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Cournot-Bertrand competition: a revisit of strategic trade policy in the third-market model
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Cournot-Bertrand Competition: A Revisit of Strategic Trade Policy in the

Third-Market Model

1. Introduction

The seminal papers of Spencer and Brander (1983) as well as Brander and Spencer (1985) have shown that R&D or export subsidies could under some conditions increase national welfare through rent-shifting if the domestic and foreign firms engage in Cournot output-setting competition in a third country market. Striking though, the robustness of the conclusion has been quickly rebutted by Eaton and Grossman (1986) simply by replacing Cournot output-setting competition with Bertrand price-setting competition. The three pioneering papers have ushered in a large body of literature of strategic trade policy, which extends the original models in various directions such as two active governments, domestic consumption of the exportable goods, multiple domestic and foreign firms, repeated games, and government budget constraint (Brander, 1995). Nonetheless, these works are flawed by a common denominator; namely, all of them assume that the two firms adopt the same strategic variable.[1] More specifically, all the firms either set outputs or prices simultaneously. However, this kind of assumption is peculiar and unjustifiable both from the theoretical and from the real world points of view.

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[1] The argument is applicable to the case of multiple firms. However, for the sake of clarity, we will use the duopoly model in our illustration at this stage.

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Consider the duopoly model as specified in Brander and Spencer (1985) or Eaton and Grossman (1986). Let the two firms be firm 1 and firm 2 and suppose that each can choose between two strategic variables, output and price. Then, there will be four possible combinations of strategies, (output, output), (output, price), (price, output), and (price, price), where the first (second) element in each vector is the strategic variable chosen by firm 1 (firm 2). It is evident that Cournot competition and Bertrand competition are nothing but two special cases out of the four possibilities. Theoretically, without further information concerning the characteristics of the firms, there is no *a priori* reason to prefer either Cournot or Bertrand to the rest two. In other words, it is perfectly likely that the two firms would choose different strategic variables in a duopoly market. As for what happens in the real world, it is useful to recognize the fact that an output setter is an unaggressive price taker while a price setter is an aggressive price maker.[2] Depending on different characteristics of the firms in a given market, it is natural that some of them act more like aggressive price makers and the others more like unaggressive price takers. Therefore, the assumption that all firms act uniformly with respect to their choice of strategic variable makes not much sense in the real life.

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[2] See Ten Kate (2014) for a cogent discussion on this point. It deserves mentioning that we discovered Ten Kate's working paper, which came out on 21 August 2014, after we finished the first draft of the manuscript. Surprisingly, the arguments in Ten Kate (2014) are broadly in line with ours, though in quite a different context.

Perhaps for those reasons, some researchers have recently begun to consider the case where one of the duopolistic firms chooses output whereas the other chooses price as strategic variable, or Cournot-Bertrand competition (Naimzada and Tramontana, 2012; Ten Kate, 2014; Tremblay and Tremblay, 2011a; Tremblay et al. 2011b; Tremblay et al. 2011c).[3] To the authors' knowledge, however, there is still no study of strategic trade policy under the Cournot-Bertrand model. The main objective of this paper is to fill this gap in the literature, and thus provides some more general results not easily detected in the original model of Brander and Spencer or Eaton and Grossman.

The rest of the paper is organized as follows. Section 2 modifies the Brander-Spencer model so that the two firms can choose different strategic variables. After analyzing both the (Cournot, Bertrand) and (Bertrand, Cournot) cases, we combine the results with those of Brander and Spencer (1985) as well as Eaton and Grossman (1986) (hereinafter BS and EG) to arrive at a more general conclusion in Section 3. Section 3 also extends the duopoly models to the case with multiple foreign firms which may choose different strategic variables. Section 4 concludes the paper.

2. The Models

Our models follow all the assumptions of Brander and Spencer (1985) with only two

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[3] As pointed out by a reviewer, an important earlier contribution concerning this kind of "mixed duopoly" is Singh and Vives (1984).

modifications: (1) the two firms produce differentiated products, and (2) the two firms choose different strategic variables. As a result, each of our models is a two-stage game. In the first stage the home government decides the optimal export tax or subsidy to maximize social welfare, and in the second stage each firm chooses the level of its strategic variable to maximize its profit.

2.1 Domestic firm sets output and foreign firm sets price

Suppose a domestic firm (firm 1) and a foreign firm (firm 2) produce differentiated products, q_1 and q_2, and export all their outputs to a third country where the prices of q_1 and q_2 are p_1 and p_2, respectively. Firm 1 (Firm 2) behaves as a Cournot-type output (Bertrand-type price) setter. Let the inverse demand functions be $p_i = p_i(q_1, q_2)$, $\partial p_i / \partial q_i < 0$, where $i = 1,2$ from now on. Making use of the inverse demand functions, the profit function of firm i, π^i, can be expressed in terms of the strategic variables q_1 and p_2 as:

$$\pi^1(q_1, p_2, s) = p_1(q_1, p_2)q_1 - c_1 q_1 + s q_1, \tag{1}$$

$$\pi^2(q_1, p_2) = p_2 q_2(q_1, p_2) - c_2 q_2(q_1, p_2), \tag{2}$$

where c_i is the fixed marginal cost of firm i and $s > 0$ (< 0) is the specific subsidy (tax) of the home government on the exports of firm 1.[4]

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[4] For simplicity, we assume no fixed costs of production.

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Assuming that the second-order conditions hold, the first-order conditions for

profit maximization are:

$$\frac{\partial \pi^1}{\partial q_1} = \frac{\partial p_1}{\partial q_1} q_1^* + p_1 - c_1 + s = 0, \qquad (3)$$

$$\frac{\partial \pi^2}{\partial p_2} = q_2 + \frac{\partial q_2}{\partial p_2}(p_2^* - c_2) = 0. \qquad (4)$$

Equations (3) and (4) implicitly define the reaction functions, $R_1(p_2)$ and $R_2(q_1)$,

of the two firms with slopes:

$$\left. \frac{dp_2}{dq_1} \right|_{R_1} = -\left(\frac{\partial^2 \pi^1 / \partial q_1^2}{\partial^2 \pi^1 / \partial q_1 \partial p_2} \right) > 0,$$

$$(5)$$

$$\left. \frac{dp_2}{dq_1} \right|_{R_2} = -\left(\frac{\partial^2 \pi^2 / \partial p_2 \partial q_1}{\partial \pi^2 / \partial p_2^2} \right) < 0.^5 \qquad (6)$$

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[5] It is easy to show that $\dfrac{\partial^2 \pi^1}{\partial q_1 \partial p_2} = \dfrac{\partial^2 p_1}{\partial q_1 \partial p_2} q_1 + \dfrac{\partial p_1}{\partial p_2}$ and $\dfrac{\partial^2 \pi^2}{\partial p_2 \partial q_1} = \dfrac{\partial^2 q_2}{\partial p_2 \partial q_1}(p_2 - c_2) + \dfrac{\partial q_2}{\partial q_1}$,

where $\dfrac{\partial p_1}{\partial p_2} > 0$ and $\dfrac{\partial q_2}{\partial q_1} < 0$ when the two goods are substitutes. Note that $\dfrac{\partial^2 p_1}{\partial q_1 \partial p_2} =$

$\dfrac{\partial^2 q_2}{\partial p_2 \partial q_1}(p_2 - c_2) = 0$ if the demand functions are linear, though their signs are in general

indeterminate. We will assume that sign $\left(\dfrac{\partial^2 \pi^1}{\partial q_1 \partial p_2}\right)$ = sign $\left(\dfrac{\partial p_1}{\partial p_2}\right)$ and sign $\left(\dfrac{\partial^2 \pi^2}{\partial p_2 \partial q_1}\right)$ = sign

$\left(\dfrac{\partial q_2}{\partial q_1}\right)$ in our analysis. These assumptions along with the second-order conditions thus determine

the slopes of $R_1(p_2)$ and $R_2(q_1)$.

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$R_1(p_2)$ is positively sloped because the marginal profit of firm 1, $\partial\pi^1/\partial q_1$, increases as p_2 rises, leading the optimal output level to increases. By the same token, $R_2(q_1)$ has a negative slope. The intersection of $R_1(p_2)$ and $R_2(q_1)$ determines the equilibrium $(q_1^*(s), p_2^*(s))$.[6] Figure 1 depicts such an equilibrium with the inverse demand functions $p_1 = a - q_1 - bq_2$ and $p_2 = a - q_2 - bq_1$, $0 < b < 1$.

Substituting $q_1^*(s)$ and $p_2^*(s)$ into equations (3) and (4), we are able to see how changes in home government's policy, s, affect the strategic variables. Totally differentiating equations (3) and (4), we get:

$$\begin{bmatrix} \dfrac{\partial^2\pi^1}{\partial q_1^2} & \dfrac{\partial^2\pi^1}{\partial q_1\partial p_2} \\ \dfrac{\partial^2\pi^2}{\partial p_2\partial q_1} & \dfrac{\partial^2\pi^2}{\partial p_2^2} \end{bmatrix} \begin{bmatrix} \dfrac{dq_1}{ds} \\ \dfrac{dp_2}{ds} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

(7)

Solving (7) by Cramer's rule gives us:

$$\frac{dq_1^*}{ds} = -(\frac{\partial^2\pi^2/\partial p_2^2}{D}) > 0,$$
(8)

$$\frac{dp_2^*}{ds} = \frac{\partial^2\pi^2/\partial p_2\partial q_1}{D} < 0,$$
(9)

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[6] For simplicity, we will always deal with the case with a unique, stable equilibrium.

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where $D = \begin{vmatrix} \dfrac{\partial^2 \pi^1}{\partial q_1^2} & \dfrac{\partial^2 \pi^1}{\partial q_1 \partial p_2} \\[3mm] \dfrac{\partial^2 \pi^2}{\partial p_2 \partial q_1} & \dfrac{\partial^2 \pi^2}{\partial p_2^2} \end{vmatrix} > 0$ by the second-order conditions and signs of $\dfrac{\partial^2 \pi^1}{\partial q_1 \partial p_2}$

and $\dfrac{\partial^2 \pi^2}{\partial p_2 \partial q_1}$. As expected, an increase (a decrease) in home government's subsidy

(tax) increases domestic firm's optimal output level while depresses foreign firm's

optimal price.

Now let's turn to the optimal policy of the home government, which is to

maximize the country's welfare. Since there is no domestic consumption, the

welfare is the profit of firm 1 net of (plus) export subsidy (tax revenue). That is,

$w(s) = \pi^1(q_1^*(s), p_2^*(s); s) - s q_1^*(s)$. The first-order condition for welfare

maximization is:

$$\frac{\partial w}{\partial s} = \frac{\partial \pi^1}{\partial p_2} \frac{\partial p_2^*}{\partial s} - s^* \frac{\partial q_1^*}{\partial s} = 0 . \tag{10}$$

Since $\partial \pi^1 \big/ \partial p_2 = \partial p_1 \big/ \partial p_2 \, q_1 > 0$ and, $\dfrac{\partial p_2^*}{\partial s} < 0$ by (9), we have:

$$\left. \frac{\partial w}{\partial s} \right|_{s=0} = \frac{\partial \pi^1}{\partial p_2} \frac{\partial p_2^*}{\partial s} < 0 . \tag{11}$$

As a result, starting with free trade the home government can improve domestic

welfare by taxing the exports of firm 1. This result, along with (8) and (10) gives us

the home country's optimal tax as follows:

$$s^* = \frac{\partial \pi^1}{\partial p_2} \frac{\partial p_2^*}{\partial s} \Big/ \frac{\partial q_1^*}{\partial s} < 0 . \tag{12}$$

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In other words, home government's optimal policy is an export tax when the domestic firm sets output and the foreign firm sets price. This is depicted in Figure 2, where E and E' denote equilibria of zero and the optimal export tax, respectively.[7] Notice that firm 1's iso-profit curve is tangent to firm 2's reaction curve, R^2, at E', implying that E' is a Stackelberg equilibrium under free trade when firm 1 behaves as a leader while firm 2 behaves as a follower.[8]

2.2 Domestic firm sets price and foreign firm sets output

This case differs from what discussed in the last section only in that the two firms exchange their strategic variables. Consequently, we will not repeat all the details to save space. The profit functions in terms of the strategic variables p_1 and q_2 are:

$$\pi^1(p_1, q_2, s) = p_1 q_1(p_1, q_2) - c_1 q_1(p_1, q_2) + s q_1(p_1, q_2),$$ (13)

$$\pi^2(p_1, q_2) = p_2(p_1, q_2)q_2 - c_2 q_2.$$ (14)

The first-order conditions for profit maximizations are:

$$\frac{\partial \pi^1}{\partial p_1} = (p_1^* - c_1 + s)\frac{\partial q_1}{\partial p_1} + q_1 = 0,$$ (15)

$$\frac{\partial \pi^2}{\partial q_2} = \frac{\partial p_2}{\partial q_2}q_2^* + p_2 - c_2 = 0.$$ (16)

The reaction functions implicitly defined by (15) and (16) have slopes as follows:

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[7] It is easy to show that raising export tax will shift R_1 leftward and that a higher iso-profit curve represents a higher profit of firm 1.
[8] Proof of this is provided in Appendix 1. But, proofs of all other similar results below will not be repeated and are available upon request from the authors. Moreover, it can be shown that the profits of both firms are higher at E' than those at E. Firm 2's iso-profit curves are not shown to avoid jamming the figure.

$$\frac{dq_2}{dp_1}\bigg|_{R_1} = -\left(\frac{\partial^2 \pi^1 / \partial p_1^2}{\partial^2 \pi^1 / \partial p_1 \partial q_2}\right) < 0, \tag{17}$$

$$\frac{dq_2}{dp_1}\bigg|_{R_2} = -\left(\frac{\partial^2 \pi^2 / \partial q_2 \partial p_1}{\partial^2 \pi^2 / \partial q_2^2}\right) > 0 .^9 \tag{18}$$

Equations (15) and (16) determine the equilibrium ($p_1^*(s)$, $q_2^*(s)$), which is represented by the intersection of reaction curves R_1 and R_2 in Figure 3.

Making use of the first-order conditions, (15) and (16), and performing comparative statistics at the equilibrium, we obtain:

$$\frac{dp_1^*}{ds} = -\frac{\partial q_1}{\partial p_1}\left(\frac{\partial^2 \pi^2 / \partial q_2^2}{D'}\right) < 0, \tag{19}$$

$$\frac{dq_2^*}{ds} = \frac{\partial q_1}{\partial p_1}\left(\frac{\partial^2 \pi^2 / \partial q_2 \partial p_1}{D'}\right) < 0, \tag{20}$$

where, $D' = \begin{vmatrix} \dfrac{\partial^2 \pi^1}{\partial p_1^2} & \dfrac{\partial^2 \pi^1}{\partial p_1 \partial q_2} \\ \dfrac{\partial^2 \pi^2}{\partial q_2 \partial p_1} & \dfrac{\partial^2 \pi^2}{\partial q_2^2} \end{vmatrix} > 0 .^{10}$ As a result, an increase (a decrease) in home

government's subsidy (tax) not only depresses domestic firm's optimal price but also decreases foreign firm's optimal output level.

Now the government tries to choose the optimal value of s to maximize the welfare $w(s) = \pi^1(p_1^*(s), q_2^*(s); s) - sq_1^*(p_1^*(s), q_2^*(s))$. The first-order condition is:

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[9] Detailed assumptions and discussions concerning the slopes of R_1 and R_2 are available upon request from the authors.

[10] The sign of D' is determined by the argument similar to that of D .

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$$\frac{\partial w}{\partial s} = \frac{\partial \pi^1}{\partial q_2}\frac{\partial q_2^*}{\partial s} - s^*(\frac{\partial q_1^*}{\partial p_1}\frac{\partial p_1^*}{\partial s} + \frac{\partial q_1^*}{\partial q_2}\frac{\partial q_2^*}{\partial s}) = 0. \tag{21}$$

That the two goods are substitutes implies $\dfrac{\partial q_1}{\partial q_2} < 0$ and thus

$\dfrac{\partial \pi^1}{\partial q_2} = (p_1 - c_1 + s)\dfrac{\partial q_1}{\partial q_2} < 0$. Combining this with (20), we have:

$$\frac{\partial w}{\partial s}\bigg|_{s=0} = \frac{\partial \pi^1}{\partial q_2}\frac{\partial q_2^*}{\partial s} > 0. \tag{22}$$

In sharp contrast to the last case, the home government should subsidize the domestic

firm to raise the domestic welfare when firm 1 sets price while firm 2 sets output.

Finally, by (19), (20) and law of demand, equation (21) gives rise to the optimal

subsidy:

$$s^* = \frac{\partial \pi^1}{\partial q_2}\frac{\partial q_2^*}{\partial s} / (\frac{\partial q_1^*}{\partial p_1}\frac{\partial p_1^*}{\partial s} + \frac{\partial q_1^*}{\partial q_2}\frac{\partial q_2^*}{\partial s}) > 0. \tag{23}$$

Figure 4 depicts the impact of home government's optimal subsidy on the strategic

variables as well as profits of the two firms. Without subsidy the reaction curves are

R_1 and R_2, and E is the equilibrium. The curve R_1 shifts downward whenever

the home government subsidizes the exports of firm 1. The optimal subsidy is the

one that moves R_1 to R_1' so that one of firm 1's iso-profit curves is tangent to R_2

at the intersection point E', where both p_1 and q_2 are smaller than those at E.

However, in the present case the increase in the profit of firm 1 is at the cost of firm

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2's profit.   Again, it can be shown that E' is a Stackelberg equilibrium under free

trade when firm 1 behaves as a leader while firm 2 behaves as a follower.

3.   Comparative Analysis and Extensions

3.1 Comparative analysis

We now put together what we obtained above and those of BS and EG to derive a

more general conclusion about the optimal policy of the home country.   The results

are summarized in Table 1, which arranges the four combinations of strategies in two

groups according to the strategic variable of firm 2.   The first (bottom) two rows

give the relevant results when firm 2 sets output (price).   It is clear to see that the

optimal policy of the home country is an export subsidy (tax) as long as firm 2's

strategic variable is output (price).   Alternatively, the optimal policy of the home

country depends on what firm 1 anticipates what firm 2 keeps fixed when firm 1

maximizes its profit.   Therefore, we establish:

Proposition 1*: Under the assumption that the two goods are substitutes, the optimal*

policy of the home country is an export subsidy (tax) as long as the foreign firm's

strategic variable is output (price) regardless of the strategic variable of the domestic

firm.

The result is perfectly in line with the conclusion of Ten Kate (2014): "What matters

is not what you set, but what you assume the others keep fixed."   This is because

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firm 1 becomes a *de facto* monopolist in the residual market once firm 2 chose and fixed its output level. For a monopolist it makes no difference whether it chooses output or price to maximize its profit, implying that the optimal policy of the home government in the first stage stays the same. The same logic applies when firm 2 chose its price as it is tantamount to fixing some level of output. Consequently, it does not matter what firm 1's strategic variable is as long as firm 2's choice of strategic variable is made.

But, why should the home government subsidize (tax) domestic firm's exports when the strategic variable of foreign firm is output (price)? Taking firm 2's strategic variable is quantity as an example. If firm 1's strategic variable is output, then its profit function becomes $\pi^1(q_1, q_2, s) = p_1(q_1, q_2)q_1 - c_1 q_1 + s q_1$. Partially differentiate $\pi^1(q_1, q_2, s)$ with respect to q_2 gives us:

$$\frac{\partial \pi^1}{\partial q_2} = \frac{\partial p_1}{\partial q_2} q_1 < 0. \tag{24}$$

If firm 1's strategic variable is price, then partially differentiate (13) with respect to q_2 gives us:

$$\frac{\partial \pi^1}{\partial q_2} = \frac{\partial q_1}{\partial q_2}(p_1 - c_1 + s) < 0. \tag{25}$$

Equations (24) and (25) suggest that any policy leading to decreases in firm 2's output would raise firm 1's profit and thus the welfare of the home country. Consequently,

the optimal policy of the home government must be an export subsidy as it depresses

q_2 under the assumption that the goods are substitutes. In a similar vein, it is

readily to show that, when firm 2's strategic variable is price, an export tax would

raise p_2 and increase firm 1's profit as well as domestic welfare.

Proposition 1 provides us with a very simple, general rule to determine the

optimal policy of the home government for all four combinations of strategic

variables. However, it does not give us any information about the size of the

optimum subsidy or tax. Fortunately, based on our results in Section 2 and those of

BS and EG, we can prove:

Corollary 1*: When the two goods are substitutes, the optimal policy of the home*

country moves the equilibrium to what would be the Stackelberg equilibrium where

the domestic firm behaves as the leader while the foreign firm behaves as the follower

under free trade.

It is also noteworthy from Table 1 that firm 2's output and profit decrease when the

home government subsidizes exports of firm 1 (the first two rows). In other words,

when firm 2's strategic variable is output, the home government can take the

advantage of the first-mover advantage vis-à-vis the firms to help firm 1 shift rent

from firm 2 to maximize domestic welfare while hurt the welfare of foreign country.

Conversely, firm 2's price and profit increase when the home government taxes

exports of firm 1 (the bottom two rows). Under the circumstances, imposing export tax by the home government facilitates collusion between the two firms, leading to higher profits (welfare) for both firms (countries) at the cost of the third country's consumers.

3.2 Extensions

While simple and intuitively appealing, proposition 1 is based on Brander and Spencer's original third-market duopoly model. Since the optimal policy depends exclusively on the foreign firm's strategic variable, a natural question to ask is whether a similar conclusion can be obtained if there is more than one firm in the foreign country and not all of them choose the same strategic variable. Before proceeding to the general case, let's consider two simplest cases where foreign country owns two firms, firm 2 and firm 3, with firm 2 (firm 3) choosing output (price) as its strategic variable while that of the firm 1 is output or price. It is evident that proposition 1 is not directly applicable here. Nevertheless, we might envisage that the result is very likely to depend on the relative strength of output effect of firm 2 and price effect of firm 3. This is what we would like to demonstrate below.

Case 1: Firm 1 sets output

As in the duopoly case and using similar notations, the profits of the three firms in terms of the strategic variable q_1, q_2 and p_3 are, respectively:

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$$\pi^1(q_1, q_2, p_3, s) = p_1(q_1, q_2, p_3)q_1 - c_1q_1 + sq_1, \tag{26}$$

$$\pi^2(q_1, q_2, p_3,) = p_2(q_1, q_2, p_3)q_2 - c_2q_2, \tag{27}$$

$$\pi^3(q_1, q_2, p_3) = p_3q_3(q_1, q_2, p_3) - c_3q_3(q_1, q_2, p_3). \tag{28}$$

Let $q_1^*(s)$, $q_2^*(s)$ and $p_3^*(s)$ denote the solutions of maximizing the three firms'

profits for given s, then the home government shall choose s in the first stage to

maximize the following welfare:

$$w = \pi^1(q_1^*(s), q_2^*(s), p_3^*(s); s) - sq_1^*(s). \tag{29}$$

The optimal export subsidy or tax turns out to be:

$$s^* = \frac{\pi_{p_3}^1 \,\partial p_3^* \big/ \partial s + \pi_{q_2}^1 \,\partial q_2^* \big/ \partial s}{\partial q_1^* \big/ \partial s}, \tag{30}$$

where $\pi_{p_3}^1 = \partial \pi^1 \big/ \partial p_3 = \partial p_1 \big/ \partial p_3 \, q_1 > 0$, $\partial p_3^* \big/ \partial s < 0$, $\pi_{q_2}^1 = \partial \pi^1 \big/ \partial q_2 = \partial p_1 \big/ \partial q_2 \, q_1 < 0$,

$\partial q_2^* \big/ \partial s < 0$, and $\partial q_1^* \big/ \partial s > 0$.

Comparing (30) with (12) and (23), we find that in each case the denominator

captures the impact of export subsidy or tax on firm 1's output, which is always

positive.[11]   The first [second] term in the numerator looks exactly the same as that of

(12) [(23)].   Loosely speaking, (30) can be regarded as the combination of (12) and

(23).   Since $\pi_{p_3}^1 \dfrac{\partial p_3^*}{\partial s}$ stands for the impact of the government policy on the domestic

firm's marginal profit via the strategic variable of firm 3, a negative $\pi_{p_3}^1 \dfrac{\partial p_3^*}{\partial s}$ implies

that the optimal policy is an export tax.   However, this is not the only effect in the

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[11] Of course, the numerical values are different.   This is also true when we discuss about the two
terms in the numerator.

present case; the government policy also affects the marginal profit of firm 1 via firm

2's strategic variable. This effect is captured by the positive $\pi_{q_2}^1 \dfrac{\partial q_2^*}{\partial s}$, suggesting an

export subsidy as the optimal policy. As a result, the sign of (30) and thus the home

government's optimal policy is generally indeterminate. For simplicity, let us call

$\pi_{p_3}^1 \dfrac{\partial p_3^*}{\partial s}$ ($\pi_{q_2}^1 \dfrac{\partial q_2^*}{\partial s}$) the indirect price (output) effect. Then, whether the optimal

policy is an export tax or an export subsidy depends on the relative strength of the two

effects. It is an export subsidy (tax) if the indirect output (price) effect dominates the

indirect price (output) effect. Last but not least, the s^* in (30) can be shown to

move the equilibrium to what would be the Stackelberg equilibrium where firm 1

behaves as the leader and the two foreign firms behave as followers under free trade.

Case 2: Firm 1 sets price

Without repeating the details, let $p_1^*(s)$, $q_2^*(s)$ and $p_3^*(s)$ be the solutions of

maximizing the three firms' profits for given s, and the home government's welfare

maximization policy is:[12]

$$s^* = \frac{\pi_{p_3}^1 \left.\dfrac{\partial p_3^*}{\partial s}\right. + \pi_{q_2}^1 \left.\dfrac{\partial q_2^*}{\partial s}\right.}{\dfrac{\partial q_1^*}{\partial p_1^*}\dfrac{\partial p_1^*}{\partial s} + \dfrac{\partial q_1^*}{\partial q_2^*}\dfrac{\partial q_2^*}{\partial s} + \dfrac{\partial q_1^*}{\partial p_3^*}\dfrac{\partial p_3^*}{\partial s}}, \tag{31}$$

where $\dfrac{\partial p_1^*}{\partial s} < 0$, $\dfrac{\partial q_2^*}{\partial s} < 0$, $\dfrac{\partial p_3^*}{\partial s} < 0$, $\pi_{p_3}^1 = (p_1 - c_1 + s)\dfrac{\partial q_1}{\partial p_3} > 0$, and

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[12] The welfare function in this case is $w = \pi^1(p_1^*(s), q_2^*(s), p_3^*(s), s) - s q_1^*(p_1^*(s), q_2^*(s), p_3^*(s))$.

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$\pi_{q_2}^1 = (p_1 - c_1 + s)\dfrac{\partial q_1}{\partial q_2} < 0$.   Like (30), the numerator of (31) consists of the indirect

output and price effects.   However, a complication comes in the denominator that

represents the effect of home government's policy on firm 1's output.   While the first

two terms are positive when the goods are substitutes, the third term turns out to be

negative.   This is a salient feature in any duopoly or oligopoly model like we have

here with two firms using prices as their strategic variables.[13]   For the present 3-firm

model, we follow the argument of Eaton and Grossman (1986) to assume that the first

two terms dominate the third one so that the denominator of (31) is positive.[14]

Barring this caveat, the result is the same as that of Case 1: the optimal policy is an

export subsidy (tax) if the indirect output (price) effect dominates the indirect price

(output) effect, and it results in the Stackelberg equilibrium where firm 1 behaves as

the leader and the two foreign firms behave as followers under free trade.

Finally, (30) and (31) can readily be extended to cases where there are n

foreign firms, of which the first k (the rest $n-k$) firms choose outputs (prices) as

their strategic variables.[15]   Depending on the strategic variable of firm 1, we have:

$$s^* = \dfrac{\sum\limits_{i=k+2}^{n+1} \pi_{p_i}^1 \,\partial p_i^* \big/ \partial s + \sum\limits_{j=2}^{k+1} \pi_{q_j}^1 \,\partial q_j^* \big/ \partial s}{\partial q_1^* \big/ \partial s} \quad , \tag{30'}$$

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[13] For instance, this also arises in Eaton and Grossman (1986)

[14] This is always true in a linear demand model.   However, the assumption becomes less likely to hold when more and more foreign firms have prices as their strategic variables as we will see below.

[15] Note n should not be so large that the strategic interactions among the $n+1$ firms disappear.

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$$ s^* = \frac{\sum\limits_{i=k+2}^{n+1} \pi_{p_i}^1 \left.\partial p_i^*\right/\partial s + \sum\limits_{j=2}^{k+1} \pi_{q_j}^1 \left.\partial q_j^*\right/\partial s}{\dfrac{\partial q_1^*}{\partial p_1^*}\dfrac{\partial p_1^*}{\partial s} + \sum\limits_{j=2}^{k+1} \dfrac{\partial q_1^*}{\partial q_j^*}\dfrac{\partial q_j^*}{\partial s} + \sum\limits_{i=k+2}^{n+1} \dfrac{\partial q_1^*}{\partial p_i^*}\dfrac{\partial p_i^*}{\partial s}} . \tag{31'} $$

Under similar circumstances discussed in (30) and (31), we thus establish:

Proposition 2*: Under the assumption that all goods are substitutes and* k $(n-k)$ *foreign firms choose outputs (prices) as their strategic variables, the optimal policy of the home country is an export subsidy (tax) as long as the indirect output (price) effect dominates the indirect price (output) effect regardless of the strategic variable of the domestic firm.*

And

Corollary 2*: When all goods are substitutes and* k $(n-k)$ *foreign firms choose outputs (prices) as their strategic variables, the optimal subsidy or tax of the home country moves the equilibrium to what would be the Stackelberg equilibrium where the domestic firm behaves as the leader while the n foreign firms behave as followers under free trade.*

It is now clear that the very general Proposition 2 and Corollary 2 subsume BS and EG as two special cases with $n = k = 1$ as well as $n = 1$ and $k = 0$, respectively.

4.   Conclusions

A common but peculiar practice in a duopoly or oligopoly analysis is to assume that all firms choose the same strategic variable.   In the case where two firms choose

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between setting output or setting price we end up with the celebrated Cournot (output, output) model and the Bertrand (price, price) model. While straightforward and intuitively appealing, this kind of practice is flawed both from the theoretical and from the real world points of view. Not only is there no *a priori* theoretical justification to exclude the possibility that firms may choose different strategic variables, but we cannot observe that firms indeed behave uniformly in the markets of real world. It is therefore interesting and important to bridge this knowledge gap and see what would happen when duopolistic or oligopolistic firms choose to use different strategic variables. This paper is intended to address the problem in the context of strategic trade policy using Brander and Spencer's third-market model. Starting with the duopoly models and incorporating the results of BS and EG, we arrive at the very general, simple rule to determine the optimal policy of the home government for any combination of strategic variables: regardless of the strategic variable of the domestic firm, the optimal policy of the home country is an export subsidy (tax) as long as the foreign firm's strategic variable is output (price). Moreover, the optimal subsidy or tax of the home country moves the equilibrium to what would be the Stackelberg equilibrium where the domestic firm behaves as the leader while the foreign firm behaves as the follower under free trade.

With appropriate interpretations and a suitable caveat, the above simple results

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are shown to hold when there is more than one firm in the foreign country, of which some choose outputs while others choose prices as their strategic variables. We have not dealt with cases where there are multiple firms in the home country as it is well-known that a sufficient large number of domestic firms might lead to an opposite result even in the relatively simple case where there is only one foreign firm and all of the firms set outputs simultaneously (Brander, 1995). Despite of this, we believe that the insight derived in this paper that it is what the foreign firms' strategic variables that determine the home government's optimal policy has significantly improved our understanding of the essence of strategic trade policy in the third-market model. Finally, it is worth noting that our paper can be extended to take into consideration the possibility of the duopolists/oligopolists to endogenously choose the strategic variable according to profit gained.[16] This is the issue we are to investigate in the near future.

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[16] We are indebted to two referees for pointing out this very promising direction for our future study.

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Appendix 1: Proof that E' in Figure 2 is a Stackelberg equilibrium under free trade when firm 1 behaves as a leader while firm 2 behaves as a follower

Proof: Suppose that firm 1 in Section 2.1 is the Stackelberg leader while firm 2 the follower without a government subsidy. The reaction function $R_2(q_1)$ can be obtained from (4) and expressed as $p_2 = p_2(q_1)$. Then, with $s = 0$, firm 1's profit function becomes $\pi^1(q_1, p_2(q_1), 0) = \pi^1(q_1, p_2(q_1))$, and the first-order condition for a profit maximum is:

$$\frac{\partial \pi^1}{\partial q_1} + \frac{\partial \pi^1}{\partial p_2} \frac{dp_2}{dq_1} = 0, \tag{A1}$$

where $\dfrac{dp_2}{dq_1}$ is the slope of $R_2(q_1)$, namely (6). Using (6), (8) and (9), we have:

$$\frac{dp_2}{dq_1} = -\frac{\partial^2 \pi^2 / \partial p_2 \partial q_1}{\partial^2 \pi^2 / \partial p_2^2} = \frac{\partial p_2^* / \partial s}{\partial q_1^* / \partial s}. \tag{A2}$$

With $s = 0$, partially differentiating (1) with respect to q_1 and p_2 gives us::

$$\frac{\partial \pi^1}{\partial q_1} = \frac{\partial p_1}{\partial q_1} q_1 + p_1 - c_1, \tag{A3}$$

$$\frac{\partial \pi^1}{\partial p_2} = q_1 \frac{\partial p_1}{\partial p_2}. \tag{A4}$$

Substituting (A2), (A3), and (A4) into (A1), we get firm 1's first-order condition for profit maximization when it behaves as the Stackelberg leader:

$$\frac{\partial p_1}{\partial q_1} q_1^* + p_1 - c_1 + q_1^* \frac{\partial p_1}{\partial p_2} \left(\frac{\partial p_2^* / \partial s}{\partial q_1^* / \partial s} \right) = 0 \tag{A5}$$

Finally, substitute the optimal subsidy given by (12) into the first-order condition (3), the result is exactly the same as (A5). This completes the proof.
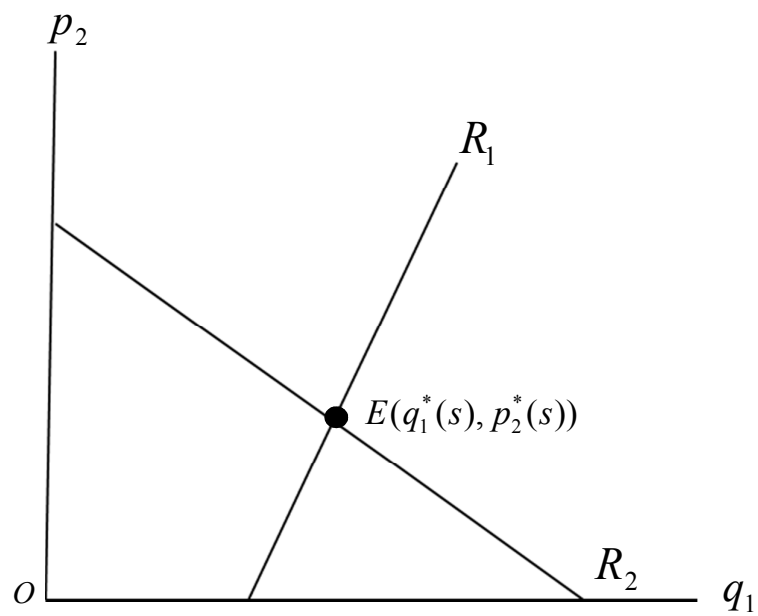
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p_2

R_1

$E(q_1^*(s), p_2^*(s))$

R_2

O

q_1

Figure 1: Reaction functions when firm 1 sets output and firm 2 sets price

p_2

R_1' R_1

E'

E

R_2

O q_1
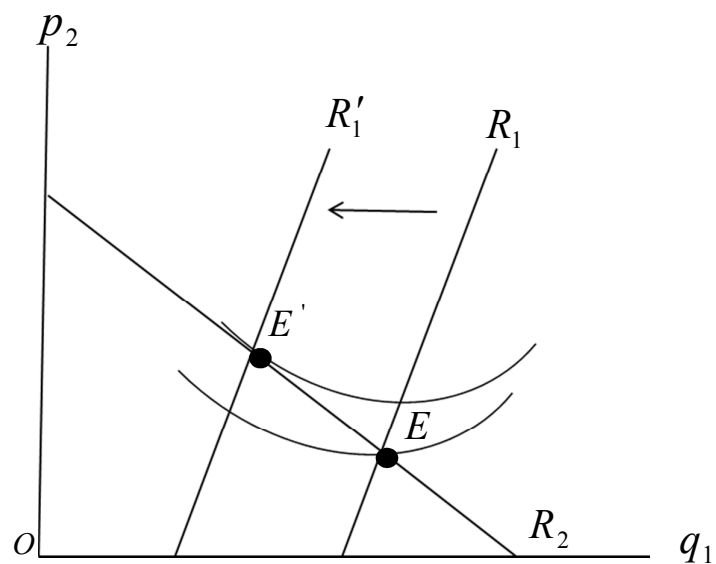
Figure 2: Optimal tax by home government when firm 1 sets output and
firm 2 sets price

2

q_2

R_2

$E(p_1^*(s), q_2^*(s))$

R_1

o

p_1
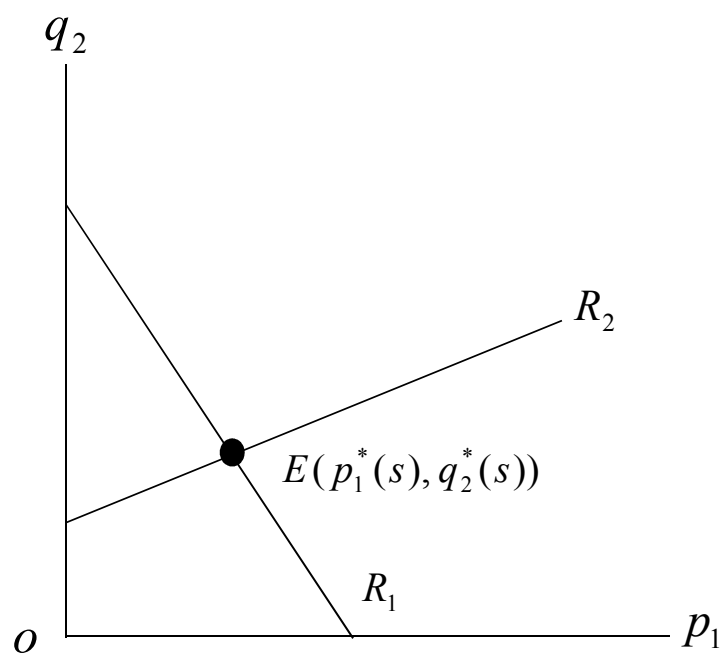
Figure 3: Reaction functions when firm 1 sets price and firm 2 sets output

q_2

R_2

E

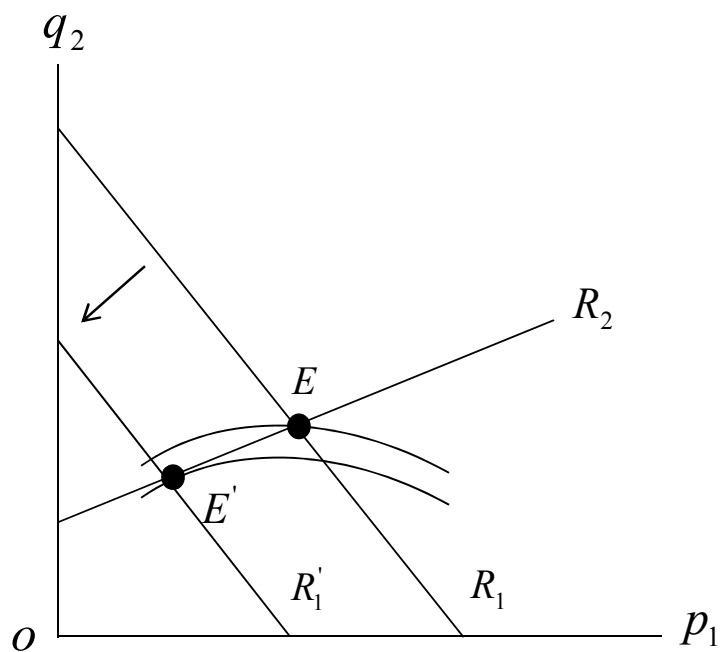E'

R_1' R_1

o p_1

Figure 4: Optimal subsidy by home government when firm 1 sets price
and firm 2 sets output

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Table 1: Comparative analysis of the four combinations of strategies

Strategic variable		Home optimal policy	Change in strategic variable		Change in profit	
Firm 1	Firm 2		Firm 1	Firm 2	Firm 1	Firm 2
Output (q_1)	**Output** (q_2)	**Subsidy** $(s^* > 0)$	$\Delta q_1 > 0$	$\Delta q_2 < 0$	$\Delta \pi^1 > 0$	$\Delta \pi^2 < 0$
Price (p_1)	**Output** (q_2)	**Subsidy** $(s^* > 0)$	$\Delta p_1 < 0$	$\Delta q_2 < 0$	$\Delta \pi^1 > 0$	$\Delta \pi^2 < 0$
Output (q_1)	Price (p_2)	Tax $(s^* < 0)$	$\Delta q_1 < 0$	$\Delta p_2 > 0$	$\Delta \pi^1 > 0$	$\Delta \pi^2 > 0$
Price (p_1)	Price (p_2)	Tax $(s^* < 0)$	$\Delta p_1 > 0$	$\Delta p_2 > 0$	$\Delta \pi^1 > 0$	$\Delta \pi^2 > 0$