset{IID}{\sim} f_{X_i}^{Gamma}(x; \alpha, \beta)$ for $x \geq 0$, where $\alpha > 0$, $\beta > 0$, and $\Gamma(\cdot)$ is the Gamma function, then $S_n \sim f_{S_n}^{Gamma}(x; n\alpha, \beta)$.

According to Ramsay (2008), we have the explicit form for distribution of the sums of Pareto random variables: If $x_i \overset{IID}{\sim} f_{X_i}^{Pareto}(x; \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{\beta}{x+\beta}\right)^{\alpha+1}$ for $x \geq 0$, then $S_n \sim f_{S_n}^{Pareto}(x; n, \alpha, \beta)$, where $f_{S_n}^{Pareto}(x; n, \alpha, \beta) = \frac{1}{\beta}\int_0^\infty e^{-\frac{xu}{\beta}}\chi_n(u, \alpha, n)\,du$,

$$\chi_n(t, \alpha, n) = \sum_{r=0}^{\frac{n-1}{2}} \frac{(-1)^r}{\pi} C_{2r+1}^{n} \left[R(t, \alpha)\right]^{n-2r-1} \left[I(t, \alpha)\right]^{2r+1},$$

$$R(t, \alpha) = 1 + \sum_{r=1}^{\infty} \frac{t^r}{(\alpha-1)\cdots(\alpha-r)} - \frac{\pi t^\alpha e^{-t}}{\Gamma(\alpha)}\cot(\pi\alpha), \quad I(t, \alpha) = \frac{\pi t^\alpha e^{-t}}{\Gamma(\alpha)}.$$

The distribution of a sum of n independent random variables with Burr distribution and Weibull distribution are referenced in Kortschak and Albrecher (2010) and Yilmaz and Alouini (2009), respectively. For the sum of random variables with lognormal distribution, which that does not have an exact explicit distribution form, we can adopt the moment-matching method to obtain the approximated distribution, similar to how Borovkova et al. (2007) used the generalized family of lognormal distributions to obtain the approximated distribution and Milevsky and Posner (1998) used distributions from the Johnson (1949) family to approximate four moments of the sum of random variables with lognormal distribution[2].

The loss process of catastrophe is a kind of jump risk in an incomplete market, and a unique risk neutral measure does not exist. However, we adopt the theorem of Merton (1976): assuming that the entire economy is only affected by regional catastrophes then the loss number process and the loss values are the specific influences on the capital market, which means that catastrophe impacts are nonsystematic risks and thus have no risk premium. According to the assumptions of Cummins and Geman (1995), Cox and Pedersen (2000), and Lee and Yu (2002) the loss process is the same under the risk neutral measure.

As mentioned previously, the credit risk of CAT bonds comes from the SPV due to the potential insolvency of the TRS counterparty. The Poisson process is the basic credit-event-counting process for modeling low-frequency default events (Smithson, 2003). Therefore, we define $\lambda_d(t)$ as the arrival rate of default process of the TRS counterparty and τ_D as the time when the TRS counterparty of the CAT bond defaults. We further assume the default process follows a Poisson process with the homogeneous arrival rate of default process in our model, that is, the $\lambda_d(t) = \lambda_d$ is a constant in the entire CAT bond risk period. Hence, the default probability of TRS counterparty default can be expressed as $\Pr(\tau_D > t) = e^{-\lambda_d t}$. This default probability should be incorporated in the pricing model for CAT bonds and we will show how it affects the CAT bond price in the next section.

For the interest rate risks, we adopt the stochastic interest rate model in Cox et al. (1985, CIR) as follows:

$$dr_t = \kappa(\theta - r_t)\,dt + \sigma_r\sqrt{r_t}\,dW_{r,t}, \tag{4}$$

where r_t stands for the instantaneous interest rate at time t, κ represents the mean-

[2]Approximation methods are continuously being innovated. However, the performance of the various approximation methods is not the focus in this study, so we do not make a conclusion regarding the optimal approximated distribution for pricing CAT bonds. What we need in our pricing model is an explicit distribution form.

reverting force measurement, θ represents the long-term mean interest rate, σ_r denotes the volatility of interest rate, and $W_{r,t}$ is the Wiener process of the instantaneous interest rate. The price of the T-maturity zero coupon bond (ZCB), $B(t,T)$, in the CIR interest rate model is given by

$$B(t,T) = C(t,T)\, e^{-D(t,T)r_t} \tag{5}$$

where

$$C(t,T) = \left[\frac{2he^{\frac{(h+\kappa+\eta)(T-t)}{2}}}{2h + (h+\kappa+\eta)\left(e^{h(T-t)} - 1\right)}\right]^{\frac{2\kappa\theta}{2}},$$

$$D(t,T) = \frac{2\left(e^{h(T-t)} - 1\right)}{2h + (h+\kappa+\eta)\left(e^{h(T-t)} - 1\right)}, \quad h = \sqrt{(\kappa+\eta)^2 + 2\sigma_r^2},$$

and η is the market price of interest rate risk.

3.2 CAT Bond Pricing Model

To determine the non-arbitrage CAT bond price from the cash flow of the bond holder, we specify the incomes associated with the expected principal of a CAT bond. Given the coupon payment times, $t_1 < \cdots < t_m$, the bond holder will receive coupon payment corresponding to the residual principal at each payment date. According to the frequency and loss distribution of catastrophe events, we can further evaluate the unconditional expected principal for any time $t_k \in [t_1, t_m]$ as follows:

$$\mathrm{E}(t_k) = \mathrm{E}\left\{\sum_{n=0}^{\infty}\left[\Pr\left(\Phi_{t_k} = n|\Lambda_{t_k}\right)\int_0^{\infty} f_{S_n}^{LossDist}(L|n)\, W(t_k|L)\, dL\right]\cdot 1_{\{\tau>t_k\}}\middle|\mathcal{F}_{t_k}\right\}, \tag{6}$$

where $\mathcal{F}_t^c = \sigma\left[\lambda_c(u), 0 \le u \le t\right]$ and $\mathcal{F}_t^d = \sigma\left[\lambda_d(u), 0 \le u \le t\right]$ are the filtrations generated by $\lambda_c(u)$ and $\lambda_d(u)$, respectively. The collective filtration $\mathcal{F}_t = \mathcal{F}_t^c \vee \mathcal{F}_t^d$ represents the smallest σ-field containing both $\mathcal{F}_t^c$ and $\mathcal{F}_t^d$, that is, containing the information of catastrophic events processes and default process, respectively. $f_{S_n}^{LossDist}(\cdot|n)$ is the density function for the sum of n random variables with the selected severity distribution.

The expected principal can be evaluated with the following steps:

1. Set the payment time t_k.

2. Determine the stochastic arrival rate of catastrophic events $\lambda_c(s)$ for $0 < s \le t$ and integrate the arrival rate to obtain the integrated intensity Λ_{t_k}.

3. Calculate the corresponding probability of the number of occurrences of the counting process Φ_{t_k} under Poisson distribution with Λ_{t_k}[3].

4. For the n-times disasters, we have the aggregate losses distribution $f_{S_n}^{LossDist}(L|n)$, for instance, $f_{S_n}^{Pareto}(L|n,\alpha,\beta)$ or $f_{S_n}^{Gamma}(x|n\alpha,\beta)$ based on the severity distribution for the individual catastrophe.

5. Compute the residual principal for the CAT bond for the corresponding catastrophe loss L.

6. Multiply the survival probability that represents the credit risk of the TRS counterparty.

The present value of the interest payment in proportion to the expected principal that bondholders will receive at the payment date is:

$$\sum_{k=1}^{m}(s+l_{k-1})(t_k-t_{k-1})\,\mathrm{E}(t_k)\,B(t_k), \tag{7}$$

where $B(t_k)=B(0,t_k)$ denotes the price of ZCB with maturity t_k in equation (5). $t_0=0$ is initial time of CAT bond issuance and the coupon rate includes floating LIBOR l_{k-1} and fixed spread s.

The relation between LIBOR and the price of zero coupon bonds is as follows:

$$l_{k-1}=\frac{1}{t_k-t_{k-1}}\left[\frac{B(t_{k-1})}{B(t_k)}-1\right], \tag{8}$$

Hence, the interest payment of the CAT bond can be further written as:

$$s\sum_{k=1}^{m}(t_k-t_{k-1})\,\mathrm{E}(t_k)\,B(t_k)+\sum_{k=1}^{m}\mathrm{E}(t_k)\left[B(t_{k-1})-B(t_k)\right]. \tag{9}$$

Thus, we have the fair CAT bond price as the discounted sum of interest payments at each payment time and the expected residual principal at maturity as follows:

$$P(0,T)=\sum_{k=1}^{m}\left[s(t_k-t_{k-1})B(t_k)+B(t_{k-1})-B(t_k)\,\mathrm{E}(t_k)+\mathrm{E}(t_k)\,B(t_k)\right]. \tag{10}$$

In this section, we derive an analytic solution for CAT bond prices that includes catastrophe risks, credit risks, and interest rate risks and allowing multiple disasters in

[3] The number of occurrences of catastrophe events theoretically should be calculated from 0 to infinitely. However, we can set an upper bound for Poisson probabilities under the given integrated intensity and truncate the number of occurrences as the Poisson probability exceeds the upper bound.

the CAT bond risk period. This model is flexible and can be widely used for the variety of CAT bonds in the market. We can choose the loss severity distribution and counting process for the occurrences of catastrophe events according to the underlying perils. In our model, we use DPSS for modeling the frequency of catastrophe, so we need to simulate the arrival rate process.

4 Sensitivity Analysis

This section uses sensitivity analysis to examine the pricing formula under different parameters. We analyze the CAT bond for per unit dollar (assume the principal of CAT bond is one dollar) when the frequency follows DSPP, the severity distribution is assumed to be Pareto distribution and Gamma distribution, the interest rates are described by the CIR model, and credit defaults are described by the Poisson model.

The parameters of base valuation are set as follows: the CAT bond is issued with a 3-year tenor ($T = 3$) and pays the 3-month LIBOR plus a spread $s = 400$ basis points (bps) quarterly. The parameters of stochastic intensity of DSPP are set to be $\mu_\lambda = 0.05$, $\sigma_\lambda = 0.1$, and $\lambda_c(0) = 0.2$. Following the empirical results calibrated by Braun (2011), the parameters of loss severity distribution are set with shape parameter $\alpha = 0.662$ and scale parameter $\beta = 1.13$ for the Pareto distribution. The attachment point and exhaustion point are set at $a_L = 20$ and $a_H = 50$, respectively. The parameters of the CIR interest rate model refer to Episcopos (2000) and Nowak and Romaniuk (2013) and are calibrated from the one-month interbank rate for the United States: $r_0 = 0.0614$, $\kappa = 0.0241$, $\theta = 0.053942$, $\sigma_r = 0.014142$, $\eta = 0$. Finally, the arrival rate of default process is set as $\lambda_d = 0.1155\%$, which is the average investment-grade global corporate default rate over last 20 years (1993-2013)[4].

Figure 3 provides an analysis of the attachment point and exhaustion point of the CAT bond to CAT bond prices. In the case of a fixed attachment point, the bond price decreases as the exhaustion point decreases because the lower the exhaustion point is, the lower the threshold of catastrophe losses that cause bondholders to lose their entire principal. Similarly, for a fixed exhaustion point, the bond price decreases as the

[4]Data Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

attachment point decreases because the lower the attachment point is, the lower the threshold at which the principal begins to be deducted, which decreases the value of the CAT bond.

[**Insert Figure 3 Here**]

Table 1 depicts the CAT bond prices controlled by the time to maturity and the spreads. From the panel A of Table 1, we observe that in the case of low spreads (e.g., spread is 100 bps), the bond price decreases with the bond's maturity, that is, the longer the time to maturity, the lower the bond price. However, in the case of higher spreads (e.g., spread is 1000 bps), the bond price increases with bond's maturity instead. We further analyze the components of bond price, the sum of the interest (Panel B) and the residual principal (Panel C), to understand these opposing results. In Panel C, we observe that the spread is independent of residual principal, which is inversely related with bond maturity because the longer the maturity is, the higher the catastrophe risks. From Panel B of Table 1, we observe that the interest increases with bond maturity and the increment in interest becomes greater as the spreads widen. By merging the effects of spreads and bond maturity on the interest and residual principal, we discover that the increment in interest is less than the decrement in residual principal that accompanies longer maturity in the case of low spreads. However, when the spreads are wide enough, the increment in interest will cover the decrement in residual principal that accompanies longer maturity, so that the CAT bond price increases.

[**Insert Table 1 Here**]

The impact of catastrophe risk on the CAT bond price is analyzed using different coefficients of loss distribution and frequency distribution of disasters. As mentioned in previous studies, the loss distribution plays an important role in CAT bond prices. Figure 4 and Figure 5 display parameters sensitivity analysis for the loss severity distribution under the Gamma and Pareto distributions, respectively. From Figure 4 and Figure 5, we can observe that the different loss distributions have different curvatures and characteristics affecting the bond price. This finding emphasizes the importance of choosing a proper distribution for pricing CAT bonds. However, the Gamma and Pareto distributions affect bond prices in one common way: The parameters make the loss distribution more heavy tailed (higher shape and scale parameters for Gamma; lower shape parameter and higher scale parameter for Pareto), which lowers bond prices. This result reveals the CAT bond is more vulnerable to severe catastrophe events, which lower the bond price.

[**Insert Figure 4 Here**]

[**Insert Figure 5 Here**]

In addition to the loss distribution, the frequency of catastrophe events is a crucial factor in the bond price. In our model, we adopt the DSPP for modeling the occurrence of disasters, which is driven by the stochastic arrival rate. Table 2 shows the scenario analysis of the instantaneous change rate (μ_λ) and the volatilities of change rate (σ_λ) of arrival rate to the bond prices. As the instantaneous change rate increases, the bond prices decrease because the frequency of catastrophes increases, which also increases the catastrophe risks. When the volatility of change rate of arrival rate increases, the bond prices are decreased slightly because the aggregate intensity of DPSS is integration of arrival rate, and this smooths the effects of the volatility change rate.

[**Insert Table 2 Here**]

Credit risk is also involved in our pricing model, and we analyze the effects of the default intensity of the TRS counterparty and bond maturity on the CAT bond price in Table 3. We adopt a Poisson process to describe the default event so that the higher the default intensity, the higher the default risk, which leads to lower bond prices for the same maturity length. On the other hand, for fixed default intensity, the bond price is positively correlated to bond maturity. Although the default intensity increases the default probability over time so that the expected principal decreases at each payment date (including residual principal), the spreads set in the base case (400 bps) are high enough to make the increment in interest larger than the decrement in residual principal that accompanies longer maturity, so that the bond price is positively correlated with maturity.

[**Insert Table 3 Here**]

[**Insert Table 4 Here**]

Before we analyze the influence of stochastic interest rates on the CAT bond price, we should first understand how the parameters of the CIR model affect the ZCB price. As documented in Cox et al. (1985), the bond price is a decreasing convex function of the mean interest rate level and an increasing concave function of the interest rate variance. Then, we examine the influence of stochastic interest rate on the CAT bond price from the point of view of ZCB. Table 4 shows how the long-term mean and the volatility of the instantaneous interest rate affect the bond price. From Panel B of Table 4, we observe that the long-term mean interest rate is positively correlated with the interest of CAT bonds but the volatility of interest rate is negatively correlated with the same. Because the coupon rate of CAT bonds is composed of the fixed spread and LIBOR and the LIBOR is positively correlated with the instantaneous interest rate, the higher the stochastic interest rate, the higher the CAT bond interest.

The opposite scenario results for the residual principal (Panel C of Table 4): the higher the mean level, the lower the residual principal, and the higher the interest rate volatility, the higher the residual principal. This result is due to the ZCB price being the discounted factor of residual principal, which causes the residual principal to have the same variation characteristics as ZCB price that controlled by the mean and volatility of the stochastic interest rate. Overall, from Panel A of Table 4, which shows the sum of the interest payments and the residual principal of CAT bonds, we know the mean stochastic interest rate is negatively correlated with bond price and the volatility has no significant influence on CAT bond prices.

5 Conclusion

CAT bonds are an important instrument in (re)insurance that transfers catastrophe risk onto capital markets, and CAT bond markets have witness tremendous growth in recent years. With the growth of CAT bond markets, constructing a comprehensive and flexible pricing model becomes crucial to providing a fair CAT bond price. In this paper, we use the dynamic reduced-form model that incorporates catastrophe risks, interest rate risks, and credit risks to evaluate CAT bonds. In contrast to Jarrow (2010), who adopted a reduced-form model for pricing CAT bonds as well, our pricing model can be applied to different payoff functions of CAT bonds and explicitly evaluate the coupon corresponding to the expected residual principal. In addition, we further remove the single disaster catastrophe event restrictions to allow for multiple disasters in the contract period. Finally, we incorporate the default risk of TRS counterparties into CAT bond prices to describe credit events such as the Lehman bankruptcy.

We illustrate the flexibility and rationality of our model via scenario analysis for the various provisions and risk models of CAT bonds. First, the CAT bond prices exhibit various and reasonable changes with changing bond maturity, spreads, and attachment and exhaustion thresholds in the provisions. Further, the rational price movement supports the validity of our pricing model. Next, the scenario analysis shows that the catastrophe risks that are controlled by loss distribution and the counting process of catastrophe events have a significant influence on bond prices. The various loss distributions have various distribution characteristics that affect interest and residual principal of CAT bonds. However, the loss distributions affect bond prices in one common way: with the distribution parameters for scenarios, the fat tailed distribution brings more catastrophe risks and decreases the value of CAT bonds. In addition, scenario parameters of the DSPP for catastrophe occurrence also sufficiently reflect the how the stochastic arrival rate affects the bond price. Finally, the impact of credit risks and interest rate risks on CAT bond prices are verified though the sensitivity analyses of default intensity and the parameters of the CIR model, respectively. The numerical results reveal that the CAT bond prices indeed will be affected by these risks. In particular, the credit risks, which are often ignored in the CAT bond valuation, may have a significant influence on bond price as the CAT bonds have long maturity.

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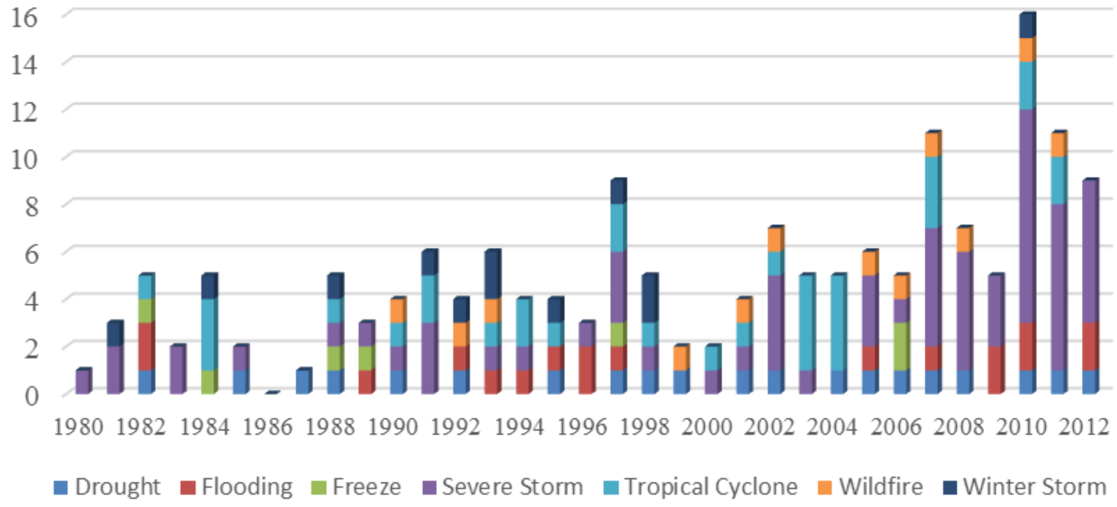

Figure 1: U.S. billion-dollar disaster events from 1980 to 2013.

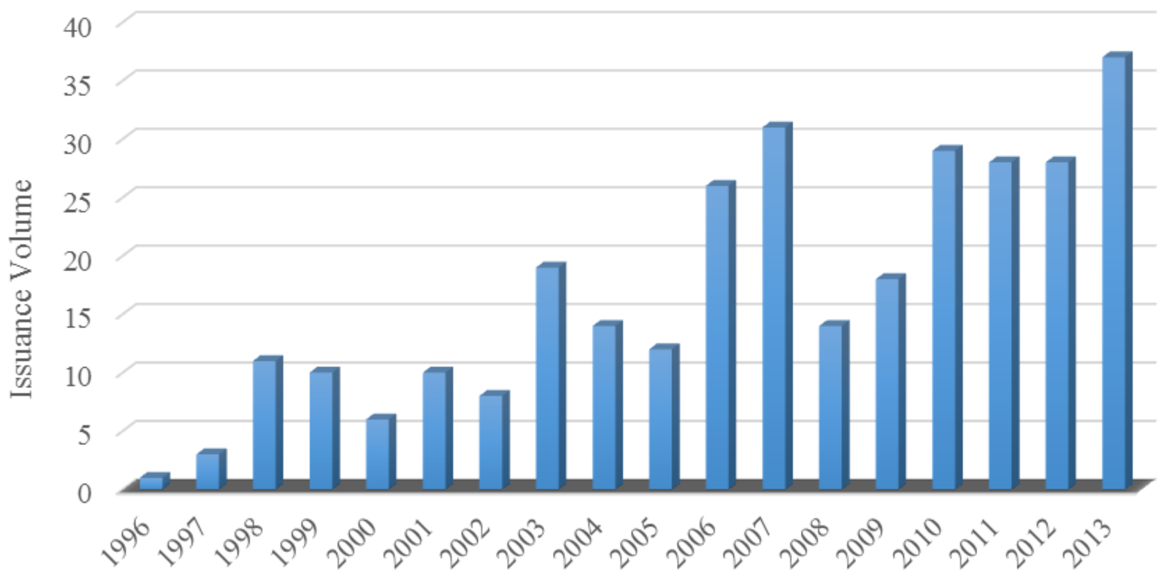

Figure 2: The trend of the global CAT bond issuant volume from 1996 to 2013.

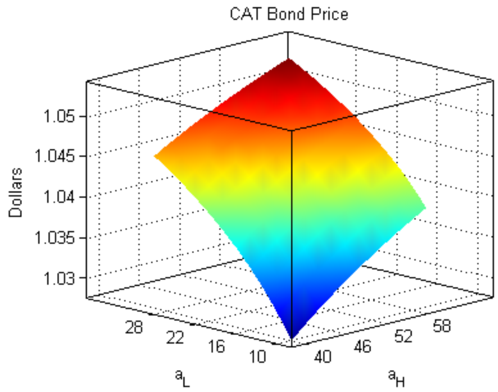

Figure 3: Scenario Analysis: CAT bond prices as the function of a_L and a_H.

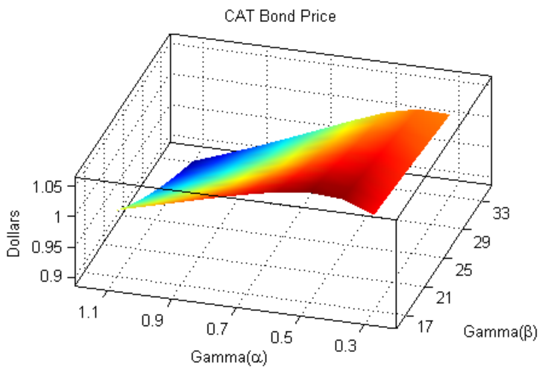

Figure 4: The CAT bond prices as the function of shape and scale parameters of Gamma distribution.

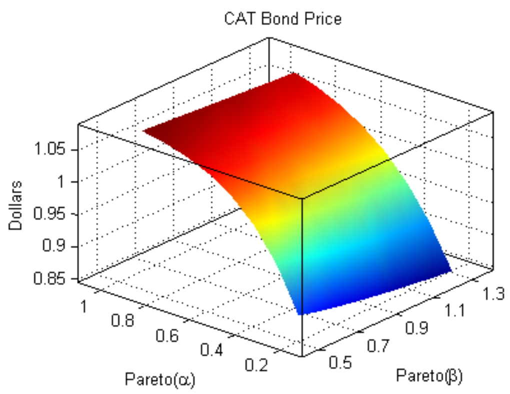

Figure 5: The CAT bond prices as the function of shape and scale parameters of Pareto distribution.

Table 1: A Scenario Analysis of CAT Bond Prices with Maturities and Spreads

Time-to-Maturity	Spread (bps)				
(Years)	100	300	500	800	1000
	Panel A: CAT Bond Price (Dollars)				
2.0	0.9776	1.0141	1.0506	1.1053	1.1418
2.5	0.9717	1.0164	1.0611	1.1281	1.1728
3.0	0.9657	1.0183	1.0708	1.1497	1.2022
3.5	0.9597	1.0198	1.0798	1.1699	1.2300
4.0	0.9535	1.0208	1.0881	1.1890	1.2563
5.0	0.9410	1.0218	1.1026	1.2237	1.3045
	Panel B: Interest (Dollars)				
2.0	0.1308	0.1673	0.2038	0.2585	0.2950
2.5	0.1601	0.2048	0.2495	0.3165	0.3612
3.0	0.1881	0.2407	0.2932	0.3720	0.4246
3.5	0.2149	0.2750	0.3351	0.4252	0.4853
4.0	0.2406	0.3079	0.3752	0.4761	0.5434
5.0	0.2885	0.3693	0.4501	0.5712	0.6520
	Panel C: Residual Principal (Dollars)				
2.0	0.1308	0.1673	0.2038	0.2585	0.2950
2.5	0.1601	0.2048	0.2495	0.3165	0.3612
3.0	0.1881	0.2407	0.2932	0.3720	0.4246
3.5	0.2149	0.2750	0.3351	0.4252	0.4853
4.0	0.2406	0.3079	0.3752	0.4761	0.5434
5.0	0.2885	0.3693	0.4501	0.5712	0.6520

Table 2: A Scenario Analysis of CAT Bond Prices with Stochastic Arrival Rates

	σ_λ				
μ_λ	0.01	0.05	0.1	0.15	0.2
0.02	1.047	1.0470	1.0468	1.0468	1.0468
0.04	1.0454	1.0454	1.0452	1.0452	1.0452
0.06	1.0438	1.0437	1.0436	1.0435	1.0435
0.08	1.042	1.0420	1.0418	1.0418	1.0417
0.1	1.0403	1.0403	1.0402	1.0400	1.0399

Table 3: A Scenario Analysis of CAT Bond Prices with Maturities and Default Intensities

Time-to-Maturity	λ_d				
(Years)	0.0001	0.0012	0.0045	0.0078	0.0100
	Panel A: CAT Bond Price (Dollars)				
2.0	1.0343	1.0322	1.0260	1.0198	1.0157
2.5	1.0412	1.0386	1.0310	1.0234	1.0183
3.0	1.0475	1.0444	1.0354	1.0265	1.0206
3.5	1.0531	1.0497	1.0393	1.0291	1.0223
4.0	1.0582	1.0543	1.0427	1.0312	1.0236
5.0	1.0627	1.0584	1.0456	1.0329	1.0246
	Panel B: Interest (Dollars)				
2.0	0.1857	0.1855	0.1848	0.1842	0.1837
2.5	0.2274	0.2271	0.2261	0.2251	0.2245
3.0	0.2674	0.2669	0.2656	0.2642	0.2633
3.5	0.3056	0.3050	0.3032	0.3015	0.3003
4.0	0.3422	0.3415	0.3392	0.3370	0.3355
5.0	0.3772	0.3763	0.3736	0.3708	0.3690
	Panel C: Residual Principal (Dollars)				
2.0	0.8486	0.8467	0.8412	0.8356	0.8320
2.5	0.8138	0.8115	0.8049	0.7982	0.7939
3.0	0.7801	0.7775	0.7699	0.7623	0.7573
3.5	0.7475	0.7446	0.7361	0.7276	0.7220
4.0	0.7159	0.7128	0.7034	0.6942	0.6881
5.0	0.6854	0.6821	0.6720	0.6621	0.6556

Table 4: A Scenario Analysis of CAT Bond Prices with Stochastic Interest Rates

	σ_r				
θ	0.005	0.010	0.025	0.035	0.045
	Panel A: CAT Bond Price (Dollars)				
0.01	1.0447	1.0447	1.0447	1.0447	1.0447
0.03	1.0446	1.0446	1.0446	1.0446	1.0446
0.05	1.0446	1.0446	1.0446	1.0446	1.0446
0.07	1.0445	1.0445	1.0445	1.0445	1.0445
0.09	1.0445	1.0445	1.0445	1.0445	1.0445
	Panel B: Interest (Dollars)				
0.01	0.2635	0.2634	0.2633	0.2632	0.2631
0.03	0.2651	0.2651	0.2649	0.2648	0.2647
0.05	0.2667	0.2667	0.2666	0.2664	0.2663
0.07	0.2683	0.2683	0.2682	0.2680	0.2679
0.09	0.2699	0.2699	0.2698	0.2696	0.2695
	Panel C: Residual Principal (Dollars)				
0.01	0.7812	0.7812	0.7813	0.7814	0.7816
0.03	0.7796	0.7796	0.7797	0.7798	0.7800
0.05	0.7779	0.7779	0.7780	0.7781	0.7783
0.07	0.7763	0.7763	0.7764	0.7765	0.7767
0.09	0.7746	0.7746	0.7747	0.7749	0.7750

科技部補助專題研究計畫執行國際合作與移地研究心得報告

計畫編號	MOST 102-2410-H-004-028-MY2		
計畫名稱	考量流動性風險下巨災債券與颶風衍生性商品之評價、實證與風險管理		
出國人員姓名	林士貴	服務機構及職稱	國立政治大學金融學系教授
出國時間	104/10/12 至 104/10/18	出國地點	加拿大溫哥華
出國研究目的	□ 實驗 □ 田野調查 □ 採集樣本 ■ 國際合作研究 □ 使用國外研究設施		

1 執行國際合作與移地研究過程

後學此次至 Department of Statistics and Actuarial Science, Simon Fraser University 拜訪 Cary Tsai 教授，除了報告生死合險 (endowment contract)，並且發展具有流動性風險下巨災債券的評價。

2 研究成果

研究成果有兩個：

1. 生死合險之相關實證已經完成，現在正在寫作論文。
2. 流動性風險下巨災債券正在數值模擬。

3 建議

本次至 Simon Fraser University 進行移地研究，除了與加拿大學者有更進一步交流，更在討論學術論文中吸取別人對於行銷文章與投稿等經驗。國際合作與移地研究是一個與外國學者合作的好方法，不僅可交換學術觀點與意見，也可得知外國相關市場之機制、法規，並比較台灣保險商品市場之現況，後續將持續針對保險商品之評價、風險管理進行研究與討論，以讓台灣本土之保險商品市場更加多元性與穩健性。

4 本次出國若屬國際合作研究，雙方合作性質係屬

□ 分工收集研究資料。

□ 交換分析實驗或調查結果。

■ 共同執行理論建立模式並驗證。

□ 共同執行歸納與比較分析。

□ 元件或產品分工研發。

□ 其他（請填寫）__________。

科技部補助專題研究計畫執行國際合作與移地研究心得報告

計畫編號	MOST 102-2410-H-004-028-MY2		
計畫名稱	考量流動性風險下巨災債券與颶風衍生性商品之評價、實證與風險管理		
出國人員姓名	林士貴	服務機構及職稱	國立政治大學金融學系教授
出國時間	104/10/28 至 104/10/31	出國地點	新加坡
出國研究目的	☐ 實驗 ☐ 田野調查 ☐ 採集樣本 ■ 國際合作研究 ☐ 使用國外研究設施		

1 執行國際合作與移地研究過程

後學此次至國立新加坡大學拜訪 Steve Kou 教授，在了解 Steve Kou 教授之相關研究方向後，針對巨災債券 (catastrophe bonds)、溫度衍生性商品 (temperature derivatives) 爲主軸進行討論。由於巨災發生時債券之債務人必須根據巨災所帶來之經濟損失而調整其本金，並發放調整後債息給予債權人，然而當發行公司面臨過於龐大之巨災損失時可能因資金不足而有違約或破產風險，使得債權人無法如期獲得債息亦無法在期末取回本金，因此關於巨災債券之部分，引入信用風險與利率風險並進一步針對巨災債券評價與風險管理。另一方面，不像巨災債券通常要滿足契約觸及條件 (trigger) 才有所理賠，溫度衍生性商品可直接給予投資人因受到溫度變化而有所損失之保障，由於溫度指數存在季節性波動度、波動叢集、非預期效果、跳躍風險等性質，在關於溫度衍生性商品之部分，引入 S-MR-S-GARCH-Jump 模型進行實證分析，並進一步針對該商品評價與風險管理。

2 研究成果

根據國立新加坡大學拜訪 Steve Kou 教授之意見，上述二種商品雖然可有效規避巨災或溫度等天然災害或氣候變遷所帶來之風險，但是卻存在流動性不足 (illiquid) 之問題，即實際買價高於目標價格、實際賣價低於目標價格，面對此種流動性不足引發之成本，此類衍生性商品必須引入流動性因子以修正理論價格。針對此議題，後學將研讀有關股票選擇權、債券等文獻中針對流動性模型之設定，未來將進一步引入至巨災商品與氣溫衍生性商品，使得評價結果更爲精確亦可提供相關機構發行時進行各風險敏感度之參考依據。

3 建議

本次至國立新加坡大學進行移地研究，除了與新加坡學者有更進一步交流，更在討論學術論文中吸取別人對於行銷文章與投稿等經驗。國際合作與移地研究是一個與外國學者合作的好方法，不僅可交換學術觀點與意見，也可得知外國相關市場之機制、法規，並比較台灣衍生性商品市場之現況，後續將持續針對衍生性商品之評價、風險管理進行研究與討論，以讓台灣本土之衍生性商品市場更加多元性與穩健性。

4 本次出國若屬國際合作研究，雙方合作性質係屬

□ 分工收集研究資料。

□ 交換分析實驗或調查結果。

■ 共同執行理論建立模式並驗證。

□ 共同執行歸納與比較分析。

□ 元件或產品分工研發。

□ 其他（請填寫）__________。

科技部補助計畫衍生研發成果推廣資料表

日期:2016/01/11

科技部補助計畫	計畫名稱: 考量流動性風險下巨災債券與颶風衍生性商品之評價、實證與風險管理
	計畫主持人: 林士貴
	計畫編號: 102-2410-H-004-028-MY2 學門領域: 財務

無研發成果推廣資料

102年度專題研究計畫研究成果彙整表

<table>
<tr><td colspan="3">計畫主持人：林士貴</td><td colspan="5">計畫編號：102-2410-H-004-028-MY2</td></tr>
<tr><td colspan="8">計畫名稱：考量流動性風險下巨災債券與颶風衍生性商品之評價、實證與風險管理</td></tr>
<tr><td colspan="3" rowspan="2">成果項目</td><td colspan="3">量化</td><td rowspan="2">單位</td><td rowspan="2">備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）</td></tr>
<tr><td>實際已達成數（被接受或已發表）</td><td>預期總達成數（含實際已達成數）</td><td>本計畫實際貢獻百分比</td></tr>
<tr><td rowspan="12">國內</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（本國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（外國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td colspan="2">其他成果
（無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）</td><td colspan="6">無</td></tr>
</table>

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估
   ■達成目標
   □未達成目標（請說明，以100字為限）
      □實驗失敗
      □因故實驗中斷
      □其他原因
   說明：

2. 研究成果在學術期刊發表或申請專利等情形：
   論文：□已發表 ■未發表之文稿 □撰寫中 □無
   專利：□已獲得 □申請中 ■無
   技轉：□已技轉 □洽談中 ■無
   其他：（以100字為限）
   本研究內容與計畫提案大致相符，除了流動性風險因市場資料取得問題尚未納入本研究計畫，其餘巨災債券所應考慮的主要風險皆可透過本研究之評價模型進行評價。本研究將結構式評價模型應用至巨災債券之評價，提出一個亦於使用且可同時包含巨災風險與財務風險的預期目標相符。

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限）
   就學術方面而言，本研究在縮減式模型的架構下提出了一個一般化的巨災債券評價模型，同時包含了巨災風險、違約風險以及利率風險，將信用衍生性商品的評價方法應用至保險市場中，以利於在巨災衍生性商品評價過程中達到評價技術創新之結果。在社會影響的層面，本研究模型具有相當的彈性，可廣泛地適用於各種支付函數與不同契約條款的巨災債券商品，也可以透過標的災害在特定地區所適用的損失幅度分配以及災害發生頻率來刻畫最重要的巨災風險，同時，信用風險與利率風險也具體地建構於本研究的評價模型之中，因此能具體地應用在實務市場中，雖然本研究主要以發展評價理論為主，但其研究結果相當具有應用價值。事實上本研究之評價模型可加入流動性溢酬，然而本研究發現巨災債券缺乏交易所之次級市場交易資訊，由於店頭市場資料取得上之困難，故暫不考慮此風險，留待向特定經紀商或巨災債券發行公司取得特定債券之報價後再進一步發展實證研究。