Variation of floor rent differentials of high-rise buildings*

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Abstract. With rapid urbanisation, the demand for office floor space also increases. Although high-rise buildings are very common in urban areas, study of the rent for different stories in such buildings is relatively rare. Liu (1988) and Grimaud (1989) separately applied agglomeration economies to study the density variation in urban areas. However, these studies assumed that the floor rent in the same building is the same for each story. Such an assumption about floor rent may not fully reflect actual floor rent and, from the perspectives of theoretical completeness and practical usefulness, formulation for different floor rents in high-rise buildings, and their effect on urban density variation should also be considered. This research note extends Ogawa and Fujita's locational potential function (1980) into two dimensions: flat dimension and story dimension. The modification of the locational potential function is then applied to revise Liu's model. Finally, the office distribution in a CBD (central business district) is then discussed.

JEL classification: L74, R14, R33

Key words: Urban density, office buildings, floor rent, locational potential function

1 Introduction

Production within an urban area, especially within the central business district (CBD), is often characterised by "agglomerative economies". Most existing models used to describe these economies are based on applying the concept of spatial

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Table 1. German method developed by Schirmer

Story	Weight	%
Fifth floor	0.9	13.0
Fourth floor	0.9	13.0
Third floor	1.0	14.5
Second floor	1.0	14.5
First floor	2.5	36.2
Basement 1	0.3	4.4
Basement 2	0.3	4.4
Total	6.9	100.00

Source: Inn (1992)

externalities to generate internal forces of spatial agglomeration (Borukhov and Hochman 1977; Solow and Vickery 1971; Hartwick and Hartwick 1974; O'Hara 1977; Ogawa and Fujita 1980; Imai 1982). Authors of these studies assume a uniform density of firms. However, if we allow for capital-land substitution, non-uniform distributions of firms will obviously prevail. Thus Tabuchi (1986), Liu (1988), Grimaud (1989) and Liu and Fujita (1991) developed variable density models. Their studies, however, assume floor rent in a given building is the same at all floor levels. This assumption is not a satisfactory depiction of conditions in modern cities, as there can be large differences in floor rents within the same high-rise building. Therefore, a model of different floor rents in high-rise buildings and the effect on the density distribution should be considered from the perspectives of both theoretical completeness and practical usefulness.

Table 2. The relative utility of a story (Rules for the Compensation of Land Acquisition of Public Utilities)

Story	A	B	C1	C2	C3
Ninth floor	32.8		30.0	30.0	30.0
Eighth floor	32.9		30.0	30.0	30.0
Seventh floor	33.0		30.0	30.0	30.0
Sixth floor	36.9	67.4	30.0	30.0	30.0
Fifth floor	40.1	70.0	30.0	30.0	30.0
Fourth floor	42.8	72.7	30.0	30.0	30.0
Third floor	44.1	75.4	60.0	30.0	30.0
Second floor	61.5	79.4	70.0	70.0	30.0
First floor	100.0	100.0	100.0	100.0	100.0
Basement 1	55.7	52.9	60.0	60.0	60.0
Basement 2	33.1		40.0	40.0	40.0

Note : C1: Fourth floor to ninth floor are residential housing; C2: Third floor to ninth floor are residential housing; C3: Second floor to ninth floor are residential housing
Source: Niitani-yodo (1986)

Empirical studies of the valuation of real estate suggest that the difference in value among stories of a high-rise building can be divided into three factors: accessibility, view and security (Lin 1998). Such empirical studies suggest that view and security are psychological factors, and accessibility is an economic factor. In the assessment of the high-rise buildings in Germany and Japan, the highest economic value weight for stories is at the first floor, and decreases with the rise in stories (see Tables 1 and 2). However, there is no theoretic rationale supplied for these rules. The purpose of this research note is to establish a model that enables us to discuss the economic value of stories in a CBD and the influence on urban structure.

Generally speaking, the configuration of a CBD is the result of a combination of attractive and repulsive forces, that is, attractive forces compel business firms to concentrate, while repulsive forces prevent them from over-concentration. The configuration of a CBD is conceived as the physical manifestation of the balance between attractive and repulsive forces. Thus Ogawa and Fujita (1980) developed a model based on the claim that the major attractive force for business firms is the tendency to reduce the transaction cost of face-to-face transactions. Conversely, the major repulsive force is the wish to avoid higher land rent due to the concentration of business firms together in the same area. The total transaction cost incurred by a business firm is the aggregation of the transaction cost with every other business firm in the city: it is a density weighted linear function of distance. In Ogawa and Fujita's model this density is constant. Liu (1988) and Grimaud (1989) introduced a variable density into this model. However, the transaction cost in their studies is still a density-weighted linear function of distance in that the floor rent of different stories in the same building is the same. Moreover, their model extension cannot fully reflect the repulsive force; and so the density distribution in their models may not fully represent the configuration of the CBD.

The present study extends Ogawa and Fujita's locational potential function (1980) in the CBD into two dimensions: a flat dimension and a story dimension.[1] The modified locational potential function is then used to revise Liu's model. Section 2 provides an outline of the assumptions of the basic model proposed in this note. The application of the model under the condition of uniform floor rent in the same building is shown in Sect. 3. The case of a non-uniform floor rent in the same building is covered in Sect. 4 and conclusions follow.

2 The basic model

A simple model of locational decisions within a CBD can be formulated by means of the following assumptions on the CBD, the business firm, and the constructor sector:

2.1 The CBD

A CBD is assumed as developed on a long strip of homogenous agricultural land. Because the city area is very long, transaction costs across the width are negligible.

[1] There is a two-sector model in Ogawa and Fujita's study. We only consider one sector in our model.

Thus the CBD can be treated as a line. Each location in the city is represented by a point x on the line on which the geographic centre is the origin. The story of a building is represented by y. The first story is 1 and the highest story at location x is $b(x)$.

2.2 The business firm

There are M identical business firms in the CBD. All of these are profit maximisers. They produce some type of services or goods that will be exported at a constant price p. Each business firm uses one unit of office space and one unit of other fixed cost in order to produce a fixed positive amount of output, say Q.

Business firms are required to conduct transactions, such as information exchange during production. We assume that business firms transact with each other with equal probability. The cost of transactions between any two-business firms is not only linearly proportional to the horizontal distance, but also to the story distance between them. Each transaction requires a separate trip, and all other firms are equally likely to be the other participant in the transaction. The travel occasioned by transactions produces no congestion. The round-trip cost of such travel is t per unit horizontal distance, and τ per unit story distance. Thus the total transaction cost of a business firm locating at location x and story y is given as:

$$\begin{aligned} T(x,y) = & \int_{-f}^{f} \int_{1}^{b(u)} \{t\left[\|x-u\|\right] + \tau\left[(y-1)+(v-1)\right]\} \, dvdu \\ & -\tau \int_{1}^{b(x)} \left[(y-1)+(v-1)\right] dv + \tau \int_{1}^{b(x)} |y-v| \, dv \ , \quad (2.1) \end{aligned}$$

where $T(x,y)$ is total transaction cost of a business firm at location x and story y; t is unit transaction cost of horizontal distance; τ represents unit transaction cost between stories; $b(x)$ is the stories of building at location x; and $-f, f$ represent the left and right urban fringes, respectively.

The first term on the right-hand side represents the transaction cost, which includes the horizontal distance (from location x to location u) and the vertical distance. The latter includes distance in the original building (from yth floor to first floor), and the terminal building (from first floor to vth floor). But for transactions within the same building, there is no need to go down the first floor then back up again. The second term on the right-hand side therefore excludes the redundant vertical transaction cost in the same building, and the third term on the right-hand side calculates the correct vertical transaction cost within the same building. The slope and curvature of the total transaction cost function are given by:

$$\frac{\partial T(x,y)}{\partial x} = t\left[\int_{-f}^{x} b(u)du - \int_{x}^{f} b(u)du - 2x\right] + 2\tau b'(x)(1-y), \quad (2.2)$$

$$\frac{\partial^2 T(x,y)}{\partial x^2} = t\left[2b(x) - 2\right] + 2\tau b''(x)(1-y)\ . \quad (2.3)$$

We assume that the city is symmetric (Ogawa and Fujita 1980). Hence from (2.2) we see that the transaction cost has an extreme point at (0,1). From (2.3), the second derivative at the extreme point is positive. Therefore, the first floor of a CBD centre building has the minimum transaction cost. Now, when we consider the first and second order partial derivatives with respect to y:

$$\begin{aligned} \frac{\partial T(x,y)}{\partial y} &= \tau \int_{-f}^{f} b(u)du + 2\tau\left[y - f - b(x)\right] \\ &= \tau M + 2\tau\left[y - f - b(x)\right]\ , \quad (2.4) \\ \frac{\partial^2 T(x,y)}{\partial y^2} &= 2\tau\ . \quad (2.5) \end{aligned}$$

Derivatives (2.4) and (2.5) are always positive, therefore the higher the floor is in a building, the more transaction cost the business firm shall pay, and the increment of the transaction cost will increase with the story.[2] Therefore, the profit of firm located at location x and the yth floor is given as:

$$\pi(x,y) = PQ - C - OR(x,y) - T(x,y), \quad (2.6)$$

where $OR(x, y)$ represent floor rent at location x and yth floor; C represents other fixed cost and other variables, as defined above.

The objective of each business firm is to choose an optimal location x and story y by taking into account the locations of all other business firms in the city so as to maximise its profit given by (2.6). Since all the business firms are assumed to be identical, the profit level of all firms must be the same at equilibrium regardless of location. Floor rent is therefore given as:

$$OR(x, y) = PQ - C - \pi - T(x, y) \ . \tag{2.7}$$

2.3 The construction sector

We assume that absentee developers supply office space to business firms. All capital and labour used for the construction of office space comes from outside the city. The construction cost function is assumed as a function of office density as follows:[3]

$$K\,[b(x)] = \alpha[b(x)]^2 \ , \tag{2.8}$$

where K is the construction cost function and α is the construction cost parameter ($\alpha > 0$).

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[2] If the density distribution is flat (all are single-story buildings), this equation is equal to 0. If there are buildings with more than one story in the CBD, M will be at least double $2f$, therefore the derivative (2.4) is positive.

[3] See Tabuchi (1986), Liu (1988) and Grimaud (1989).

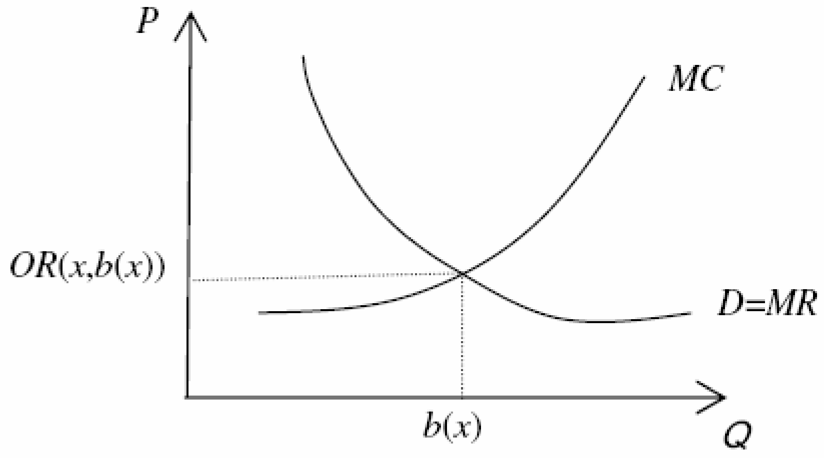

Fig. 1. First-degree price discrimination

Since the market for office space is assumed as perfectly competitive, the profit of each developer should be zero in equilibrium:

$$\pi_c(x) = \int_1^{b(x)} OR(x,y)dy - \alpha[b(x)]^2 - R(x), \tag{2.9}$$

where $\pi_c(x)$ is unit profit of a developer at x and $R(x)$ is unit land rent at x.

When developers bid successfully for the land at location x, they become the monopolist at that location. The rent for office space at each location x and the $b(x)$th story equals the marginal cost of providing office space:

$$MC = \frac{\partial \left\{\alpha[b(x)]^2 + R(x)\right\}}{\partial b(x)} = MR\ . \tag{2.10}$$

From Equations (2.4), (2.5) and (2.7), we know that at the same location the floor rent varies between the different stories. The highest floor rent is on the first floor of the building, and the lowest floor rent is in our model on the top floor.[4] This circumstance can be treated as first-degree price discrimination. The marginal cost therefore equals the marginal revenue, which determines the building height at location x, $b(x)$, and the top floor rent of the building (Fig. 1).

Thus,

$$MR = OR(x, b(x))\ . \tag{2.11}$$

Equation (2.10) can be written as:

$$OR(x, b(x)) = \frac{\partial \left\{\alpha[b(x)]^2 + R(x)\right\}}{\partial b(x)} = 2\alpha b(x)\ . \tag{2.12}$$

3 The case of a uniform floor rent in the same building

In order to compare the difference between the model of a uniform floor rent in the same building and the model of a floor rent that varies across stories in the same building, we introduce in this section an application of the model where there is a

[4] It is accepted that the top floor may, in reality, command a premium rent.

uniform floor rent distribution in the same building and consider the effect on the density distribution. All assumptions from the previous section are the same, except that the transaction cost function now only takes account of distance between flat locations. Therefore, the transaction cost is:

$$T(x) = t \int_{-f}^{f} b(z) \left| x - z \right| dz \ , \tag{3.1}$$

where $T(x)$ is the transaction cost at location x and $b(z)$ is the firm density distribution at location z. Under symmetry, the slope and curvature of the total transaction cost function are given by:

$$\frac{dT(x)}{dx} = t \left[\int_{-f}^{x} b(z)dz - \int_{x}^{f} b(z)dz \right] = t \left[2 \int_{-f}^{x} b(z)dz - M \right] \ , \tag{3.2}$$

$$\frac{d^2T(x)}{dx^2} = 2tb(x) > 0 \ . \tag{3.3}$$

Since $b(x) > 0, T(x)$ is strictly convex. From Equation (2.10) we obtain:

$$\begin{aligned} MR &= OR(x) = PQ - C - T(x) - \pi \\ MC &= 2\alpha b(x) \\ PQ - C - T(x) - \pi &= 2\alpha b(x) \ . \end{aligned} \tag{3.4}$$

Differentiating Equation (3.4) with respect to x and taking Equations (3.2) and (3.3) into account, we have:

$$-t\left[2\int_{-f}^{x} b(y)dy - M\right] = 2\alpha b'(x)\ , \tag{3.5}$$

$$-2tb(x) = 2\alpha b''(x)\ . \tag{3.6}$$

The density of the business firm distribution is strictly concave everywhere and the highest location is at the centre, $x = 0$.

4 The case of a non-uniform floor rent in the same building

In this section we apply the model to the case of non-uniform floor rent in the same building and consider the effect on the density distribution. From Equation (2.7), we have:

$$OR(x, b(x)) = PQ - C - \pi - T(x, b(x))\ . \tag{4.1}$$

From previous assumptions, PQ, C, and π will be the same regardless of location. Hence from Equations (2.12) and (4.1):

$$2\alpha\, b(x) = F - T(x, b(x))\ , \tag{4.2}$$

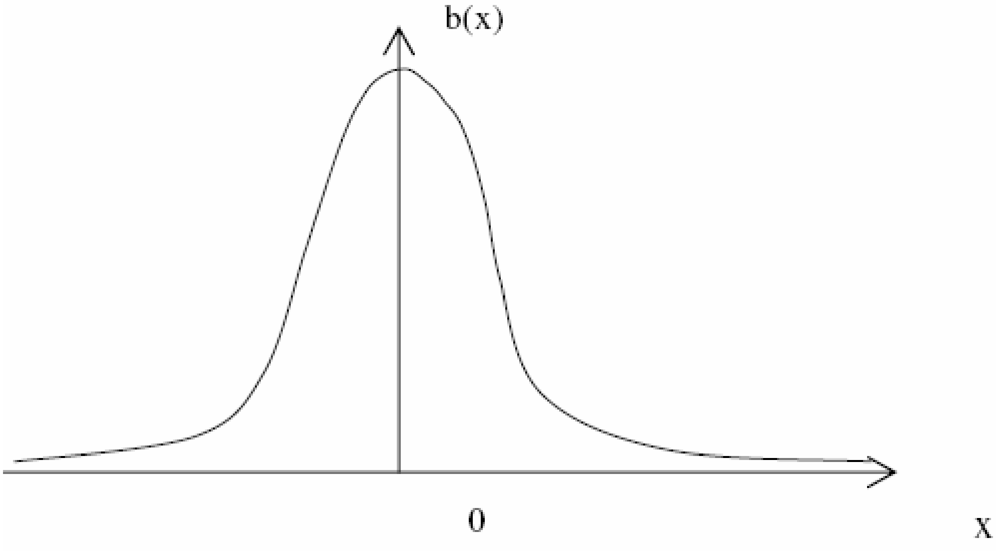

Fig. 2. Density distribution of business firm

where $F = PQ - C - \pi$. Differentiating Equation (4.2) with respect to x, we have:

$$2\alpha b'(x) = -t\left[2\int_{-f}^{x} b(u)du - M - 2x\right] - 2\tau b'(x)[1 - b(x)], \quad (4.3)$$

$$2\alpha b''(x) = -t[2b(x) - 2] - 2\tau b''(x) + 2\tau b''(x)b(x) + 2\tau b'(x)b'(x) \ . \quad (4.4)$$

Rearranging (4.3) and (4.4), we obtain:

$$b'(x) = \frac{-t\left[2\int_{-f}^{x} b(u)du - M - 2x\right]}{2\alpha - 2\tau[b(x) - 1]} , \quad (4.5)$$

$$b''(x) = \frac{-t[b(x) - 1] + \tau[b'(x)]^2}{\alpha - \tau[b(x) - 1]} \ . \quad (4.6)$$

Under the assumption of a symmetric city configuration, we see from (4.5) that the business firm density has an extreme point at the centre, $x = 0$. It is reasonable to assume that the unit construction cost α is much higher than the unit transaction cost among stories τ. The denominator in Equation (4.6) is thus positive. From (4.5), $b'(x)$ is equal to 0 in the centre, and increases with $|x|$, and the $b(x)$ decreases with the $|x|$. Therefore, the numerator in Equation (4.6) is negative around the centre and positive in the periphery of the CBD. Hence the curvature of the density distribution is concave in the centre and convex in the periphery (Fig. 2).

5 Conclusion

This research note has discussed the transaction cost of firms located in different buildings and on different floors. Unlike the case of a uniform floor rent in the same building, consideration of different floor rents shows that the firm distribution is concave in the centre of the CBD and convex at the periphery of the CBD. This difference can be explained as follows. The agglomeration effect is higher in the CBD centre or at lower stories of the same building. Business firms will choose the high story buildings around the centre of the CBD, and will avoid high story buildings at the periphery. Therefore, the height of a building will decrease more slowly in the centre of CBD than at the periphery.

To conclude this research note, we would like to point out two possible extensions of our model. First, we have assumed that each business firm occupies a fixed amount of floor space. It would, however, be desirable to generalise the model to allow for business firms to choose optimal amounts of floor space. There could also be a consideration of the impact of the view and prestige of the floor as well as security factors in determining floor rent. A more complete model for describing the CBD configuration should consider these factors.

References

Borukhov E, Hochman O (1977) Optimal and market equilibrium in a model of a city without a predetermined center. *Environment and Planning A* 9: 849–855

Grimaud A (1989) Agglomeration economies and building height. *Journal of Urban Economics* 25: 17–31

Hartwick EJ, Hartwick JM (1974) Efficient resource allocation in a multinucleated city with intermediate goods. *Quarterly Journal of Economics* 88: 340–352

Imai H (1982) CBD hypothesis and economics of agglomeration. *Journal of Economic Theory* 28: 275–299

Inn C (1992) *A study of the relationship between the highway land right and the usage of different stories.* Published by The Research Institution of Chinese Land Economics (in Chinese)

Lin I (1998) *The appraisal of real estate.* Wen-shen Publisher (in Chinese)

Liu H (1988) Two-sector non-monocentric urban land use model with variable density. *Environment and Planning A* 20: 477–488

Liu H, Fujita M (1991) A monopolistic competition model of spatial agglomeration with variable density. *Annals of Regional Science* 25: 81–99

Niitani-yodo S (1986) Case studies of land use for high-rise buildings. *The appraisal of real estate* (in Japanese)

Ogawa H, Fujita M (1980) Equilibrium land use pattern in a non-monocentric city. *Journal of Regional Science* 20: 455–475

O'Hara JD (1977) Location of firms within a square central business district. *Journal of Political Economy* 85: 1189–1207

Solow RM, Vickrey WS (1971) Land use in a long narrow city. *Journal of Economic Theory* 3: 430–447

Tabuchi T (1986) Urban agglomeration economies in a linear city. *Regional Science and Urban Economics* 16: 421–436