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Detecting and modelling the jump risk of CO_2 emission allowances and their impact on the valuation of option on futures contracts

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Modelling CO_2 emission allowance prices is important for pricing CO_2 emission allowance linked assets in the emissions trading scheme (ETS). Some statistical properties of CO_2 emission allowance prices have been discovered in the literature ignoring price jumps. By employing real data from the ETS, this research first detects the jump risk using a jump test and then verifies jump effects in modelling CO_2 emission allowance prices by comparing the in-sample and out-of-sample model performance. We suggest a model which can capture the statistical properties of autocorrelation, volatility clustering and jump effects is more appropriate for modelling CO_2 emission allowance prices. We establish a general framework for pricing CO_2 emission allowance options on futures contracts with these properties and find that the jump risk significantly affects the value of the CO_2 emission allowance option on futures contracts. More importantly, we demonstrate that the dynamic jump ARMA–GARCH model can provide more accurate valuations of the CO_2 emission allowance options on futures than other models in terms of pricing error.

Keywords: Emission allowances; Dynamic jump model; Jump test; Conditional Esscher transform

JEL Classification: C51, G13

1. Introduction

In recent years, scientists have become increasingly concerned with the concentration of greenhouse gases (GHGs) in the atmosphere and the phenomenon of global warming. The effect of global warming has potentially catastrophic consequences on the environment as can be seen from the ice caps that are melting faster, the sea levels that are rising and weather patterns that are changing. Thus, managing the risks caused by GHGs is becoming more and more important for successive governments and international society as a whole. The Kyoto protocol, which was initially adopted on 11 December 1997 in Kyoto, Japan and came into force on 16 February 2005, is a well-known agreement that is a response to these risks.§ Today, the Kyoto protocol covers over 190 counties. The industrialized countries under the protocol agreement are to reduce their collective emissions of GHGs by 5.2% compared with those for the year 1990. The goal of the Kyoto agreement is to lower overall emissions from six GHGs—carbon dioxide, methane, nitrous oxide, sulphur hexafluoride, HFCs and PFCs—calculated as an average over the five-year period of 2008–2012. National targets range from 8% reductions for the European Union and some others to 7% for the US, 6% for Japan, 0% for Russia, and permitted increases of 8% for Australia and 10% for Iceland.

Developing the financial market for trading emission allowances or permits, primarily carbon dioxide (CO_2), is one of the main mechanisms for reducing GHGs. Several national and

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§The Kyoto protocol contains three mechanisms, namely, international emission trading (IET), joint implementation (JI) and the clean development mechanism (CDM) that are designed to support Annex I members to meet their targets by purchasing emission credits from other parties, in particular:

- for the IET mechanism, these credits are AAUs from other Annex I members.
- for the JI mechanism, the credits are the so-called ERUs (emission reduction units) and RMUs (specific removable units) which are obtained from projects within the Annex I area.
- for the CDM mechanism, the credits are the so-called CERs (certified emission reductions), obtained from GHG reduction within non-Annex I countries.

regional emission markets have been established. For example, the European Union Emissions Trading Scheme (EU ETS) formally entered into operation in January 2005. Since then, the EU ETS has become one of the important trading markets for reducing worldwide emissions of carbon dioxide (CO_2) within the Kyoto Protocol. Under the EU ETS scheme, the firms must hold the required amount of emission permits at the end of the year to meet their emissions of CO_2 over the previous year. However, the ETS allows firms to trade the amount of emission permits that they hold. Thus, within this framework, the prices of CO_2 emission allowances can affect the global economy because they give rise to an increasingly significant impact on the prices of power, gas and other emission-related commodities and economic activities, respectively (e.g. Kara *et al*. 2008, Carmona *et al*. 2010). According to a 2012 report of the World Bank, the global transactions for 2011 exceeded 10.2 billion tons of CO_2 worth approximately $176 billion.†

To date, the EU ETS has experienced three phases. Phase I, from 2005–2007, has proved to be a relatively turbulent introductory period, and the system has passed Phase II (2008–2012) with Phase III started in 2013. The EU ETS is now the world's largest market for CO_2 emission allowances, accounting for approximately 98% of the global transactions for 2011. Thus, CO_2 emission allowances have become a new asset class in financial markets. Understanding the price behaviour and dynamics of CO_2 emission allowances is of major importance in the market for ETS. In recent years, there has been a growing interest in 'carbon finance' and many academic papers have started to examine the CO_2 emission allowance prices. For example, Paolella and Taschini (2008) find that a generalized asymmetric *t* innovation distribution particularly suits the stylized facts of CO_2 emissions data. Seifert *et al*. (2008) study the features of the EU ETS and analyse the resulting CO_2 spot price dynamics within a stochastic equilibrium model. Benz and Truck (2009) examine the limited sample of over-the-counter emission allowance spot prices using a regime-switching model. Chesney and Taschini (2012) develop an endogenous model for describing the emission allowance price dynamics process. The above studies have discussed the various stochastic properties of the CO_2 emission allowance price.

Due to abnormal events, analysing jump risk in asset prices has attracted a great deal of attention. However, most of the existing literature explores the jump risk for pricing options with equity returns (Merton 1976, Jorion 1988, Chan and Maheu 2002, Kou 2002, Maheu and McCurdy 2004, Duan *et al*. 2006, Daal *et al*. 2007). In regard to the emission allowance price, the above studies have not introduced jumps in their modelling with the exception of Daskalakis *et al*. (2009) and Borovkov *et al*. (2011). Borovkov *et al*. (2011) point out some of the situations in which the jumps may appear due to the way in which the EU ETS are currently managed. For instance, the member states negotiate their allowance allocations and the market responds by adapting to the new situation. As a result, a revised decision on the amount of allocated certificates leads to a jump in the market prices of the allowances. In addition, a sudden change in the demand for and/or price of fuel can result in the pollution levels changing dramatically, which in turn impacts the allowance prices. Thus, modelling jumps in emission allowance prices must be considered if the market emission price exhibits jumps. Daskalakis *et al*. (2009) is a pioneering work that considers the jump effects in modelling CO_2 spot price dynamics and has received much attention. They first use a jump-diffusion spot price model and a mean reverting stochastic convenience yield to describe the relationship between spot and futures markets for contracts written within the trial period (Phase I) that expire in the Kyoto commitment period. After that, Borovkov *et al*. (2011) further deal with both continuous time diffusion and jump-diffusion models with emission markets using a mathematical model based on market equilibria derived from the corresponding partial differential equations. Unfortunately, the authors do not use real-world emission market EU ETS data to examine the jump effects. In order to confirm the theoretical models of the jump effects, this study intends to fill the gap by carrying out an in-depth analysis based on the available empirical evidence.

To analyse the jump effects using market data, we employ the price data for the EU ETS in Phase II. Bredin and Muckley (2011) point out that the EU ETS in Phase I is characterized by the uncertainty and volatility associated with the market price of CO_2 emission allowances and Phase II is a maturing market driven by the fundamentals. Montagnoli and de Vries (2010) also explore the efficient markets hypothesis (EMH) in both Phases I and II of the EU ETS and test for the weak form efficiency using the random walk hypothesis and variance ratio tests.‡ The results point to inefficiency in Phase I, but identify efficiency at the beginning of Phase II, showing signs that the EU ETS is maturing. The EU ETS in Phase I is an immature market where the characteristics and the statistical structures of the market data in Phase I may not be appropriate for analysis and forecasting. In addition, these markets for the EU ETS tend to react sharply to world events and more mature markets (in Phase II) may either absorb or adjust to new information. Therefore, this research focuses on Phase II of the EU ETS and we extend Daskalakis *et al*. (2009) to work with CO_2 spot prices in Phase II in order to investigate the jump risk in CO_2 emission allowance prices and value their linked assets,§ which we demonstrate with CO_2 options on futures contracts. Our paper differs from Daskalakis *et al*. (2009) in three respects. First, we perform a jump test to formally examine the jumps in the CO_2 spot price. After proving the existence of the jumps, we take into account the jumps in modelling the CO_2 spot price. Second, we allow the structure of the jump intensity to be both time-varying (referring to dynamic jumps) and constant and examine the most appropriate setting for jumps using actual CO_2 emission allowance price data.¶ Third, in addition to jumps, we also consider the important properties of autocorrelation in the log-return and persistence in volatility to capture the dynamics of CO_2 emission allowance prices. Thus, the jump dynamics are

†State and trends of the carbon market in 2012. World Bank CF Research Report, Washington DC.

‡The EMH states, in its weak form, that a market's prices fully reflect all available information. This implies that investors cannot outperform the market by exploiting past information (e.g. Fama 1970).

§The current CO_2 linked assets are spot, futures and options contracts.

¶For the CO_2 allowance spot price, we focus on the most liquid platforms. BlueNext is the market place dedicated to CO_2 allowances based in Paris and has become the most liquid platform for spot trading; 72% of the volume of spot contracts are traded on BlueNext.

Figure 1. The price of the spot emission allowance in Europe.

built on an ARMA–GARCH model, which is referred to as a jump ARMA–GARCH model in this paper.

To provide more robust findings in modelling the CO_2 emission allowance prices, we compare the fitting accuracy of the proposed jump ARMA–GARCH model with that of various other jump diffusion models such as the Merton, Kou jump and mean reverting jump diffusion models as well as the models proposed in the literature regarding pricing CO_2 emission allowance options on futures, such as the Black–Scholes, mean reverting, ARMA–GARCH and ARMA–EGARCH models. Both in-sample and out-of-sample tests are examined. The empirical investigation shows that the jump ARMA–GARCH model with a dynamic jump specification gives the best model fit and forecasting performance. In addition, the dynamic jump ARMA–GARCH model shows significant persistence in the conditional jump. This indicates that the risk associated with jumps in the CO_2 price return is systematic. Thus, taking into account the dynamic jump effect in pricing CO_2 emission allowance options on futures is important. We further develop a framework for pricing CO_2 emission allowance options on futures contracts to demonstrate the impact of different properties of the CO_2 emission allowance on its price. Our numerical analysis shows that different properties of the CO_2 emission allowance affect the price of a CO_2 emission allowance option on futures contract to different extents. Taking into consideration the jump effect increases the values of options on futures. The value under the proposed dynamic jump ARMA–GARCH model gives rise to the most significant effects compared with other CO_2 emission allowance return models.

The above findings are important to the ETS market. This article contributes to the literature on CO_2 emission allowance options on futures pricing in the following ways. First, we detect the presence of jumps in the CO_2 emission allowance price and evaluate various jump models for modelling the CO_2 emission allowance price. Secondly, to value the CO_2 emission allowance option on futures, we employ the conditional Esscher transform technique to derive the risk neutral valuation framework under the CO_2 emission allowance price return dynamics following a jump ARMA–GARCH model. Finally,

Table 1. Descriptive statistics of the EUA spot price (S) and logarithmic return (R).

	Prices	Returns
Observations	1205	1204
Mean	13.8865	−0.0010
Maximum	28.7300	0.2038
Minimum	5.7200	−0.1080
Std. Dev.	5.1345	0.0278
Skewness	0.7420***	0.2695***
Excess Kurtosis	0.2695**	4.8062***
Jarque–Bera	114.2379***	1173.4474***

Notes: The skewness and excess kurtosis statistics include a test of the null hypotheses where each is zero (the population values if the series is i.i.d. normal). The Jarque–Bera statistic is used to test for normality based on the skewness and kurtosis measures combined. The symbol ** and *** denote significance at the 5% and 1% levels, respectively.

the fair prices of the options on futures based on various CO_2 price return models are calculated and analysed numerically. It is shown that ignoring the autocorrelation, volatility clustering and jump risk with the CO_2 emission allowance price return would underprice the CO_2 emission allowance option on futures.

The remainder of this paper is organized as follows. Section 2 provides an in-depth analysis of the econometrics of the CO_2 emission allowance price. In section 3, we construct a jump ARMA–GARCH model and perform an empirical analysis to investigate the jump effect in the CO_2 emission allowance price. Section 4 shows the pricing of CO_2 emission allowance options on futures under risk neutral valuation. Section 5 presents the conclusions. Finally, most proofs are given in the appendix.

2. Detecting jumps with CO_2 emission allowance prices in the EU ETS

2.1. Statistical analysis of the CO_2 emission allowance price

Various exchanges offer spot trading of CO_2 allowances. In this paper, we focus on CO_2 spot contracts exchanged on the most liquid and largest platform, BlueNext, which is the market place dedicated to CO_2 allowances in Paris. It was created in 2005 and became the most liquid platform for spot trading where the global volume has reached an average of 446,000 tons per day and more than 70% of the overall volumes of spot contracts are traded on BlueNext. To model the CO_2 emission allowance price, we use the CO_2 emission allowance price from BlueNext. The EU ETS envisages several time phases. The first one extends from 2005 to 2007 and can be regarded as a trial period aimed at getting the scheme 'up and running'. The second allocation phase is planned for the period 2008–2012, which coincides with the Kyoto commitment period. From then on, there are consecutive seven-year periods (starting from the 2013–2020 trading period). The trading of the BlueNext spot price started on 24 June 2005. However, from October 2006 until December 2007, CO_2 spot prices were decreasing towards zero due to the banking restrictions implemented between 2007 and 2008 (Daskalakis *et al*. 2009). Due to this erratic non-reliable behaviour of spot prices during Phase I, we have chosen to work only with Phase II CO_2 spot prices in this article.

The data-set consists of daily closing prices for the period 26 February 2008–28 December 2012 (1205 observations). The trend of the emission allowance price during this period is depicted in figure 1 and the descriptive statistics of the prices and returns of the CO_2 emission allowances are presented in table 1.† We can observe that the prices of the CO_2 emission allowances rise rapidly to their maximum level of nearly 29 euros and then fall abruptly to less than 6 euros per EUA.‡ As shown in figure 2, the historical pattern of the daily returns is found to change more than three standard deviations over the sample period. Thus, it also reveals that the jump risk appears in CO_2 emission allowances. The spot market for the CO_2 emission allowances seems to have high volatility. Moreover, the skewness and kurtosis coefficients suggest a leptokurtic distribution with positively skewed returns in the spot market for CO_2 emission allowances. This leptokurtic effect is also confirmed by the Jarque–Bera test.

2.2. *Detecting jumps*

Based on the statistical analysis, we find that jumps might appear with the price of the CO_2 emission allowance. We further examine whether the jumps exist or not. To formally test for the presence of jumps, we use the jump test proposed by Huang and Tauchen (2005).§ Furthermore, Bollerslev *et al.* (2008) use this approach to develop the co-jump test. We first use the jump test by Huang and Tauchen (2005) to detect the jumps in the CO_2 emission allowance price.

The results of the jump test for the CO_2 emission allowance price based on the period 26 February 2008–28 December 2012 are shown in figures 3–5. The data-set consists of daily observations for the CO_2 emission allowance price. To detect the jump effect, we present the results based on the total returns measured at three different intervals for a robustness check. That is, we examine weekly, monthly and two monthly intervals separately. According to figure 3 on a weekly basis, we reject the no jump null hypothesis of 5% critical value for the 10, 21, 151, 181, 194 weekly observations.¶ Figure 4 on monthly interval shows that we reject the no jump null hypothesis for 3, 11, 38, 49 and 53 monthly observations under 10% critical value.‖ In addition, figure 5 concludes that we reject the no jump null hypothesis for 2, 12 and 19 two monthly observations under 10% critical value.†† Thus, the results presented in figures 3 and 4 and figure 5 all reveal that the CO_2 emission allowance returns exhibit jumps. Furthermore, the results indicate that jumps are statistically important components of CO_2 emission allowance movements.

3. The dynamics of CO_2 emission allowance returns with jumps

Based on the empirical investigation, we further consider the jump effect in modelling CO_2 emission allowance return dynamics. Since the properties of the autocorrelation effect and volatility clustering have already been found with CO_2 emission allowance return dynamics (Paolella and Taschini 2008, Benz and Truck 2009),‡‡ we build up the jump specification with the ARMA(s,m)–GARCH(p,q) process for modelling the CO_2 emission allowance returns.

3.1. *The jump ARMA(s, m)–GARCH(p, q) model*

Let $\left(\Omega;\ \Phi;\ P;\ (\Phi_t)_{t=0}^{N}\right)$ be a complete probability space, where P is the data-generating probability measure with specifications for the conditional mean and conditional variance. The spot price of CO_2 emission allowance over trading day t is denoted as S_t and its corresponding return (R_t) is defined as the difference of the natural logarithm of the general price levels ($R_t = \ln(S_t) - \ln(S_{t-1})$) The jump ARMA($s$, m)–GARCH (p, q) model governing the return dynamics can be expressed as follows:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \mu_t + \epsilon_t, \tag{1}$$

†We define the log return of CO_2 emission allowance at time t as shown in section.

‡One allowance exchanged on the EU ETS corresponds to 1 ton of CO_2 released in the atmosphere, and is called the European Union Allowance (EUA).

§The theory behind the jump test can be referred to Barndorff-Nielsen and Shephard (2004), Barndorff-Nielsen *et al*. (2005) and Huang and Tauchen (2005).

¶Under the 10% critical value, we can reject the no jump null hypothesis critical value for 1, 10, 21, 30, 44, 65, 89,118, 135, 151, 181, 194 and 211 weekly observations.

‖We can reject the no jump null hypothesis critical value for 3, 11, 38, 49 and 53 monthly observations when we use around 22 observations in this case.

††We can reject the no jump null hypothesis critical value for 2, 12 and 19 two monthly observations when we use around the 44 observation in this case.

‡‡In this study, we can find that CO_2 emission allowance return has strong autocorrelation effect and volatility clustering, when we use Ljung–Box (LB) Q statistics (West and Cho 1995) and Engle (1982) test. Although not reported here, the parameter estimates of the models are available upon request.

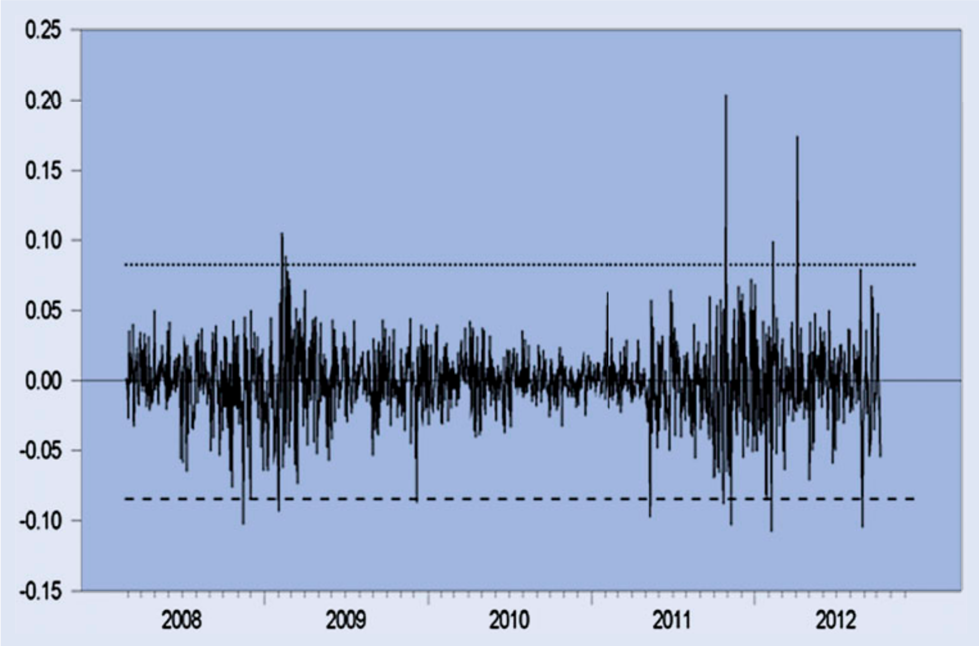

Figure 2. Historical daily returns of CO_2 emission allowances.

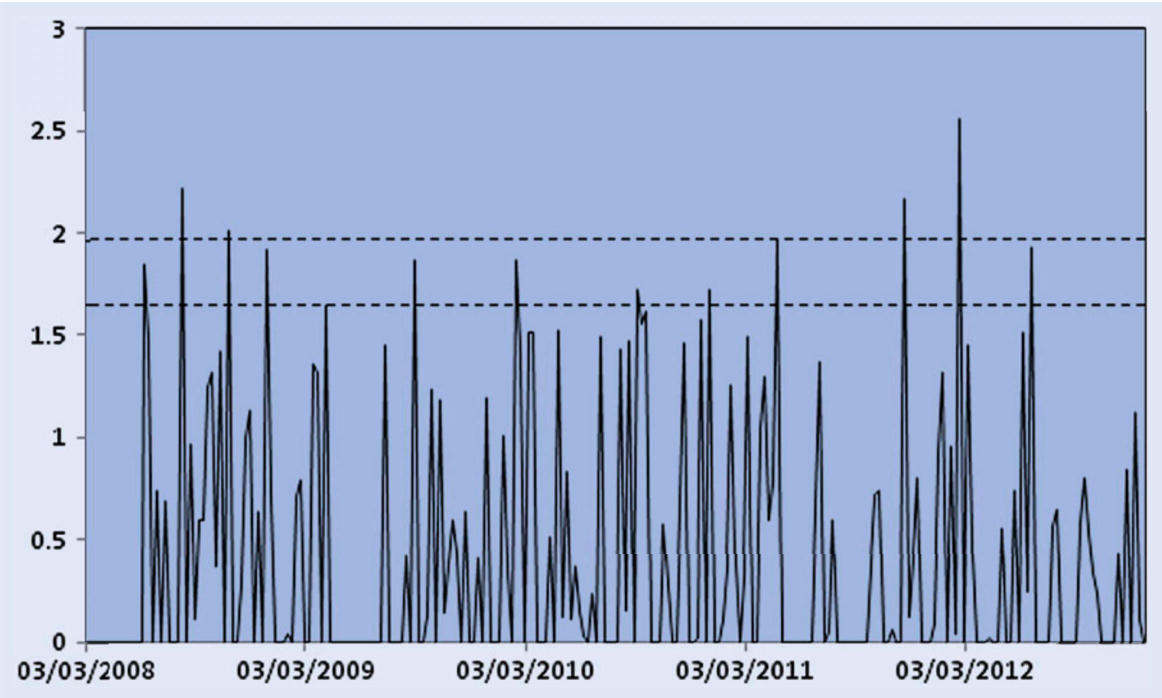

Figure 3. Results of the jump test for the weekly price of the CO_2 emission allowance. The solid line represents observed jumps and the dashed line represents critical value of 10% and 5% levels, respectively.

The mean return follows an autoregressive moving average (ARMA) process as

$$\mu_t = c + \sum_{i=1}^{s} \vartheta_i R_{t-i} + \sum_{j=1}^{m} \zeta_j \epsilon_{t-j}, \tag{2}$$

where s is the order of the autocorrelation terms; m is the order of the moving average terms; ϑ_i is the ith-order autocorrelation coefficient; ζ_j is the jth-order moving average coefficient. ϵ_t is the return innovation observable at time t, which is

$$\epsilon_t = \epsilon_{1,t} + \epsilon_{2,t} \tag{3}$$

Extending from Maheu and McCurdy (2004),† we set two stochastic innovations in which the first component $(\epsilon_{1,t})$ captures smoothly evolving changes in the conditional variance

†Maheu and McCurdy (2004) consider the jump setting under a constant conditional mean of GARCH model. We deal with a jump ARMA–GARCH model and the likelihood function for parameter estimation is reconstructed.

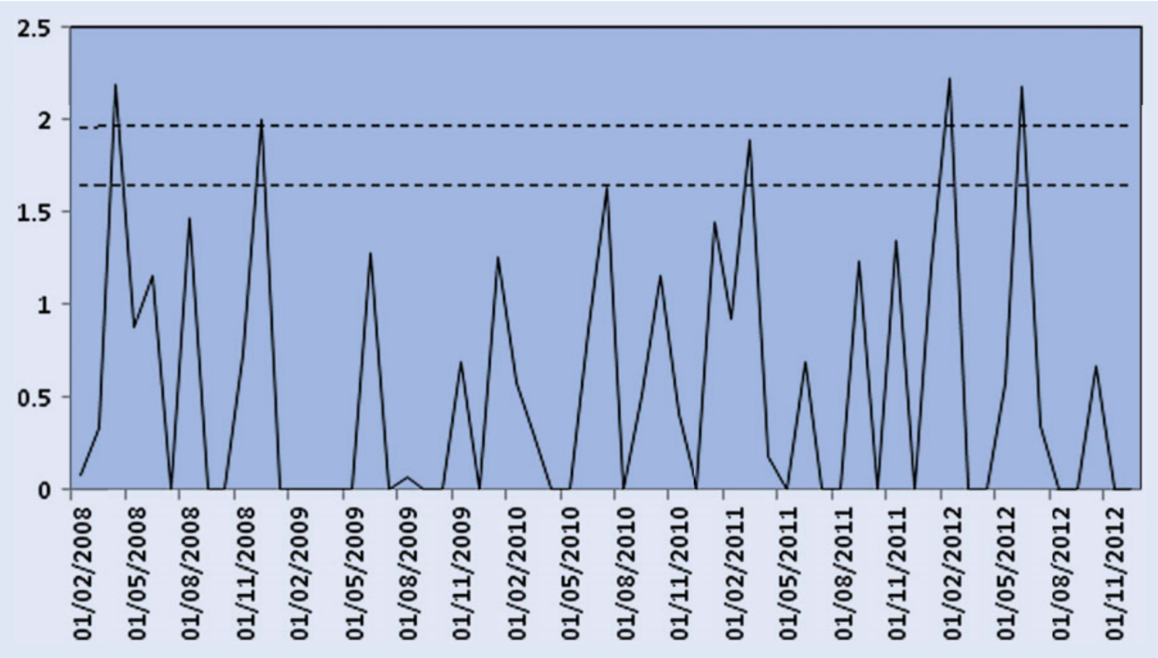

Figure 4. Results of the jump test for the monthly price of the CO_2 emission allowance. The solid line represents observed jumps and the dashed line represents the critical value of 10% and 5% levels, respectively.

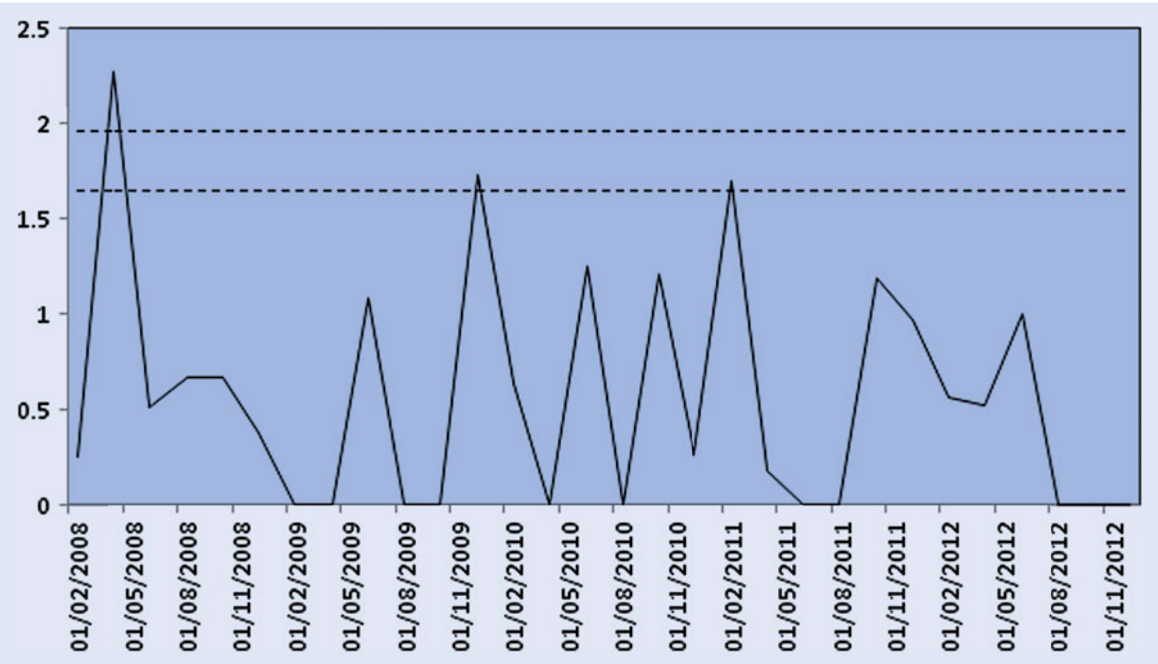

Figure 5. Results of the jump test for the two monthly price of the CO_2 emission allowance. The solid line represents observed jumps and the dashed line represents the critical value of 10% and 5% levels, respectively.

of returns and the second component $(\epsilon_{2,t})$ causes infrequent large moves in returns and are denoted as jumps. $(\epsilon_{1,t})$ is set as a mean-zero innovation $\left(E\left[\epsilon_{1,t}|\Phi_{t-1}\right]=0\right)$ with a normal stochastic forcing process as

$$\epsilon_{1,t}=\sqrt{h_t}z_t, \quad z_t \sim NID\,(0,1)\,, \tag{4}$$

And h_t denote the conditional variance of the innovations, given an information set of Φ_{t-1}

$$h_t=\omega+\sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2+\sum_{j=1}^{p}\beta_j h_{t-j}, \tag{5}$$

where p is the order of the GARCH terms; q is the order of the ARCH term; α_i is the ith-order ARCH coefficient; and β_j is the jth-order GARCH coefficient. $\epsilon_{1,t}$ is contemporaneously independent of $\epsilon_{2,t}$. $\epsilon_{2,t}$ is a jump innovation that is also conditionally mean zero $\left(E\left[\epsilon_{2,t}|\Phi_{t-1}\right]=0\right)$ and we describe $\epsilon_{2,t}$ in next subsection.

3.2. *The setting of jump dynamics*

To capture the jump risk, the second component of innovation is employed to reflect the large change in price and modelled as

$$\epsilon_{2,t} = \sum_{k=1}^{N_t} V_{t,k} - \phi\lambda_t, \quad \text{for } k = 1, 2, \ldots \tag{6}$$

where $V_{t,k}$ denotes the jump size for the kth jump with the jump size following the normal distribution with parameters, (ϕ, θ^2), that is $V_{t,k} \sim NID(\phi, \theta^2)$. Thus, $\sum_{k=1}^{N_t} V_{t,k}$ represents the jump component affecting returns from $t-1$ to t (period t) with the jump frequency of N_t over the interval $(t-1, t)$. We assume the distribution of jumps is to be Poisson with a time-varying conditional intensity parameter (λ_t). The conditional density of N_t is

$$P(N_t = j | \Phi_{t-1}) = \frac{\exp(-\lambda_t)\lambda_t^j}{j!}, \quad j = 0, 1, 2, \ldots, \tag{7}$$

where the parameter λ_t represents the mean and variance for the Poisson random variable, also referred to as the conditional jump intensity. To facilitate the investigation of the jump effect on the returns of CO_2 emission allowances, we extend the work of Chan and Maheu (2002), Maheu and McCurdy (2004), and Daal *et al.* (2007) to specify the time-varying conditional intensity parameter (λ_t) as an ARMA form, which is

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \varsigma\Psi_{t-1} \tag{8}$$

where ρ measures jumps persistence and ς measures the sensitivity of the jump frequency (λ_t) to past shocks (Ψ_{t-1}). Ψ_{t-1} represents the unpredictable component affecting our inference on the conditional mean of the counting process, N_{t-1}, then this suggests corresponding changes. We also investigate the constant jump effect, which represents a special case of equation (8) with the restriction of constant jump intensity $(\lambda_t = \lambda_0)$; this is imposed by setting $\rho = 0$ and $\varsigma = 0$.

The conditional jump intensity in this model is time-varying, with an unconditional value under certain circumstances. In order to derive the unconditional value of λ_t, we must first recognize that Ψ_t is a martingale difference sequence with respect to Φ_{t-1}, because:

$$E[\Psi_t | \Phi_{t-1}] = E[E[N_t | \Phi_t] | \Phi_{t-1}] - \lambda_t = \lambda_t - \lambda_t = 0, \tag{9}$$

thus, $E[\Psi_t] = 0$ and $Cov(\Psi_t, \Psi_{t-i}) = 0, \quad i > 0$.

Another way of interpreting this result is to note that, by definition, Ψ_t is nothing more than the rational forecasting error associated with updating the information set; that is, $\Psi_t = E[N_t | \Phi_t] - E[N_t | \Phi_{t-1}]$. Thus, Maheu and McCurdy (2004) have pointed out several important features of conditional intensity model. First, if the conditional jump intensity is stationary, $(|\rho| < 1)$, then the unconditional jump intensity is equal to

$$E[\lambda_t] = \frac{\lambda_0}{1-\rho} \tag{10}$$

Second, forecasts of λ_{t+i}, and therefore, the conditional variance of $\epsilon_{2,t+i}$, is straightforward to calculate. For example, multi-period forecasts of the expected number of future jumps are

$$E[\lambda_{t+i} | \Phi_{t-1}] = \begin{cases} \lambda_t & i = 0 \\ \lambda_0\left(1 + \rho + \cdots + \rho^{i-1}\right) + \rho^i\lambda_t & i \geq 1 \end{cases} \tag{11}$$

Notice that the conditional jump intensity can be re-expressed as:

$$\lambda_t = \lambda_0 + (\rho - \varsigma)\lambda_{t-1} + \varsigma E[N_{t-1} | \Phi_{t-1}], \tag{12}$$

where $E[N_{t-1} | \Phi_{t-1}]$ is our ex post assessment of the expected number of jumps that occurred from $t-2$ to $t-1$, which can be obtained by

$$E[N_{t-1} | \Phi_{t-1}] = \sum_{j=0}^{\infty} jP(N_{t-1} = j | \Phi_{t-1}) \tag{13}$$

where $P(N_{t-1} = j | \Phi_{t-1})$ is the ex post inference on N_{t-1} given time $t-1$ information and referred as the filter in conducting the ex post assessment. The details of the ARMA jump setting and the process of the ex post assessment can be referred to Chan and Maheu (2002) and Maheu and McCurdy (2004). A sufficient condition for $\lambda_t \geq 0$, for all $t > 1$, is $\lambda_0 > 0$, $\rho \geq \varsigma$, and $\varsigma > 0$. To estimate the conditional jump intensity, startup value of λ_t, Ψ_t, $t = 1$ must be set. We follow Maheu and McCurdy (2004) approach to set the startup value of the jump intensity to the unconditional value in equation (10), and $\Psi_1 = 0$.

3.3. *Parameter estimation*

The parameters of the jump ARMA–GARCH model are estimated using maximum-likelihood estimation (MLE) method. We construct the likelihood function as follows. Let $F_n(\Theta)$ denote the log-likelihood function and Θ is the parameter set governing the jump ARMA–GARCH model, which implies $\Theta = (C, \vartheta_1, \vartheta_2, \omega, \alpha, \beta, \lambda_0, \rho, \varsigma, \phi, \theta)$. We aim to find the optimal parameters (Θ^*) to maximize the log-likelihood function. The log-likelihood function can be expressed as:

$$F_n(\Theta) := \sum_{t=1}^{N} \log f(R_t | \Phi_{t-1}, \Theta) \tag{14}$$

In equation (14), the conditional density of return at time t $f(R_t | \Phi_{t-1}, \Theta)$ for calculating log-likelihood function can be obtained by integrating out the number of jumps as:

$$\begin{aligned} f(R_t | \Phi_{t-1}, \Theta) &= \sum_{j=0}^{\infty} f(R_t | N_t = j, \Phi_{t-1}, \Theta)\, P(N_t = j | \Phi_{t-1}, \Theta) \\ &= \sum_{j=0}^{\infty} \frac{1}{\sqrt{2\pi\left(h_t + j\theta^2\right)}} \exp\left(-\frac{(R_t - \mu_t + \phi\lambda_t - j\phi)^2}{2\left(h_t + j\theta^2\right)}\right) \\ &\quad \times \frac{\exp(-\lambda_t)\lambda_t^j}{j!} \end{aligned} \tag{15}$$

where $P(N_t = j | \Phi_{t-1}, \Theta)$ denotes the conditional density as shown in equation (7). $f(R_t | N_t = j, \Phi_{t-1}, \Theta)$ represents the conditional density of returns given j jumps occurring up to time $t-1$ and follows a Gaussian distribution, that is

$$\begin{aligned} f(R_t | N_t = j, \Phi_{t-1}, \Theta) &= \frac{1}{\sqrt{2\pi\left(h_t + j\theta^2\right)}} \\ &\quad \times \exp\left(-\frac{(R_t - \mu_t + \phi\lambda_t - j\phi)^2}{2\left(h_t + j\theta^2\right)}\right) \end{aligned} \tag{16}$$

And the conditional density of N_t ($P(N_t = j|\Phi_{t-1}, \Theta)$) is shown in equation (7). Since we assume the time-varying conditional intensity parameter (λ_t) follow an ARMA form as shown in equation (8), we need to work out the past shock (Ψ_{t-1}) that affects the inference on the conditional mean of the counting process first. Ψ_{t-1} is defined as

$$\begin{aligned}\Psi_{t-1} &= E[N_{t-1}|\Phi_{t-1}, \Theta] - \lambda_{t-1} \\ &= \sum_{j=0}^{\infty} jP(N_{t-1} = j|\Phi_{t-1}, \Theta) - \lambda_{t-1}\end{aligned} \qquad (17)$$

where $E[N_{t-1}|\Phi_{t-1}, \Theta]$ is given by equation (13). This expression could be estimated if $P(N_{t-1} = j|\Phi_{t-1}, \Theta)$ are known. Following Maheu and McCurdy (2004), the ex post probability of the occurrence of j jumps at time $t-1$ can be inferred using Bayes' formula as follows.

$$\begin{aligned}E[N_{t-1}|\Phi_{t-1}, \Theta] &= \sum_{j=0}^{\infty} jP(N_{t-1} = j|\Phi_{t-1}, \Theta) = \sum_{j=0}^{\infty} j\frac{f(R_{t-1}|N_{t-1} = j, \Phi_{t-2}, \Theta)\, P(N_{t-1} = j|\Phi_{t-2}, \Theta)}{f(R_{t-1}|\Phi_{t-2}, \Theta)} \\ &= \frac{\sum_{j=1}^{\infty} \frac{\exp(-\lambda_{t-1})\lambda_{t-1}^{j}}{j!} \frac{1}{\sqrt{2\pi(h_{t-1}+j\theta^2)}} \times \exp\left(-\frac{(R_{t-1}-\mu_{t-1}+\phi\lambda_{t-1}-j\phi)^2}{2(h_{t-1}+j\theta^2)}\right)}{f(R_{t-1}|\Phi_{t-2}, \Theta)}\end{aligned} \qquad (18)$$

The details of Bayes' inference on calculating $E[N_{t-1}|\Phi_{t-1}, \Theta]$ are presented in appendix 1. Thus, by iterating on (8), (15) and (18), we can construct the log-likelihood function and obtain the maximum-likelihood estimators. In addition, equations (15), (17) and (18) involves an infinite summation depending on the jumps.† We find that truncation of the infinite sum in the likelihood at 10 captures all the tail probabilities and gleans sufficient precision in the estimation procedure.

3.4. *Analysis of goodness-of-fit and forecasting accuracy of the jump ARMA(s, m)–GARCH(p, q) models*

We examine the performance of the Jump ARMA–GARCH Model using time series data for the CO_2 emission allowance price index. In particular, we investigate the jump intensity to be time-varying or constant. Based on the data from BlueNext for the period 26 February 2008–28 December 2012 (1205 observations), we select the ARMA(2,0)–GARCH(1,1) model using the Box and Jenkins (1976) approach.‡

We further investigate the jump dynamics for both the constant and dynamic jump models under the ARMA(2,0)–GARCH(1,1) model. The parameter estimates and the goodness-of-fit for these two jump ARMA(2,0)–GARCH(1,1) models are presented in table 2.

We evaluate the performance of the jump model based on the log-likelihood, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), respectively.§ According to the log-likelihood, AIC and BIC criteria, the dynamic jump ARMA–GARCH model provides better goodness-of-fit results. The persistence parameter (ρ) in the dynamic jump ARMA–GARCH model is significant and is estimated to be 0.8961. This result suggests that a high probability of many (few) jumps today tends to be followed by a high probability of many (few) jumps tomorrow. Recall that Ψ_t is the measurable shock constructed by the econometrician using the ex post filter. In a correctly specified model, Ψ_t should not display any systematic behaviour.

To investigate the importance of the jump effect for modelling CO_2 emission allowance price returns, we also fit the existing models proposed in the literature (Benz and Truck 2009, Daskalakis *et al*. 2009) such as the Black–Scholes model (BSM), and mean reverting (MR), ARMA–GARCH (AMG) and ARMA–EGARCH (AMEG) models. In addition, we compare the performance of the jump ARMA–GARCH model with that of the other jump diffusion models such as the Merton (1976), Kou (2002) and the mean reverting jump diffusion (JD, DEJD and MRJD) models, respectively. Among these models, both jump models in Merton (1976), and the Kou (2002) and mean reverting jump diffusion models allow for the jump effect but do not consider the effects of autocorrelation and volatility persistence. The AMEG allows for asymmetric effects between positive and negative asset returns.¶

Table 3 presents the goodness-of-fit results for different models. The empirical results demonstrate the superiority of the jump ARMA–GARCH model over existing CO_2 emission allowance price return models. In addition, the DJAMG is an improvement in terms of the log-likelihood, AIC and BIC over the above models. Although the diffusion models proposed in JD, DEJD and MRJD consider the jump effect, the models' performance is even worse than that of the time series models that consider the effects of autocorrelation and volatility clustering. Therefore, the CO_2 emission allowance price return model that considers the three properties proves to be important. The results in table 3 somewhat contradict the common finding of mean reverting behaviour observed in commodities and energy (see Schwartz 1997). Due to the addition of mean-reversion appears to decrease the goodness-of-fit, especially in the case of the JD and MRJD. These results are consistent with those of Daskalakis *et al*. (2009).

In order to compare the models forecast performance, we also examine the out-of-sample forecasting accuracy. Recall that the in-sample period is the estimation period from 26

†Equation (15), (17) and (18) involve an infinite sum over the possible number of jumps, N_t, In practice, for our model estimated we found that the conditional Poisson distribution had zero probability in the tail for values of $N_t \geq 10$ and the likelihood and the parameter estimates converge.

‡The details of the time series analysis and the parameter estimates for the model are available upon request.

§AIC $= 2/N \ln(\text{likelihood}) + 2/N \times$ (number of parameters), (Akaike 1973); BIC $= -2/N \ln$ (likelihood) $+$ ((number of parameters) $\times \ln(N))/N$. N is the sample size.

¶The stochastic processes of these models are available upon request.

Table 2. Estimates of the CJAMG and DJAMG Model for the CO_2 emission allowance spot price.

Parameter	CJAMG	DJAMG
C	2.49e−004	8.70e−004
	(6.07e−004)	(6.17e−004)
ϑ_1	0.0349	6.63e−003
	(0.0301)	(0.0303)
ϑ_2	−0.0519*	−0.0693**
	(0.0299)	(0.0297)
ω	5.51e−006**	3.16e−006*
	(2.75e−006)	(1.77e−006)
α	0.0829***	0.0427***
	(0.0165)	(9.39e−003)
β	0.8929***	0.9325***
	(0.0194)	(0.0132)
λ_0	0.0540*	0.0215*
	(0.0322)	(0.0124)
ρ		0.8961***
		(0.0586)
ς		0.1453
		(0.1035)
ϕ	−9.54e−003	−0.0105**
	(8.59e−003)	(4.24e−003)
θ	0.0486***	0.0325***
	(0.0125)	(5.94e−003)
AIC	−4.6373	−4.6508
BIC	−4.5842	−4.5859
Log-likelihood	2786.9906	2795.1158

Notes: The symbols *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. Standard errors are in parentheses.

Table 3. Model selection, 26 February 2008–28 December 2012.

Model	Log-likelihood	AIC	BIC
BSM	2605.1910	−4.3243	−4.3158
MR	404.0948	−0.6657	−0.6530
AMG	2751.1712	−4.5776	−4.5423
AMEG	2758.8114	−4.5790	−4.5498
JD	2681.1440	−4.4454	−4.4243
DEJD	2700.0151	−4.4981	−4.4750
MRJD	2643.1281	−4.3997	−4.3765
CJAMG	2786.9906	−4.6373	−4.5842
DJAMG	2795.1158	−4.6508	−4.5859

Notes: Previous models in the literature, e.g. the Black–Scholes model (BSM), the mean reverting (MR), ARMA–GARCH (AMG), ARMA–EGARCH (AMEG), jump diffusion (JD), double exponential jump diffusion (DEJD), mean reverting jump diffusion (MRJD), constant jump ARMA–GARCH (CJAMG) and dynamic jump ARMA–GARCH (DJAMG) models.

February 2008 to 29 December 2011, while the out-of-sample period covers the forecasting horizon, which is the one-year period from 2 January 2012 to 28 December 2012. The out-of-sample performance is measured by mean squared percentage error (MSPE) and mean absolute percentage error (MAPE).† In addition, we assess the statistical validation of various models according to Diebold and Mariano (1995) test.‡ All of the evaluations for each model are based on forecasting the daily CO_2 emission allowance price.

Table 4 presents the results of the forecast performance. Based on the two criteria of MSPE and MAPE, we find that the DJAMG model has a better forecasting accuracy compared with other models. In addition, the Diebold and Mariano (1995) test also gives the statistical validation of the out-of-sample comparisons that the DJAMG model outperforms other models significantly. Our results are in line with Daskalakis *et al.* (2009)** that the addition of jumps improves performance significantly because the EUA spot prices are subject to large movements that cannot be explained by standard diffusion processes. Moreover, the proposed DJAMG model outperforms the JD model investigated in Daskalakis *et al.* (2009).§ Therefore, our study contributes two further findings. First, the three properties of jump component, autocorrelation in the conditional mean and the time variation in the conditional variance component are critical in modelling daily CO_2 emission allowance price. Taking into account the jumps but ignoring the other two properties of autocorrelation in the conditional mean and the time variation in the conditional variance component still results in a larger forecasting error. Second, the performance measures of MSPE or MAPE show both DJMAG and CJMAG models outperform other models in terms of forecasting errors and the persistence parameter in the dynamics jump ARMA–GARCH model is significant. Although we cannot prove DJAMG model is better than the CJAMG model statistically, our proposed jump model allows us to deal with the CO_2 emission allowance prices when relating to economic and energy policies change.

4. Valuation of CO_2 emission allowance option on futures

4.1. CO_2 *emission allowance option on futures*

To better describe the CO_2 emission allowance option on futures, we define the following notation first:

- F_t: the value of the CO_2 emission allowance futures price at time t.
- S_t: the value of the CO_2 emission allowance spot price at time t.
- r: the risk-free interest rate using the 90-day Euribor rate.
- δ: the current marginal net rate of convenience yield.
- T: the futures contract maturing at time T.

The futures price $F_t(S_t, T)$ at time t for a contract on S_t maturing at time T is given by the standard cost-of-carry relationship:

$$F_t(S_t, T) = S_t e^{(r-\delta)(T-t)} \tag{19}$$

†The measurements which are used can be defined as: $MSPE = 1/N \sum_{t=1}^{N} \left(\left(S_t - \hat{S}_t\right)/S_t\right)^2$, $MAPE = 1/N \sum_{t=1}^{N} \left|\left(S_t - \hat{S}_t\right)/S_t\right|$ where S_t is the actual value of the CO_2 emission allowance price, $\hat{S}_t$ is the forecast value of the daily CO_2 emission allowance price and N is the number of observations.

‡The Diebold and Mariano (1995) test is used to compare the predictive accuracy of loss functions of different models. The null hypothesis of the Diebold and Mariano (1995) test is that the two models have the same MSPE or MAPE.

§Daskalakis *et al.* (2009) ignore the two properties of autocorrelation in the conditional mean and the time variation in the conditional variance for modelling CO_2 emission allowance price.

Table 4. Forecasting error for different models, 2 January 2012–28 December 2012.

	MSPE	MAPE
Panel A: Model		
BSM	0.02477	0.13579
MR	0.03554	0.16789
AMG	0.00985	0.08072
AMEG	0.00941	0.07815
JD	0.01002	0.08290
DEJD	0.00982	0.08178
MRJD	0.02092	0.11138
CJAMG	0.00831	0.07714
DJAMG	0.00819	0.07613
Panel B: Model comparison under D&M Statistics		
BSM vs. DJAMG	4.583***	4.703***
MR vs. DJAMG	5.113***	5.821***
AMG vs. DJAMG	1.842*	1.886*
AMEG vs. DJAMG	1.689*	1.736*
JD vs. DJAMG	2.121**	2.145**
DEJD vs. DJAMG	2.030**	2.041**
MRJD vs. DJAMG	2.728***	3.204***
CJAMG vs. DJAMG	1.285	1.390

Notes: Previous models in the literature, e.g. the Black–Scholes model (BSM), the mean reverting (MR), ARMA–GARCH (AMG), ARMA–EGARCH (AMEG), jump diffusion (JD), double exponential jump diffusion (DEJD), mean reverting jump diffusion (MRJD), constant jump ARMA–GARCH (CJAMG) and dynamic jump ARMA–GARCH (DJAMG) models. The null hypothesis is that the two forecasts have the same MSPE or MAPE in the panel B. D&M refer to the Diebold and Mariano (1995) test and the positive values indicate superiority of our dynamic jump ARMA–GARCH model. ***, ** and * denote significant relative to the asymptotic null distribution at the 1%, 5% and 10%, respectively.

Furthermore, the CO_2 emission allowance option on futures can be expressed as:

$$C\left(t, T_1, T\right) = e^{-r_1(T_1-t)} E^Q\left[\left(F_t\left(S_t, T\right) - K\right)^+\right]. \quad (20)$$

where K is the option strike price. The value $C\left(t, T_1, T\right)$ at time t of a call option expiring at time T_1 (with $T \geq T_1$) and r_1 is the 30-day Euribor rate.

4.2. *Risk neutral valuation framework under a jump ARMA–GARCH model*

The no arbitrage value of the CO_2 emission allowance option on futures depends on the dynamics of the CO_2 emission allowance price return dynamics. The empirical investigation has found that the jump effect and the effects of autocorrelation and volatility clustering cannot be ignored in modelling CO_2 emission allowance price returns. As for the empirical findings in this research, we propose a jump ARMA–GARCH model. However, taking into account these realistic properties makes the valuation problem for the option on the futures contract in equation (20) more complicated. To achieve our goal, we employ the conditional Esscher transform technique developed by Bühlmann *et al.* (1996) to derive the risk neutral valuation framework under the CO_2 emission allowance price returns following a jump ARMA–GARCH model.

The Esscher transform was introduced by Esscher (1932) and has been widely applied to pricing financial and insurance securities in an incomplete market. Gerber and Shiu (1994) create an equivalent martingale measure by the Esscher transform which is justified by maximizing the expected power utility of an economic agent. Bühlmann *et al.* (1996) generalize the Esscher transform to stochastic processes and introduce the concept of the conditional Esscher transform. Siu *et al.* (2004) employ the conditional Esscher transform to price derivatives, assuming that the underlying asset returns follow GARCH processes. In this paper, we utilize the technique of the conditional Esscher transform to price the CO_2 emission allowance price return under a jump ARMA(s,m)–GARCH(p,q) model.

The exponential titling of X with respect to the reference variable R is defined as

$$f_X^*(x) = f_X(x)\frac{E\left[\exp(aR)\,|X = x\right]}{E\left[\exp(aR)\right]}, \quad (21)$$

where f_X and f_X^* represent the probability density function of X before and after the exponential tilting, respectively. Chen *et al.* (2010) have pointed out if choosing the reference R to be the risk X itself, the exponential tilting is reduced to the Esscher transform:

$$\begin{aligned} f_X^*(x) &= f_X(x)\frac{E\left[\exp(aR)\,|X = x\right]}{E\left[\exp(aR)\right]} \\ &= f_X(x)\frac{\exp(ax)}{E\left[\exp(aX)\right]} \end{aligned} \quad (22)$$

Bühlmann *et al.* (1996) generalize the Esscher transform to a stochastic process and introduce to concept of the conditional Esscher transform. In terms of probability density functions, the conditional Esscher transform is defined as

$$f_{X_t}^*(x|\Phi_{t-1}) = f_{X_t}(x|\Phi_{t-1})\frac{\exp(a_t x)}{E\left[\exp(a_t X_t)\,|\Phi_{t-1}\right]} \quad (23)$$

Thus, define a sequence $\{\Lambda_t\}_{t=0}^N$ with $\Lambda_0 = 1$, and for $t \geq 1$,

$$\Lambda_t = \prod_{k=1}^{n}\frac{\exp(a_k R_k)}{E\left[\exp(a_k R_k)\,|\Phi_{k-1}\right]} \quad (24)$$

For some constant $a_1, a_2, \ldots, a_t$. Bühlmann *et al.* (1996) prove that $\{\Lambda_t\}_{t=0}^N$ is a martingale. Finally, we can adapt this pricing framework when the underlying asset returns under a risk neutral measure, Q, $R_t|\Phi_{t-1} \sim \left(r - \frac{1}{2}h_t^*, h_t^*\right)$ and the CO_2 emission allowance return becomes

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{1}{2}h_t^* + \epsilon_t^Q, \quad (25)$$

where $\epsilon_t^Q = \epsilon_t - a_t h_t^*$ follows a normal distribution with mean 0 and variance h_t^* under measure Q. In other words, the CO_2 emission allowance price return dynamics under measure Q are similar in form to those under measure P, albeit with shifted parameters and with drift $r - \frac{1}{2}h_t^*$. See appendix 2 for the derivation of the conditional Esscher parameters. Using the conditional Esscher transform, we can go on to obtain the CO_2 emission allowance price returns under measure Q and then calculate the price of each option on futures contracts for the CO_2 emission allowance returns at time t under the equivalent martingale measure Q can be evaluated using Monte Carlo simulations.

4.3. *Numerical analysis for the* CO_2 *emission allowance option on futures*

We examine the value of the CO_2 emission allowance option on futures contracts in this section. To address the importance of

Table 5. Base parameter values of the CO_2 emission allowance option on futures contracts.

Parameter	Notation	Initial value
CO_2 emission allowance spot price	S_0	15.07
Option strike price	K	15
Call option expiring at time	T_1	30
Futures expiring at time	T	90
30-day Euribor rate	r_1	0.818%
90-day Euribor rate	r	1.029%
Convenience yield	δ	0

Table 6. CO_2 emission allowance option on futures for various models.

Model	Option on futures
BSM	0.9743
MR	0.9118
AMG	1.0794
AMEG	1.0887
JD	1.0475
DEJD	1.0582
MRJD	1.0314
CJAMG	1.1081
DJAMG	1.1482

Notes: Previous models in the literature, e.g. the Black–Scholes model (BSM), the mean reverting (MR), ARMA–GARCH (AMG), ARMA–EGARCH (AMEG), jump diffusion (JD), double exponential jump diffusion (DEJD), mean reverting jump diffusion (MRJD), constant jump ARMA–GARCH (CJAMG) and dynamic jump ARMA–GARCH (DJAMG) models.

Table 7. Pricing error under the different CO_2 emission allowance options on futures for various models.

Model	Pricing error (%)
BSM	15.145
MR	20.588
AMG	5.992
AMEG	5.182
JD	8.770
DEJD	7.838
MRJD	10.172
CJAMG	3.492
DJAMG	–

Notes: Previous models in the literature include the Black–Scholes model (BSM), the mean reverting (MR), ARMA–GARCH (AMG), ARMA–EGARCH (AMEG), jump diffusion (JD), double exponential jump diffusion (DEJD), mean reverting jump diffusion (MRJD) and constant jump ARMA–GARCH (CJAMG) models.

different properties of the CO_2 emission allowance in terms of pricing its option on futures, in addition to the proposed jump ARMA–GARCH model, we calculate the values of the option on futures contracts based on various models such as the BSM, MR, AMG, AMEG, JD, DEJD, MRJD and CJAMG models. The pricing formula of the CO_2 emission allowance option on futures contracts using these different models is described in appendix 3. We list the assumptions for the parameter values of the CO_2 emission allowance options on futures contracts in table 5. The corresponding values and pricing errors for the CO_2 emission allowance option on futures values for different models are displayed in tables 6 and 7, respectively.

We can discover the impact of jump risk and model risk on the price of the CO_2 emission allowance option on the futures contract according to the results in tables 6 and 7. Table 6 shows that the value of the CO_2 emission allowance option on futures ranges from 0.9118 to 1.1482. In table 7, the pricing error ranges from 20.588% for the option on futures using the MR model to approximately 3.5% when using the CJAMG model, based on the option on futures prices valued under the DJAMG model. We find that the MR model gives the lowest value of 0.9118 for the CO_2 emission allowance option on the futures contract and the pricing errors are far larger than for those in the DJAMG model. This indicates that the CO_2 emission allowance price does not have a mean reverting process, and thus there is serious model risk. The empirical results are consistent with the findings in Daskalakis *et al.* (2009).

In addition, the Black–Scholes model gives a lower value (0.9743) and larger pricing error (15.145%) for the CO_2 emission allowance option on futures compared with the other CO_2 emission allowance return models investigated in this research. It indicates that we may significantly underprice the value of the CO_2 emission allowance option on futures if we ignore the important properties of autocorrelation, volatility clustering and the jump effect for the CO_2 emission allowance price dynamics.

Regarding the jump effect, in table 7, we observe the pricing errors for using the AMG, CJAMG and DJAMG models separately. The empirical analysis in section 2 has shown that jump risk appears in the CO_2 emission allowance return data and we cannot ignore the jumps in modelling the CO_2 emission allowance price dynamics and therefore price its option on futures. Taking into account the jump effect increases the value of the option on futures and reduces the pricing error by approximately 6%. On the other hand, ignoring the dynamic jump function gives rise to a 3.5% pricing error. However, under different jump components, there is not much difference between JD, DEJD and MRJD. Furthermore, when the effects of autocorrelation and volatility clustering are considered, the pricing error decreases from 8% to 10%. Finally, the proposed dynamic jump ARMA–GARCH model gives the most significant effect compared with other CO_2 emission allowance return models.

5. Conclusion

The overwhelming majority of scientists agree that our globe is undergoing major climate change. They also agree that the level of CO_2 in the atmosphere is rising significantly. Managing the risks caused by GHGs has become more and more important globally. The financial markets in many countries have developed trading systems for a CO_2 emission allowance certificate, which is one of the most powerful mechanisms available to reduce national GHG emissions. In addition, a variety of specialized financial instruments linked to the CO_2 emission allowance, such as a CO_2 emission allowance option on futures, have been developed in several national and regional emission markets. It thus becomes essential to understand the properties of the CO_2 emission allowance prices in the carbon market.

Developing a proper model that can capture the properties of the CO_2 emission allowance prices is the first step for pricing the financial instruments linked to CO_2 emission allowances. Although many studies have not yet provided any conclusive results with regard to the most appropriate model for capturing the dynamics of the CO_2 emission allowance in pricing its futures contracts, they have made great efforts to discover the properties of the CO_2 emission allowance. However, the literature has not satisfactorily addressed the important property of jumps regarding the emission allowance price. Based on the present practice of the EU ETS, there are some driving forces that could cause jumps (Borovkov *et al*. 2011). Therefore, this study performs an empirical analysis to investigate the jump effects with a CO_2 emission allowance return series. By using the Phase II data based on CO_2 emission allowance spot prices in the EU ETS, we find that the dynamics of the CO_2 emission allowance price has the following statistical property: a strong positive autocorrelation effect among the log-returns, where the volatility of the log-returns varies with time and a jump effect that appears in the log-return series.

After validating these effects by comparing the various models with the in-sample and out-of-sample fitting performance, we propose a jump ARMA–GARCH model to capture the dynamics of CO_2 emission allowance price and further develop a framework for pricing CO_2 emission allowance options on futures contracts. Our numerical analysis shows that different properties of the CO_2 emission allowance affect the price of CO_2 emission allowance option on futures contracts to different extents. Although taking into account the jump effect increases the value of options on futures, there is not much difference under different jump components. The proposed dynamic jump ARMA–GARCH model gives rise to the most significant effect compared with other CO_2 emission allowance return models. Thus, ignoring the jump risk with the CO_2 emission allowance price return would significantly underprice CO_2 emission allowance options on futures.

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Appendix 1.

We utilize the ex post assessment to calculate $E\left[N_{t-1}|\Phi_{t-1},\Theta\right]$. As shown in equation (18), $E\left[N_{t-1}|\Phi_{t-1},\Theta\right] = \sum_{j=0}^{\infty} jP\left(N_{t-1}=j|\Phi_{t-1},\Theta\right)$. The $P\left(N_{t-1}=j|\Phi_{t-1},\Theta\right)$ is the ex post inference on N_{t-1} given the time $t-1$ information and needs to be estimated first. To calculate $P\left(N_{t-1}=j|\Phi_{t-1},\Theta\right)$, we use the Bayes rules. Thus, we can infer the ex post probability of the occurrence of j jumps at time $t-1$ as the filter. The construction the filter shows as follows. Starting with the information available at time $t-1$ $\left(\Phi_{t-1}\right)$, it contains the information on R_{t-1} and previous returns captured in Φ_{t-2}. Thus, we can obtain the ex post probability of the occurrence of j jumps at time $t-1$ to evaluate equation (17).

$$
\begin{aligned}
P\left(N_{t-1}=j|\Phi_{t-1},\Theta\right) &= P\left(N_{t-1}=j|R_{t-1},\Phi_{t-2},\Theta\right) = \frac{P\left(N_{t-1}=j,R_{t-1},\Phi_{t-2},\Theta\right)}{P\left(R_{t-1},\Phi_{t-2},\Theta\right)} \\
&= \frac{f\left(R_{t-1}|N_{t-1}=j,\Phi_{t-2},\Theta\right)P\left(N_{t-1}=j|\Phi_{t-2},\Theta\right)P\left(\Phi_{t-2},\Theta\right)}{f\left(R_{t-1}|\Phi_{t-2},\Theta\right)P\left(\Phi_{t-2},\Theta\right)} \\
&= \frac{f\left(R_{t-1}|N_{t-1}=j,\Phi_{t-2},\Theta\right)P\left(N_{t-1}=j|\Phi_{t-2},\Theta\right)}{f\left(R_{t-1}|\Phi_{t-2},\Theta\right)} \qquad \text{(A1)}
\end{aligned}
$$

Thus, the Equation A1 is consistent with the equation (24) of Maheu and McCurdy (2004).

Appendix 2.

To obtain the CO_2 emission allowance price dynamic under a risk neutral measure, we employ an equivalent martingale measure using the conditional Esscher transform developed by Bühlmann *et al.* (1996). Due to the discount CO_2 emission allowance price under the Q measure is a martingale, we have:

$$S_{t-1} = E^Q\left(\frac{B_{t-1}}{B_t}S_t|\Phi_{t-1}\right) = E^Q\left(\exp\left(-r\right)S_t|\Phi_{t-1}\right) \qquad \text{(B2)}$$

We assume that the interest rate is fixed at r. Consequently, $B_t = B_{t-1}e^r$. We obtain:

$$
\begin{aligned}
S_{t-1} &= e^{-r}E^Q\left(S_t|\Phi_{t-1}\right) = e^{-r}E^P\left(\frac{\Lambda_t}{\Lambda_{t-1}}S_t|\Phi_{t-1}\right) \\
&= S_{t-1}e^{-r}\frac{E^P\left(\exp\left(\left(a_t+\iota\right)R_t\right)|\Phi_{t-1}\right)}{E^P\left(\exp\left(\left(a_t\right)R_t\right)|\Phi_{t-1}\right)} \qquad \text{(B3)}
\end{aligned}
$$

Or equivalently,

$$e^r = \frac{E^P\left(\exp\left(\left(a_t+\iota\right)R_t\right)|\Phi_{t-1}\right)}{E^P\left(\exp\left(\left(a_t\right)R_t\right)|\Phi_{t-1}\right)} \qquad \text{(B4)}$$

To ensure the risk neutral Q to be an equivalent martingale measure, we need to have

$$E^Q\left[\exp\left(R_t\right)|\Phi_{t-1}\right] = e^r \qquad \text{(B5)}$$

Because, Maheu and McCurdy (2004) pointed out the conditional moments of return are

$$
\begin{aligned}
E\left[R_t|\Phi_{t-1}\right] &= \mu_t \\
Var\left[R_t|\Phi_{t-1}\right] &= h_t + \left(\phi^2+\theta^2\right)\lambda_t = h_t^* \qquad \text{(B6)}
\end{aligned}
$$

Thus, R_t is normally distributed with mean μ_t and variance h_t^*, given the information Φ_{t-1}, we obtain

$$
\begin{aligned}
E^Q\left[\exp\left(\iota R_t\right)|\Phi_{t-1}\right] &= \frac{\exp\left(\left(a_t+\iota\right)\mu_t + \frac{1}{2}\left(a_t+\iota\right)^2 h_t^*\right)}{\exp\left(a_t\mu_t + \frac{1}{2}a_t^2 h_t^*\right)} \\
&= \exp\left(\left(\mu_t + a_t h_t^*\right)\iota + \frac{1}{2}h_t^*\iota^2\right) \qquad \text{(B7)}
\end{aligned}
$$

Therefore,

$$E^Q\left[\exp\left(R_t\right)|\Phi_{t-1}\right] = \exp\left(\mu_t + a_t h_t^* + \frac{1}{2}h_t^*\right) \qquad \text{(B8)}$$

Through the equations (B5) and (B8), we have

$$\mu_t = r - a_t h_t^* - \frac{1}{2}h_t^* \qquad \text{(B9)}$$

Similarly, the characteristic function of ϵ_t under martingale measure Q is of the form:

$$
\begin{aligned}
&E^Q\left(\exp\left(i\varpi\epsilon_t\right)|\Phi_{t-1}\right) \\
&= E^P\left(\frac{\Lambda_t}{\Lambda_{t-1}}e^{i\varpi\epsilon_t}|\Phi_{t-1}\right) = \frac{E^P\left(e^{a_t R_t}e^{i\varpi\epsilon_t}|\Phi_{t-1}\right)}{E^P\left(\exp\left(\left(a_t\right)R_t\right)|\Phi_{t-1}\right)} \\
&= \frac{\exp\left(a_t\mu_t\right)E^P\left(e^{\left(a_t+i\varpi\right)\epsilon_t}|\Phi_{t-1}\right)}{\exp\left(a_t\mu_t + \frac{1}{2}a_t^2 h_t^*\right)} = \frac{\exp\left(\frac{1}{2}\left(a_t+i\varpi\right)^2 h_t^*\right)}{\exp\left(\frac{1}{2}a_t^2 h_t^*\right)} \\
&= \exp\left(i\varpi a_t h_t^* - \frac{1}{2}\varpi^2 h_t^*\right) \qquad \text{(B10)}
\end{aligned}
$$

Consequently, ϵ_t under the measure Q become normally distributed, with mean $a_t h_t^*$ and variance h_t^*, given the information Φ_{t-1}. That is, given the information Φ_{t-1}, $\epsilon_t^Q = \epsilon_t - a_t h_t^*$ follow normally mean 0 and variance h_t^* under measure Q. Finally, the equation (1) can be rewritten as:

$$
\begin{aligned}
R_t &= \ln\left(\frac{S_t}{S_{t-1}}\right) = \mu_t + \epsilon_t = r - a_t h_t^* - \frac{1}{2}h_t^* + \epsilon_t^Q + a_t h_t^* \\
&= r - \frac{1}{2}h_t^* + \epsilon_t^Q \qquad \text{(B11)}
\end{aligned}
$$

Appendix 3.

In this appendix, based on the empirical investigation, the properties of the autocorrelation effect, volatility clustering and jump effect have been investigated with CO_2 emission allowance price return dynamics. With these features, we can propose a jump ARMA–GARCH model for pricing CO_2 emission allowance options on futures, and the expectation in equation (20) can be evaluated using Monte Carlo simulations based on the risk neutral return process. For comparison purposes, we also calculate the CO_2 emission allowance options on futures based on the different CO_2 emission allowance price return models mentioned in section 3. When the underlying CO_2 emission allowance price is assumed to follow a geometric Brownian motion, the Black–Scholes formula can be applied to valuing the CO_2 emission allowance option on futures. The exact formula is

$$
\begin{aligned}
C\left(t,T_1,T\right)_{\mathrm{BSM}} = F\left(S_t,T\right)e^{-r_1\left(T_1-t\right)}N\left(d_1\right) \\
- Ke^{-r_1\left(T_1-t\right)}N\left(d_2\right), \qquad \text{(C12)}
\end{aligned}
$$

where

$$d_1 = \frac{\ln\left(\frac{F(S_t,T)}{K}\right) + \frac{\sigma^2}{2}(T_1 - t)}{\sigma\sqrt{T_1 - t}},$$
$$d_2 = d_1 - \sigma\sqrt{T_1 - t}$$

To investigate the jump effect, we also compare the price of the CO_2 emission allowance option on futures based on the well-known jump diffusion models by Merton (1976) and Kou (2002). Merton (1976) implemented the jump diffusion model using a standard time homogeneous Poisson process with parameter λ dictating the arrival of jumps, and the jump sizes follow a Gaussian distribution $\left(N\left(\phi, \theta^2\right)\right)$. m denotes the average jump size if the Poisson event occurs. When the CO_2 emission allowance price return follows the Merton jump diffusion model, the corresponding pricing formula for the option on futures can be expressed as:

$$C(t, T_1, T)_{JD} = P(t, T_1) \sum_{j=0}^{\infty} \left[\frac{e^{\left(-\lambda^Q (T_1 - t)\right)} \lambda^Q (T_1 - t)^j}{j!}\right] \times \left[F(S_t, T) N(d_1) - K N(d_2)\right], \quad \text{(C13)}$$

where $d_1 = \frac{\ln\left(\frac{F(S_t,T)}{K}\right) + \frac{\sigma^2}{2}(T_1 - t)}{\sigma\sqrt{T_1 - t}}$, $d_2 = d_1 - \sigma\sqrt{T_1 - t}$, λ^Q is the risk-adjusted parameter under the risk neutral measure and $P(t, T_1)$ is the price at time t of a zero coupon bond with maturity at time T_1.

Differing from Merton (1976), Kou (2002) assumes that the jump sizes are asymmetric double exponentially distributed to capture the leptokurtic feature. The density function of the jump size is assumed to be of the form:

$$f_Y(y) = p \cdot \eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q \cdot \eta_2 e^{\eta_2 y} 1_{\{y < 0\}}, \quad \eta_1 > 1, \eta_2 > 0$$

When the CO_2 emission allowance price dynamic follows Kou (2002)'s double exponential jump model, the pricing formula for options on futures can be expressed as:

$$\begin{aligned} &C(t, T_1, T)_{DEJD} \\ &= e^{-r_1(T_1 - t)}\{F(S_t, T)\Gamma(r_1 + \frac{\sigma^2}{2} - \lambda\varpi, \sigma, \hat{\lambda}, \hat{p}, \hat{\eta}_1, \hat{\eta}_2; \\ &\quad \log(K/F(S_t, T)) + r_1 T_1, T_1) \\ &- K\Gamma(r_1 - \frac{\sigma^2}{2} - \lambda\varpi, \sigma, \lambda, p, \eta_1, \eta_2; \log(K/F(S_t, T)) \\ &\quad + r_1 T_1, T_1)\}, \end{aligned} \quad \text{(C14)}$$

where p and q represent the probabilities of upward and downward jumps and $p, q \geq 0$, $p + q = 1$, $\hat{p} = \frac{p}{1+\varpi} \cdot \frac{\eta_1}{\eta_1 - 1}$, $\hat{\eta}_1 = \eta_1 - 1$, $\hat{\eta}_2 = \eta_2 + 1$, $\hat{\lambda} = \lambda(\varpi + 1)$, and $\varpi = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$. Equation (C14) resembles the Black–Scholes formula for options on futures in equation (C12) with $\Gamma(\bullet)$ taking the place of $N(\bullet)$. The details of the derivation of the $\Gamma(\bullet)$ function can be found in Kou (2002).