RESEARCH OF GPS HEIGHTING WITH NETWORK RTK MEASUREMENTS

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ABSTRACT: Different from the traditional levelling, the modernized height survey is an economical technology. It can obtain orthometric heights through the geoid undulation and the ellipsoidal heights from GPS observations, which is called "GPS-heighting" or "GPS-levelling". GPS-heighting mostly uses static measurements to obtain the position and ellipsoid height information. However, the Virtual Base Station - Real Time Kinematic (VBS-RTK) is more efficient than static GPS surveying. Taiwan has built up a regional network RTK system, called e-Global Navigation Satellite System (e-GNSS), which provide the real-time kinematic positioning based VBS-RTK and network Differential GNSS (DGNSS). Considering a more efficient procedure on GPS-leveling, we obtain the measurements at benchmarks by using e-GNSS instead of GPS in this paper. The geoid undulation is estimated by using gravimetric and hybrid geoid model respectively. In addition to geoid models, we verify the accuracy of e-GNSS levelling by difference method. With the high efficiency, accuracy and low costs, the GPS-heighting based on network RTK can be applied more widely on engineering and disaster prevention and rescue.

KEY WORDS: GPS-heighting, GPS-levelling, RTK, geoid undulation

1. INTRODUCTION

Orthometric height is a physical quantity that refers to the surface of the geoid and it is an elevation which has wide application on topographic maps, practical surveying and engineering, etc. Traditionally, the spirit levelling is with high accuracy on orthometric heights. However, it is manpower-, money- and time-consuming. The Global Positioning System (GPS) is a satellite-based navigation system which was developed by the U.S. Department of Defense since 1980s. The GPS provides high efficiency and precession of location and time information. The GPS positional information is a three-dimensional and Earth-centered solution, which is usually converted to latitude, longitude and height relative to an ellipsoidal Earth model. The ellipsoid heights are geometric quantities. In contrast, orthometric heights are physical quantities. By combining GPS-derived ellipsoid height and a geoid model, orthometric height can be obtained more efficient, which is called "GPS-heighting" or "GPS-levelling".

Mostly, GPS-heighting uses static measurements to obtain the high-accuracy position and ellipsoid height information. Since fast and stable internet access is available and the mature Virtual Reference Station (VRS) is developed, the network Real-Time Kinematic (RTK) GPS has become an important tool of survey engineering. Traditional RTK GPS positioning uses a single reference station, and the space relativity of positioning errors will decrease as the distance between reference and rover station. To overcome this defect, Taiwan has built up a regional network RTK system, called e-Global Navigation Satellite System (e-GNSS), which provide the RTK positioning based VBS-RTK and network Differential GNSS (DGNSS). The VBS-RTK system of the National Land Surveying and Mapping Center in Taiwan adopts regional multi-reference stations to solve the system errors, such as ionospheric and tropospheric delays. The distance between the reference stations in the network do not exceed 50 km. This network RTK system can receive the signals from GPS and GLONASS for now. The e-GNSS system based on VBS-RTK is an efficient technique to determine high precision coordinates within a short period of time. The system provides high precision positioning service, the coordinate system is named e-GNSS[2015]. This network RTK system determines legal coordinates by 3D real-time coordinate transformation. The transformed accuracy for horizontal coordinate was better than 5 cm, for ellipsoid and orthometric height was better than 10cm (Yeh et al., 2012). In this study, we obtain the measurements at benchmarks by using e-GNSS instead of GPS.

2. TEST AREA AND MODELS

In this case study, we take the Miaoli city in Taiwan to be test area. The Miaoli city is located in the northwestern part of Taiwan, and its area is approximately 1,820 km^2 (the width of East-West is about 64 km and the width of South-North is about 50 km). There is no obvious threat to land subsidence in the Miaoli city and the velocity field of area is smaller here. There are 111 first-order levelling benchmarks in the Miaoli city along five different provincial highways: Tai 1st Line (Tai_1), Tai 3rd Line (Tai_3), Tai 6th Line (Tai_6), Tai 13th Line (Tai_13) and

Tai 61th Line (Tai_61). Figure 1 depicts the geography, topography and distribution of benchmarks in Miaoli city. To deduct the destroyed, losing and bad sky visible points, there are 100 benchmarks to be our test points in this study (at benchmarks). Because different geoid models are estimates form the levelling, GPS and e-GNSS data by geometric fitting method and gravimetric geoid model, there different types of orthometic height for each test point. The models are described as follow.

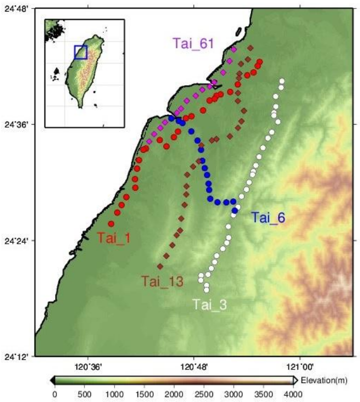

Figure 1. Geography, topography and distribution of benchmarks

The local geoid models in this study include two different types are : the gravimetric and hybrid geoid models. They are announced by the Ministry of the Interior (MOI) of Taiwan in 2014. The gravimetric geoid model is termed TWGEOID2014, and the hybrid geoid model is termed TWHYGEO2014. Both they refer to the global reference frame ITRF94 and GRS80 ellipsoid for geographical coordinates.

2.1 Gravimetric Geoid Model

Local gravimetric geoid determination requires data comprising three components: long-wavelength, medium-wavelength, and short-wavelength components. The long-wavelength component is sourced from Global Geopotential Model (GGM). The medium-wavelength is sourced from terrestrial gravity data. The short-wavelength is sourced from Digital Terrain Model (DTM) data. In TWGEOID2014, the EGM2008 to degree 2190 is used as the long wavelength part of this geoid. The terrestrial gravity include the land, shipborne, airborne and altimeter-derived gravity data. There are total 9,042 points of the land gravity data provided by difference organizations of Taiwan in 1980-2012: MOI, Academia Sinica, Chinese Society of Survey Engineering, etc. There are total 20,844 points of the shipborne gravity data provided by difference organizations of Taiwan in 1996-2013: the National Central University, National Land Surveying and Mapping Center (NLSC), etc. There are total 2,596 points of the airborne gravity data provided by the MOI in 2004-2009. The altimeter-derived gravity data is supplemented to the insufficient marine and offshore gravity data. The satellite data include Geosat/GM, ERS-1/GM, Geosat/ERM, ERS-1/35d, ERS-2/35d and T/P. The DTM data is with 90m resolution on 3”× 3” and 9”× 9” grids which is from 40m DEM using the aerial photography surveyed by the Aerial Survey Office, Forestry Bureau of Taiwan.

All processed datasets were combined by the band-limited least-squares collocation. And, the Stokes formula based on the Fast Fourier Transform (FFT) technique was used to compute the gravimetric geoid model with the standard remove-computation-restore procedure. The gravimetric geoid, TWGEOID2014, is a model with 30”× 30” grid (Hwang et al., 2013).

2.2 Hybird Geoid Model

A hybrid geoid model, TWHYGEO2014, is determined by merging GPS-derived and gravimetric geoidal heights (TWGEOID2014). The gravimetric geoid model exists a system deviation from the true geoid because of the long-wavelength errors come from spherical harmonic coefficients.

The N_{GPS} data adopt the measurements from three types of points provided by MOI, NLSC, National Chiao Tung University, Ministry of Science and Technology, etc.: 1. 1,721 first-order first-class and first-order second-class levelling benchmarks for 1-3 hours GPS survey, 2. 214 levelling benchmarks for 12/24 hours GPS survey, 3. 52 e-GPS satellite positioning points for 24 hours GPS survey (Hwang et al., 2013). The distribution of these points is uniform over Taiwan, and the reference surface of the levelling heights is TWVD2001. TWVD2001 is the Taiwan vertical datum now, and it is an orthometric height system.

3. TESTING METHODLOGY

Since 2004, a regional real-time GPS network for expanding the effective range of cadastral measurement has been set up in Taiwan, and there are more than 150 stations now. The continuous and real-time satellite data (1 second) are transmitted by the satellite positioning stations. The measurements from the satellite positioning stations are determined and integrated by the control centre. Via wireless radio communication, users can obtain the high accuracy positioning information in a short time. The e-GNSS is a network RTK system based on VBS-RTK in Taiwan.

The heights in the network RTK system are ellipsoid heights. The ellipsoid heights (*h*) refer to an ellipsoid datum, and the orthometric heights (*H*) refer to the geoid. Thus, the geoid heights (*N*) are needed to convert *h* into *H* in GPS-heighting, using the following equation (Heiskanen and Moritz, 1967):

$$H = h - N \qquad (1)$$

However, there are some errors in GPS-heighting. According to different errors, the items of equation 1 can be rewritten by the following equations:

$$h = h^{GPS} + \delta h^{GPS} \quad (2)$$

$$N = N^{\text{model}} + \delta N^{\text{model}} \quad (3)$$

$$H = H^{level} + \delta H^{level} \quad (4)$$

Where, h^{GPS} is the ellipsoid height and can be obtained by GPS. N^{model} is the geoid height and can be obtained by geoid model. H^{level} is the orthometric height and can be obtained by levelling. δh^{GPS}, δN^{model} and δH^{level} are the errors come from GPS, geoid model and levelling, respectively. Instead of using the static GPS, e-GNSS measurements, TWGEOID2014 and TWHYGEO2014 geoid model data are adopted in GPS-heighting in this paper. Using the equations (1)~(3), the orthometric heights and the discrepancy in e-GNSS heighting ($H^{e\text{-}GNSS}$) are estimated as follows:

$$\begin{aligned} H^{e-GNSS} &= h^{e-GNSS} - N^{\text{model}} \\ &= H - \delta h^{e-GNSS} + \delta N^{\text{model}} \end{aligned} \quad (5)$$

$$\begin{aligned} dH &= H^{e-GNSS} - H^{level} \\ &= -\delta h^{GPS} + \delta N^{\text{model}} + \delta H^{level} \end{aligned} \quad (6)$$

In this paper, the discrepancy with levelling is regarded as the accuracy of e-GNSS heighting. Generally, the levelling can achieve mm accuracy, the GPS ellipsoid heights are cm accuracy, and the errors of the geoid model are more than 10 cm. Thus, the major error of GPS-heighting comes from the geoid model.

Mentioned in section 2.2, the gravimetric model exists a system deviation. In addition to the hybrid model, the difference method is usually applied to promote the accuracy in gravimetric model. Based on a reference benchmark in the test region, the difference between the reference point and the neighboring points can eliminate effectively this system deviation in GPS-heighting (Milbert, 1991; Featherstone, 1998; Kearsley, 1998).

$$\begin{aligned} H_{AB}^{e-GNSS} &= h_{AB}^{e-GNSS} - N_{AB}^{\text{model}} \\ &= (h_{AB} - \delta h_{AB}^{e-GNSS}) - (N_{AB} - \delta N_{AB}^{\text{model}}) \\ &= H_{AB}^{level} - \delta h_{AB}^{e-GNSS} + \delta N_{AB}^{\text{model}} + \delta H_{AB}^{level} \end{aligned} \quad (7)$$

Where the subscripts present two points (A,B); h_{AB} and N_{AB} are the differences of true ellipsoid height and geoid undulation at points. δh_{AB}^{e-GNSS}, $\delta N_{AB}^{\text{model}}$ and δH_{AB}^{level} are the difference of e-GNSS ellipsoid height errors, geoid models errors and levelling errors at points.

4. RESULTS AND ANALYSIS

In the test area (mentioned in section 2), we surveyed by e-GNSS at the 100 first-order levelling benchmarks with 1 epoch per second for 20 minutes from end of May to end of July in 2015. The network RTK measurements are compared to the static GPS, and the standard deviations of the difference on North, East and height direction are 1.9 cm, 2.1 cm, and 6.6 cm, respectively.

The experimental geoid undulations at benchmarks in gravimetric and hybrid geoid models are shown in Figure.2. Using the e-GNSS measurements (h) and the estimated geoid undulations (N) by two geoid models, the orthometric heights (H) at benchmarks by e-GNSS heighting are achieved. Figure.3 presents the results of orthometric heights by e-GNSS heighting. Finally, the differences (dH) of orthometric heights between e-GNSS heighting and levelling are estimated, shown in Figure. 4. Table. 1 shows the statistics of the estimated the geoid undulations ($N^{gravity}$, N^{hybird}), the orthometric height ($H^{gravity}$, H^{hybird}), and the orthometric height differences ($dH^{gravity}$, dH^{hybird}) at benchmarks in the two geoid models. The maximum and minimum values of the geoid heights in gravimetric model are 20.896 m (point #3069 on Tai_3) and 18.814 m (point #1067 on Tai_1). The results of the geoid heights in hybrid model are 21.069 m and 18.955 m, the event points are the same as the gravimetric model. The two models present similar results on topography. However, the differences dH^{hybird} in hybrid model have better results than the differences $dH^{gravity}$ in gravimetric model. The Root Mean Square (RMS) value of the difference in hybrid model decreases about 71%. The geoid accuracy in gravimetric model is about 19.4 cm, and the accuracy in hybrid model is about 5.7 cm. It is proven from the experimental results that the hybrid model is a better method to determinate the geoid heights.

Two difference tests in this study: Test1: Chosen the reference benchmark point with longest averaged distance with others. Test2: Chosen the reference benchmark point with shortest averaged distance with others. Figure. 5 depicts the points in the tests. Point #3078 is the reference benchmark in Test1, and point #B014 is the reference benchmark in Test2. The experimental results are shown as Table 2. Observing the 5th column in Table 2, the RMS value in Test1 is 5.7 cm and in Test2 is 5.2 cm. The accuracy of the difference method in this study is about 5.4 cm which is better than the accuracy in gravimetric model (19.4 cm). It is proven that the difference method can promote the accuracy in gravimetric model. Observing the 3th column in Table 2, the mean value in Test1 is -2.3 cm and in Test2 is 0.5 cm. The experimental results are also proven that the accuracy of difference method is relative to the distance between the reference benchmark and the neighboring points. Points with shorter distance can eliminate more system errors.

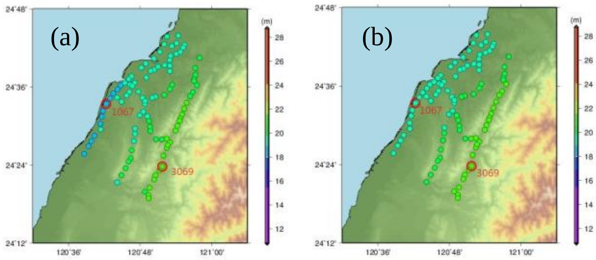

Figure 2. Geoid undulations at benchmarks:
(a) gravimetric geoid model; (b) hybrid geoid model

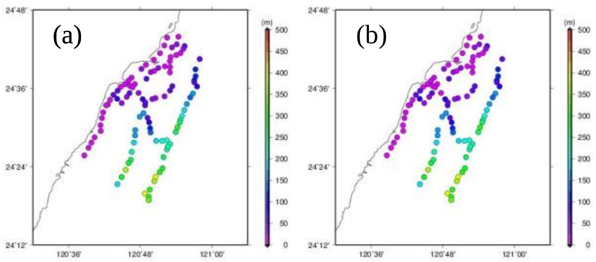

Figure 3. Orthometric heights at benchmarks:
(a) gravimetric geoid model; (b) hybrid geoid model

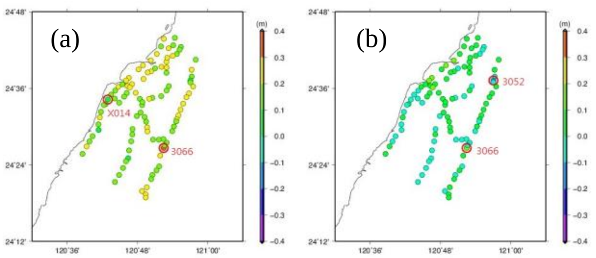

Figure 4. Discrepancy of the orthometric heights:
(a) gravimetric geoid model; (b) hybrid geoid model

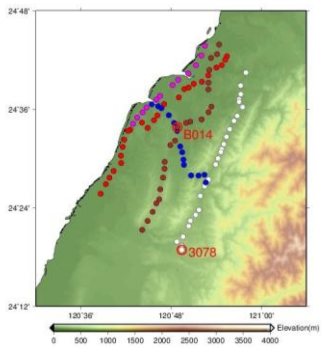

Figure 5. Reference benchmarks in the difference tests

Table 1. Statistics of geoid heights, orthometric heights and discrepancy of the orthometric heights (Unit: m)

Result	Max.	Min.	Mean	RMS
$N^{gravity}$	20.869	18.814	19.701	--
N^{hybird}	21.069	18.955	19.877	--
$H^{gravity}$	393.928	3.252	111.620	--
H^{hybird}	393.743	3.104	111.444	--
dH^{graviy}	0.319	0.042	0.187	0.194
dH^{hybird}	0.162	-0.105	0.011	0.057

Table 2. Statistics of the difference tests

Test	Average Distance/km	Mean/cm	SD/cm	RMS/cm
1	27.3	-2.3	5.2	5.7
2	12.7	0.5	5.2	5.2

5. CONCLUSIONS

The network RTK technology is applied to convert *h* into *H* in GPS-heighting, called e-GNSS heighting in this paper. Two geoid models, TWGEOID2014 and TWHYGEO2014, are adopted. The experimental results present that the hybrid model is better than gravimetric model because the system deviations in gravimetric model are corrected by merging GPS-derived and gravimetric geoidal heights. In this study, the orthometric height accuracy in gravimetric model is 19.4 cm and in hybrid model is 5.7 cm. In addition to the hybrid model, the difference method is experimented to eliminate the system errors. The accuracy in this method is about 5.4 cm and promote the accuracy in gravimetric model. With the high efficiency, accuracy and low costs, the GPS-heighting based on network RTK can be applied more widely on engineering and disaster prevention and rescue.

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