A Statistical Basis For Fuzzy Engineering Economics

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Abstract

This paper introduces a systematic way to analyze fuzzy data in both engineering fields and economics, with emphasis on fuzzy engineering economics. The approach is statistical in nature, in which fuzzy information and data are treated as bona fide random elements within probability theory. This provides, not only a coexistence for randomness and fuzziness in the complex task of handling all kinds of uncertainty in real-world problems, but also a statistical theory supporting empirical analyses in applications. This can also viewed as a complement to two usual approaches in the literature, namely, either using only fuzzy methods, or using some forms of fuzzifying statistics. We will give illustrating and motivating important examples, in the area of regression (for prediction purposes) with seemingly unobservable variables, in which, fuzzy rule-based technology provides nonlinear models for estimating unobservables (from determinants/causal variables), followed by statistics with fuzzy data in linear regression models. The main contribution of this paper is the rigorous formulation of statistics with fuzzy data using continuous lattice structure of upper semi continuous membership functions (random fuzzy closed sets) which can be used in a variety of useful applied situations where fuzziness and randomness coexist.

Keywords: Coarsening schemes, econometrics, engineering economics, fuzzy control, fuzzy logics, fuzzy rule bases, fuzzy sets, random sets, random fuzzy sets.

1. Introduction

This paper is about fuzzy technology for econometrics in the sense that it can be combined rigorously with an appropriate statistical theory to enlarge the domain of applicability of econometrics in general and of engineering economics in particular. As such, the paper addresses both engineers and econometricians.

More specifically, in one direction, we call the attention of econometricians to relevant fuzzy materials which should be used in econometrics, and in another direction, we emphasize, to engineering economists, the need to treat fuzzy data properly as random elements in a bona fide statistical setting

There exists a two-way relationship between engineering/physical science and economics/social science. Like physical sysyems, economic systems are uncertain and dynamical, and as such, econometrics has borrowed almost all tools from physical sciences, such as Kalmam filter and quantum mechanics (path integral and Hamiltonian for options and interest rates, e.g., [1]). In the reverse direction, e.g., engineering economics (previously known as "Engineering Economy", [2]) used economic theories in its projects. This is so, since engineering is an important part of the manufacturing sector of the economy (more specifically, engineering economics concerns the application of economic principles to engineering problems).

While fuzzy technology is successful in engineering, such as control (see e.g. [3]; [4]), and pattern recognition based on clustering (especially, [5]), it did not enter econometric tool box from the "front door", so to speak. Perhaps, a main reason is that econometrics is dominated by statistics, and fuzzy technology in engineering seems to offer an alternative to statistics rather than a combined methodology. Clearly, taking fuzzy data into account is beneficial for economic analysis (which, in turn, is beneficial to engineering economics), so that a combined methodology of probability and fuzziness is desirable. As far as economic analysis is concerned, empirical evidence can only provide indicative conclusions, but not necessarily conclusive conclusions. In this spirit, a statistical theory of inference with fuzzy data seems to be what is needed. There exist similar attempts in the literature, exemplified by, e.g., [6], but, in which, it is "fuzzy statistics" rather than "statistics of fuzzy data" (remember: "fuzzy logic" is not a logic which is fuzzy, but it is a logic of fuzzy concepts in natural language). In the Fuzzy Engineering Economics community (see, e.g., [7]), emphasis seems to be on fuzzy technology alone. The paper of [8] on a probabilistic approach to fuzzy engi-

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neering economics only focused on using probability for manipulating fuzzy (complex) numbers rather than as a theory of statistics for fuzzy data. Elsewhere, research focused on the "fuzzy approach to statistical analysis", as exemplified by a special issue of the Journal Computational Statistics and Data Analysis (2006), although in it, [9] were the only ones emphasizing the need to look at a "statistical approach to fuzzy data". Specifically, the guest editors of this special issue, [10] wrote "The contribution of Nguyen and Wu focuses on Fuzzy Statistics seen as "Statistics with fuzzy data". In this specific context, statistical data may be point-valued, set-valued or fuzzy-set valued observations. Random sets (viewed as elements of separable metric spaces) are proposed at the appropriate mathematical model for set-valued observations. Likewise, random fuzzy sets are suitable for analyzing random fuzzy data. Some issues related to these modelization are examined. The proposals stemming from this investigation may help in strengthening the bases of a sound methodology for the analysis of imprecise data. In particular, the notion of "coarsening" is thoroughly discussed along with the process of generation of a membership function from available information.

It is clear that randomness and fuzziness are two distinct types of uncertainty which often exist simultaneously in real-world problems. As such, and as Zadeh has emphasized, they should be treated together, rather than separately. The question is how?

But, when do we face situations where both randomness and fuzziness appear? Typically, these are situations in which the outcomes of random elements (i.e., general forms of random quantities whose outcomes could be points, vectors, curves, sets or fuzzy sets) of interest are linguistic (imprecise to the extent that only fuzzy set modeling is appropriate).

In view of all the above, we will present, in this paper, the foundations for a complete theory of statistics for supporting fuzzy data analyses for engineering economics in particular, and for econometrics and engineering in general. Our main contributions are twofold: on one hand, we exhibit various situations where fuzziness appears naturally and is subjected to statistical analysis (e.g. regression with unobservable covariates), and on the other hand, we provide the foundations for statistical analysis of fuzzy data using continuous lattice structure of fuzzy closed sets.

The paper is organized as follows. In section 2, we recall the essentials of fuzzy sets and logics for econometricians. In section 3, we illustrate how these concepts and techniques from the fuzzy theory could help statistical analyses. Section 4 is devoted somewhat to the reverse direction, namely addressing statistical aspects which could help engineering economists to validate their empirical results. Section 5 is devoted to the main contribution of the paper, namely the presentation of a statistical theory of fuzzy data. Section 6 summarizes our discussions and our main contributions.

2. Essentials of fuzzy technology

While fuzzy technology is well-known in engineering circle, it is not so for econometric community. Here is a tutorial on the basis of fuzzy technology that we will employ in this paper. According to [11], fuzzy concepts in a natural language, such as "low" (temperature), "efficiency", "happiness", can be modeled mathematically for information processing purposes. As we will see, fuzzy concepts are usually values of qualitative (linguistic) variables of interest in decision-making (e.g., not disabled, partially disabled, fully disabled are "values"/outcomes of the linguistic variable "disability status"). More importantly, fuzzy concepts are used as coarsening schemes in human intelligent behavior (e.g., in intelligent control).

Using a familiar procedure in mathematics, namely, as far as generalizations are concerned, some equivalences of a concept are more suitable for the purpose than others (e.g., an equivalent framework for deriving the Black-Scholes option pricing formula from PDE is martingales, allowing extensions to other markets), Zadeh defined fuzzy sets (i.e., mathematical objects representing fuzzy concepts) by extending the range of ordinary (crisp) set indicator functions (for a complete theory of fuzzy sets and logics, see e.g.,[12]). Let U be a set. A subset A ⊆ U is characterized by it indicator function A (.): U → {0, 1} (note that we use the same notation A for the set and its indicator function, the context will tell us clearly which is which!), where A (u) = 1 or 0 according to u ∈ A ∈ or u / ∈ A. This equivalent way to describe a set brings out the notion of membership of elements of U to subsets of U: when A (u) = 1, the element u is a member of A, whereas when A (u) = 0, u is not a member of A. Here, membership degrees are only 1 and 0 (there is no partial membership). Extending the range {0, 1} to the whole unit interval [0, 1] lead to a generalization of crisp sets to fuzzy sets. Specifically, a fuzzy subset of U is a function A (.): U → [0, 1] where A (u) ∈ [0, 1] is the membership of an elements u ∈ U in the underlying fuzzy concept. For example, for $U = R^+$, the function $A(.): R^+ \to [0,1]$ given by

$$A(u) = \begin{cases} 0 & if\ u < 20 \\ \dfrac{u-20}{55} & if\ 20 \le u \le 75 \\ 1 & if\ u > 75 \end{cases}$$

is a membership function for the fuzzy concept "high income" (where, e.g., 20 means \$20, 000, annually). The value $\frac{30}{55} \in [0, 1]$ is the degree to which the income of 50 is compatible with the meaning of "high income". It is not the probability that an income of 50 is high! Fuzziness is a matter of degree. Membership functions are not probability distributions. Note that, partial membership is allowed. The notion of partial membership is realistic in many situations, such as in coalitional games, where players, when joining a coalition, might not necessarily commit their full resources (see e.g., [13], [14] and [15]). As such, for a fair, say, capital risk allocation, it is necessary to consider fuzzy games, i.e., games in which coalitions are fuzzy subsets of the set of players, whose membership functions are determined by proportions of resources committed.

Having fuzzy data modeled as fuzzy sets, we can proceed to manipulate/process them by extending operations on the underlying set U. This is achieved by the well-known extension principle. If $\varphi: U \times V \to W$ and A, B are fuzzy subsets of U, V, respectively, then φ is extended to fuzzy subsets as

$$\varphi(A,B)(w) = \max_{\{(u,v):\varphi(u,v)=w\}} (A(u) \wedge B(v))$$

where $\wedge$ denotes minimum (and $\vee$ denotes maximum). Note that, in one hand, since any function $A(.): U \to [0,1]$ can be recovered from its level sets $A_\alpha = \{u \in U: A(u) \geq \alpha\}, \alpha \in [0,1]$ by $A(u) = \int_0^1 A_\alpha(u) d\alpha$, and on the other hand, manipulations with level sets are simpler, the following well-known result (known in the literature as Nguyen's Theorem, see [16], [17], [18]) is useful. A necessary and sufficient condition for

$$\varphi\left(A_\alpha^{(1)}, A_\alpha^{(2)}, \ldots, A_\alpha^{(n)}\right) = [\varphi(A^{(1)}, A^{(2)}, \ldots, A^{(n)})]_\alpha$$

where φ : U1 × U2 × ... × Un → V , and $A^{(i)}$ is a fuzzy subset on $U_i, i = 1,2,\ldots,n$

is that for each $v \in V$,

$$\max_{\{(u_1,u_2,\ldots,u_n)\in\varphi^{-1}(v)\}} [\wedge_{i=1}^n A^{(i)}(u_i)]$$

is attained.

While fuzzy logic connectives, including "implication operator" = ⇒ (representing "If... Then..." rules in fuzzy technology, see e.g., [3], [4]) are familiar with engineers, they appear as tools to suggest (nonlinear) models for statistics. This can be seen as follows. A statistical model, such as a linear regression model $Y_i = \theta X_i + \varepsilon_i$, $i = 1,2,\ldots,n$, is in fact a collection of "If...Then..." rules, since what they mean is that, for each i, the model reads "If X is X_i (and ε is ε_i), then Y is Y_i" (where "is" stands for "equal"). This observation allows an extension to fuzzy data: when $(X_i, Y_i), i = 1,2,\ldots,n$ are fuzzy data (linguistic labels), the "rules" become R_i:"If X is X_i, then Y is Y_i" or "$X_i \Rightarrow Y_i$" where the connective "If...Then..." is the implication ⇒ in fuzzy logic, which is a fuzzy relation on the Cartesian product, say, U × V , i.e. $f_\Rightarrow(u,v)$ is the degree to which u "implies" v. In the simplest fuzzy logic system, $f_\Rightarrow(u,v)$ in "$X_i \Rightarrow Y_i$" can be taken as $X_i(u) \wedge Y_i(v)$.

In the statistical linear regression, the goal is to arrive at a "prediction formula" from, say, given numerical data $(X_i, Y_i), i = 1,2,\ldots,n$. This is achieved by combining the "rules" $Y_i = \theta X_i + \varepsilon_i$ by using some method of estimation (e.g., least squares, when random variables have finite variances) to estimate the parameter θ to arrive as $\hat{\theta}_n X$. A counterpart of such a procedure when the data $(X_i, Y_i), i = 1,2,\ldots,n$ are fuzzy is combining the rules "$X_i \Rightarrow Y_i$" by the compositional rule of inference $\vee_{i=1}^n [X_i(u) \wedge Y_i(v)]$ to obtain the combined membership function, where $\vee_{i=1}^n [X_i(u) \wedge Y_i(v)]$ is the degree to which u implies v given the rule base. Given a value u, the implied consequence is a fuzzy subset of V given by $v \to \vee_{i=1}^n [X_i(u) \wedge Y_i(v)]$. The important point is this. Fuzzy rule bases play the role of statistical models in the presence of fuzzy data.

Remark: With respect to the "conditional" implication "$A \Rightarrow B$" and its degree of compatibility, used in the previous context, it is of course tempting to ask whether they can be given a probabilistic flavor? For example, for crisp events A, B on some probability space (Ω, A, P), can we view the degree of "$A \Rightarrow B$" as $P(B|A)$?

If $A \Rightarrow B = A^c \cup B$ (material implication), then

$$\begin{aligned} P(A \Rightarrow B) &= P(A^c \cup B) = P(A^c \cup (A \cap B)) \\ &= P(A^c) + P(A \cap B) = P(A^c) + P(B|A)P(A) \\ &= P(A^c) + P(B|A)[1 - P(A^c)] \\ &= P(B|A) + P(A^c)[1 - P(B|A)] \\ &= P(B|A) + P(A^c)P(B^c|A) \geq P(B|A) \end{aligned}$$

with equality holding if and only if $P(B^c \cap A) = 0$ or $P(A) = 1$, a rather trivial case. It is known that there is no operation ∇ on the $\sigma-$ field $\mathcal{A}$ such that $P(A \nabla B) = P(B|A)$ (see [19]). Thus, the answer is no, even for crisp events let alone for fuzzy events (a fuzzy event is a fuzzy subset of Ω whose membership function is measurable). Therefore, if we insist on $P(A|B)$ as the degree for $A \Longrightarrow B$ to be true, we have to represent mathematically the rule $A \Longrightarrow B$ differently. Since $A \Longrightarrow B \notin \mathcal{A}$, it could be an object lying outside of A for the equation $P(A \Longrightarrow B) = P(B|A)$ to hold. This is similar to complex numbers. For the solution to this problem, see [19].

Fuzzy rules and their fusion are very important for reasoning in intelligent systems. Each rule reflects common sense knowledge, and will be used (see later) to provide models for knowledge acquisition. Just like the case of default reasoning in computer science, rules could have exceptions (for reasoning with such rules, see [20]). For representation of fuzzy information on computer, see [21].

3. Fuzzy technology for statistics

We turn now to the question: how can fuzzy technology help statistics? In a sense, we will point out some significant contributions of fuzzy theory, outside the engineering fields, to statistics in particular, and to decision theory in general.

Regression analysis is the main tool of statistics for investigating relationships between economic variables. As statistical theory, based upon probability theory, seems to leave no stone unturned in its path, it addresses also the important situations where variables (response variables, regressors and covariates) could be latent or qualitative (see a Text like [22]). However, in this context, there is one stone unturned. It is the case where regressors are seemingly unobservable and need to be "estimated" to run the regression. A typical situation is the study of the effect of underground economy (u.e.) on national economy, recently investigated by [23], and [24]. Let Y denote the GDP of a country and X denote the size of the u.e. of that country. A simple linear regression model is Y = aX + b + ε. While Y is observable (say, yearly), X is not. It is not realistic to make further assumptions to proceed as in standard practice! To run such a regression, we need to "create" a time series of X. How? It is precisely here that fuzzy technology could help! But before elaborating on this possibility, let's pause and says few words about this interesting demand. Although X is unobservable, we might be able to identify some main causes of it. Then we need an ingredient to infer X, in some fashion, from these observable causes. This sounds somewhat like we are in the context of causal inference? As emphasized by [25], probability (and hence "standard" statistics) is insufficient to handle causality, noting that correlation is not causality (e.g., "symptoms are associated with diseases", but "symptoms do not cause diseases"), and *"Causal inference requires two additional ingredients: a scientific language for articulating causal knowledge, and a mathematical machinery for processing that knowledge, combining it with data and drawing new causal conclusions about the phenomenon"*. While our problem here is not about causal inference, since we assume that some causes of X are identified, and we proceed to "estimate" X from them. However, fuzzy theory seems to provide the two additional ingredients mentioned by Pearl: the "scientific language for articulating knowledge" is fuzzy logic in the form of fuzzy "if...then..." rules and mathematical machinery for processing knowledge" is the compositional rule of inference.

Important remark. *This remark emphasizes a significant contribution of fuzzy theory to causal inference that both engineers and econometricians should take a closer look at it.* When considering causality, we face two problems: finding the causes of an "effect", and studying the effects of causes of an effect. Causal inference is referred to the latter. See [26] for an excellent paper on causal inference from a statistical viewpoint. More specifically, causal inference concerns measuring (in a numerical sense) the effects of given causes on an "effect" variable. The above fuzzy procedure offers a more realistic way for conducting causal inference, even for the case where the effect variable is not observable. It does so by using (linguistic) coarsening schemes, resulting in assessing (rather than "measuring") causal effects from common sense knowledge, without imposing untested statistical assumptions. This is a very important issue to explore for future research and applications, where fuzzy technology could provide a realistic alternative to statistical approach, somewhat in line with Pearl's view. It should be remembered that when we offer an alternative (to existing approaches) we need to explain why another alternative (e.g., what are its advantages?). In view of the state-of-the-art of causal inference, an alternative to carry out causal inference by fuzzy theory should be welcome and well-justified. Indeed, as [27] put it "I would advise against regarding any one approach or blending as a complete solution or algorithm for problems of causal inference; the area remains one rich with open problems and opportunities for innovation".

In the specific case of u.e., causes of u.e. could be "taxation", "unemployment rate" and "corruption index". A typical fuzzy rule is of the form "If taxation is high, unemployment rate is low, and corruption index is medium, then the size of u.e. is medium". Given a fuzzy rule base (consisting of a finite set of fuzzy rules), the compositional rule of inference allows the derivation, from observed causes, the "estimated" size of u.e., expressed as fuzzy sets. With such "fuzzy estimates" of the regressor "size of u.e.", we then face a linear regression model $Y = aX + b + \varepsilon$ with fuzzy data $(X_i, Y_i), i = 1,2,\dots,n$ Note that (statistical) regression with fuzzy data is different than conventional "fuzzy regression" in engineering literature, in which the model is $Y = aX + b$ with coefficients a, b being fuzzy sets, X non fuzzy, resulting in fuzzy output Y.

Another situation where fuzzifying concepts (rather than data) is useful is in financial econometrics. The following example brings out also an advantage of using fuzzy methodology in investing economic problems, showing that, in some cases, fuzzy theory is really indispensable.

The capital risk allocation (CRA) is an important problem in financial risk management (see, e.g. [15]). The most recent approach to the solution of CRA is based upon coalitional game theory, since cost functions can be expressed in terms of characteristic functions of such games. Unfortunately, the Shapley value [28] cannot be inside the core of the game for coherent risk measures. It was suggested that extending (crisp) coalitional games to *fuzzy games* ([13], [14]) could lead to a solution.

Extending a coalitional game to a fuzzy game is possible by using the mathematical concept of fuzzy sets. Here, it is conceivable that members of a coalition might not always commit their full resources when joining a coalition. As such, degrees of participation in a coalition should be taken into account, for a fair capital risk allocation. When doing so, we actually consider fuzzy coalitions, thus, enlarging the coalitional game.

A similar situation of considering fuzzy data in engineering economic problems is statistical quality control (SQC) which should lead to more reliable control charts. Observations within the "in-control zone" (e.g., within six standard deviations) clearly have different "degrees", and should not be treated as the same. Unlike fuzzy coalitions, observations in SQC are then random fuzzy (closed) sets which should be subject to statistical analysis.

4. Econometrics for engineering

We illustrate in this section several important aspects of econometric analysis which seems unfamiliar to engineering economists. These aspects also serve as a prelude for extending them to fuzzy random variables. Specifically, we illustrate situations in which statistical data are sets rather than points in Euclidean spaces.

A well-known graphical method in exploratory data analysis to test ("informally") the goodness-of-fit of a sample (e.g., that the sample came from a normal distribution) is the Quantile-Quantile plot (QQ plot). Let $(X_1, X_2, \dots, X_n)$ be a random sample drawn from a population X with unknown distribution function F . To see whether the sample comes from a specific distribution F^0 we compare various quantiles of F^0 with corresponding empirical quantiles, i.e., for $\alpha \in (0,1)$, compare

$$q_\alpha(F^0) = \inf\{x \in \mathbb{R}: F^0(x) \geq \alpha\}$$

with $q_\alpha(F_n)$ where the empirical distribution function is

$$F_n(x|X_1, X_2, \dots, X_n) = \frac{1}{n}\sum_{i=1}^{n} 1_{(X_i \leq x)}$$

Specifically, we plot $q_\alpha(F_n)$ versus $q_\alpha(F^0)$ for several values of α. Note that, unlike moments, a distribution function is characterized by its quantiles, i.e., can be recovered from its quantiles. As such, coincidence of quantiles is a good indication for goodnees-of-fit. In the QQ plot, the indication of a good fit is detected when the QQ plots "hugs" a straight line through the origin at an angle of 45 degrees (as often said in the literature, this is a "quick and dirty" way of doing statistics!). We have just said that such an inspection reveals only an indicative conclusion. It is not a conclusive conclusion, i.e., not "rigorous". Again, this is a good place to remind applied statisticians about validity of empirical analyses! Empirical analyses need to be validated to draw final conclusions. How to "validate" the QQ plot? Well, if the sample did come from F^0, then the QQ plot should converge to a straight line as the sample size increases. Here the "target" is a straight line, a subset of $\mathbb{R}^2$ and the QQ plot is a sequence of random subsets of $\mathbb{R}^2$ (see section 6). As such, we need to know what we mean by "convergence of a sequence of random sets to another set?"! It was [29] who investigated this problem in a formal theory of random sets.

Remark. Historically, while random sets appeared naturally in many places, such as stochastic geometry, its formal theory was not rigorously established until 1975 (by [30]). When estimating the "size" (area, volume) of a random set, [31] did not really consider a formal concept of a random set. This so since the size of a random set $\mu(S)$ (where μ the Lebesgue measure on $\mathbb{R}^d$) is in fact a numerical random variable, although it depends on the random set S . Without a formal concept of random sets, it is not possible to find the distribution of the nonnegative random variable $\mu(S)$ which is a function of S. The clever result of Robbins is this. As far as the expected value of $\mu(S)$ is concerned, we need much less than the distribution of $\mu(S)$. Specifically, the knowledge on the coverage function of the informal random set S is sufficient to determine $E\mu(S)$, a "weaker" form of information. Note that if S is a confidence interval, which is a random set, (say, at $1-\alpha$ confidence level), then $\mu(S)$ is its length, and an optimal confidence interval is the one with smallest length (maximum precision at a given confidence level). The computation of the expected length of a random set of the form $S = [0, X]$ where $X \geq 0$ is a random variable is simple: the length of S is X , so that

$$E\mu(S) = EX = \int_0^\infty P(X > x)dx = \int_0^\infty \pi(x)dx$$

where

$$\pi(x) = P(x \in S) = P(x \in [0, X]) = P(X > x)$$

is the coverage function of the random set S . The Robbins' formula says that the above formula is in fact general: For any random set S on $\mathbb{R}^d$, we have

$$E\mu(S) = \int_{\mathbb{R}^d} \pi(x)d\mu(x)$$

where $\pi(.): \mathbb{R}^d \to [0,1]$ is the coverage function of S.

Now, while the $q_\alpha(F^0)$ are deterministic, the $q_\alpha(F_n)$ are random (depending upon the sample), and as such the points $(q_\alpha(F_n), q_\alpha(F_n))$ for various α, are random points in the plane, forming a random set (of points). Since the sample is finite, this random set is *a closed* subset of the plane $\mathbb{R}^2$. Note that a straight line in $\mathbb{R}^2$ is also a closed set, and hence we are facing the problem of, say, almost sure convergence of *random closed sets*.

As statistical quality control (SQC) is within engineering economics, let's mention the current research in SQC. In its most general form, it is about estimating level sets of (multivariate) probability density functions.

Current research in Statistical Quality Control (SQC) addresses the more realistic statistical models in which characteristics of manufacturing products need not follow multivariate normal distributions. In other words, the research aims at deriving tolerance regions (leading to control charts) in the setting of multivariate, nonparametric models. This is carried out by recognizing that traditional tolerance regions are nothing else than level sets of probability density functions. The recent paper by [32] brings out the usefulness of using copulas in modern SQC. The multivariate SQC (see [33]) is essentially based on (parametric) normal distributions.

In the univariate case, Shewhart in 1924 first observed that, if the (single) product characteristic is modeled by a random variable X (due to its possible variations), then we can detect whether it is "out of range" (out-of-control) if the new value is far away from its mean $\mu = EX$ by 3 standard deviation $\sigma = \sqrt{Var(X)} = \sqrt{E(X-\mu)^2}$, by using Chebyshev's inequality:

$$P(|X-\mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}$$

For example, for $k = 3$,

$$P(|X-\mu| \leq 3\sigma) \geq 0.8889$$

Remark. Using extension of Chebyshev's inequality in higher dimensions (i.e., for random vectors), similar assessments can be obtained. If we insist that X is normal $N(\mu, \sigma^2)$, then the above lower bound is more accurate, namely

$$P(|X-\mu| \leq 3\sigma) \geq 0.997$$

so that the interval $[\mu - 3\sigma, \mu + 3\sigma]$ could be used as a "tolerance" zone for the variations of X. Specifically, since $(P(|X-\mu| > 3\sigma)$ is so small, it is unlikely that a value of X in $|X-\mu| > 3\sigma$ could come from X. Of course, false alarms could arise!

The following observation is essential for considering multivariate SQC when traditional multivariate normal distribution assumption is dropped.

If we look at the tolerance interval $[\mu - 3\sigma, \mu + 3\sigma]$, we realize that it is precisely the set

$$\{x \in \mathbb{R}: f(x) \geq c\}$$

where

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2\sigma^2}(x-\mu)^2\}$$

and $c = f(\mu + 3\sigma)$, with $\mu + 3\sigma$ being a quantile of X.

Thus, for general multivariate (joint) density function , a tolerance region is of the form

$$\{x \in \mathbb{R}^d: f(x) \geq c\}$$

is that

$$P(f(X) \geq c_\alpha = \alpha$$

i.e., the probability that a new observation, say, $Xn + 1(\omega)$, is in the level set is some predetermined α.

Now, of course, the joint density f on $\mathbb{R}^d$ (e.g., when the manufacturing product depends on d (related) characteristics) is unknown. As such, the (population) "parameter" $\{x \in \mathbb{R}^d: f(x) \geq c\}$, which is a set, needs to be estimated (by some set statistics, i.e., random sets). Such a set statistic is the statistical tolerance region for deriving multivariate control charts. The problem is how to estimate the level set $A(c) = \{x \in \mathbb{R}^d: f(x) \geq c\}$? Plug-in estimator $\{x \in \mathbb{R}^d: f_n(x) \geq c\}$, where $f_n(x)$ is some nonparametric estimator of f requires lots of analytic assumptions, and might be computational intractable in high dimensions. An alternative, suggested by [34], when instead, some qualitative information is given, such as the shape of level sets (e.g., closed convex sets, ellipsoids), is as follows. Recall the way *extremum estimators* in statistics are derived: if a population parameter optimizes a theoretical objective function, then a plausible estimator for it is the statistic optimizing the empirical counterpart of that objective function. Let μ denote the Lebesgue measure on $\mathbb{R}^d$ and consider the signed measure $(dF - c\mu)(.) = \varepsilon_c(.)$ on $\mathcal{B}(\mathbb{R}^d)$. For $B \in \mathcal{B}(\mathbb{R}^d)$, writing $B = [A(c) \cap B] \cup [A^c(c) \cap B]$, we have $\varepsilon_c(B) \leq \varepsilon_c(A(c))$, so that $A(c)$ maximizes the objective function $B \to \varepsilon_c(B)$. The empirical counterpart of $\varepsilon_c(.)$ is $\varepsilon_{c,n}(.) = (dF_n - c\mu)(.)$, so that, a plausible estimator of $A(c)$ could be the random set statistic $A_n(c)$ maximizing $\varepsilon_{c,n}(B)$ over all $B \in \mathcal{C} \subseteq \mathcal{B}(\mathbb{R}^d)$, where $\mathcal{C}$ is some specified class of Borel sets, such as closed convex sets, ellipsoids. How to "solve" this set-function optimization? See, [35], To establish the consistency of $A_n(c)$, we need a formal theory of random sets (see section 6).

The set estimation of level sets of a probability density function can be used to estimate the density itself (nonparametrically). Indeed, since

$$f(x) = \int_0^\infty 1_{A(\alpha)}(x) d\alpha$$

it suffices to take

$$f_n(x) = \int_0^\infty 1_{A_n(\alpha)}(x) d\alpha$$

as an estimator of $f(x)$. It remains of course to show that such an estimator is at least consistent. A formal theory of random sets is needed.

Now, if our observed data are more general than sets, namely fuzzy sets, then we face "random fuzzy sets" as data which are values of "linguistic variables". On this issue, let's look back at how statisticians used to handle linguistic variables, in the popular tool of linear regression (see a cook book like [22]).

Linguistic variables could be intrinsically linguistic (e.g., ability in mathematics, skill of workers), i.e., variables whose values can only be described in linguistic terms, or due to coarsening schemes (for more on coarsening, see [4]). The second kind of linguistic variables is very important in data analysis. An essential aspect of human intelligence, say, in making everyday life decisions, is coarsening domains of numerical variables. When we cannot guess with precision the temperature at some location, we coarsen a domain [a, b] of the variable "temperature", i.e., transforming it into a fuzzy partition, such as "very cold , cold, medium, hot, very hot", in order to obtain a correct, but imprecise, useful information. Formally, a fuzzy partition of a set U is a collection $\{A_i : i = 1, 2, \ldots n\}$ of fuzzy subsets of U such that the sum of their membership functions is one: $\sum_{i=1}^{n} A_i(u) = 1$, for all $u \in U$. This is clearly a generalization of the ordinary concept of a (crisp) partition of a set. This explains why in statistics we consider linguistic values like "very low, low, normal, high, excessive" for the variable "unemployment rate" in regression analysis, for example. By doing so, we actually transform a quantitative variable into a fuzzy variable (in the sense that the values of the latter are fuzzy sets). But, that transformation, in classical statistics, is only for the purpose of classification to collect (counting) data, and not viewing these linguistic values as data per se.

Here is an example of using "quantitative indicators" in regression with a qualitative predictor. In the regression of advertising expenditures X (quantitative) on the quality of sales management Y (qualitative with two values "low, high"), the quantitative indicator of Y is

$$Y = \begin{cases} 1 \; if\ the\ quality\ of\ sales\ management\ is\ high \\ 0 \; otherwise \end{cases}$$

Well, how "high" is defined here to obtain the indicator of Y?! Of course, "high" is defuzzified by using some threshold.

What we have in mind when talking about fuzzy data is at least twofold. First, even in classical problems as above, defuzzification might entail loss of information. Is there a better way than sharp defuzzification? e.g., some smooth procedures. This is perhaps the main reason in using fuzzy modeling in the newly developed Regression-Discontinuity Analysis, see [36].

Secondly, as we will illustrate below, there are important situations where fuzzy data need to be treated as data, i.e., just like a random sample of numerical observations, so that manipulation, processing of them are necessary. This is not considered in classical regression with qualitative variables. In fact, that is impossible since there is no fuzzy modeling available.

5. How to provide a statistical basis for fuzzy data analysis?

As far as the applications of econometrics to engineering economics are concerned, it seems desirable that empirical analyses should be placed within statistical theory. This is so since we wish to interpret them or draw conclusions from them with theoretical justifications. Specifically, if our data are fuzzy sets obtained at random, then we should place them properly within an appropriate statistical framework from which logical inference and decisions could be made.

While it is possible to provide directly a statistical setting for fuzzy data, we will start out with the special case of (crisp) set data for the benefits of those who are not familiar with set-valued observations.

Statistical observations in standard statistics are points or vectors in Euclidean spaces $\mathbb{R}^d$, or curves (e.g., stock market fluctuations) in infinite dimensional function spaces. We have seen that sets as statistical observations occur in many practical problems. In view of their different nature (noneuclidean), their statistical setting is somewhat unfamiliar to statisticians, let alone econometricians and engineering economists.

Roughly speaking, a "random set" is a set obtained as random. Just like the case of random variables which take values in $\mathbb{R}^d$ to formulate set-valued variables as random elements, we need to specify a $\sigma -$ field of subsets of their range spaces. Here is the outline of Matheron's theory of random closed sets on Euclidean spaces $\mathbb{R}^d$.

The general framework of probability is this. We always consider an abstract probability space (Ω, A, P) on which all random elements are defined. To specify a type of random elements X , we specify a measurable space $(U, \mathcal{U})$ consisting of a set U which is the range of the random element X we have in mind, and $\mathcal{U}$ a suitable $\sigma -$field of subsets of U (for the domain of the probability measures governing the random evolution of X , elements of $\mathcal{U}$ are events).

For example, if $U = 2^V$ (power set of V), with V being a finite set, i.e., U is the set of all subsets of a finite set V , then just take $\mathcal{U}$ = power set of 2^V since probability measures can be defined on such $\mathcal{U}$: simply assign $Q(A) \in [0, 1]$ such that $\sum_{A \subseteq V} Q(A) = 1$, and define $P(.)$ on 2^{2^V} by $P(\mathbb{A}) = \sum_{A \in \mathbb{A}} Q(A)$.

As for $U = \mathbb{R}$, an infinite, uncountable set, the situation is more delicate.

The power set of $\mathbb{R}$ is too big to define probability measures on it. We seek a largest collection $\mathcal{U}$ of subsets of $\mathbb{R}$ (but strictly contained in the power set of $\mathbb{R}$) to be the domain of all probability measures. Inspired by measure theory in real analysis, it turns out that there is a canonical way of getting such $\mathcal{U}$. We equip $\mathbb{R}$ with a topology (i.e., declare a collection $\mathcal{O}$ of subsets as "open" sets). For $\mathbb{R}$, the canonical topology is the smallest collection of subsets containing the open intervals (a, b). Then take the smallest σ −field containing all open sets, denoted as B($\mathcal{O}$) (we also say that it is the σ −field generated by $\mathcal{O}$). The "canonical" σ−field obtained this way is referred to as the Borel σ−field associated with the topology $\mathcal{O}$. Just to be self-contained, a σ−field is a collection of subsets, suitable for defining probability measures on it. A collection $\mathcal{B}$ of subsets (events) of a set U is a σ−field if it satisfies the conditions (i) $U \in \mathcal{B}$, (ii) If $A \in \mathcal{B}$ then its complement $A^c \in \mathcal{B}$, and (iii) For any countable collection of elements of $\mathcal{B}$, $\{A_n, n \geq 1\}, \cup_{n\geq 1} A_n \in \mathcal{B}$.

Now, consider $U = \mathcal{F}(\mathbb{R}^d)$, the set of closed subsets of $\mathbb{R}^d$. We will proceed to equip U with a topology τ and take $U = \mathcal{B}(\tau)$.

Let $\mathcal{F}, \mathcal{G}, \mathcal{K}$ denote the classes of closed, open and compact subsets of $\mathbb{R}^d$, respectively. For $A \subseteq \mathbb{R}^d$, let

$$\mathcal{F}_A = \{F \in \mathcal{F}: F \cap A \neq \emptyset\}, \mathcal{F}^A = \{F \in \mathcal{F}: F \cap A = \emptyset\}$$

$$\mathcal{F}^K_{G_1, G_2, \ldots, G_n} = \mathcal{F}^K \cap \mathcal{F}_{G_1} \cap \mathcal{F}_{G_2} \cap \ldots \cap \mathcal{F}_{G_b}$$

$$\mathbb{B} = \{\mathcal{F}^K_{G_1, G_2, \ldots, G_n}: K \in \mathcal{K}, G_i \in \mathcal{G}, n \neq 0\}$$

Let τ be the topology generated by the base $\mathbb{B}$. This topology is called the hit-or-miss topology of $\mathcal{F}$. The associated Borel σ −field is denoted as $\mathcal{B}(\mathcal{F})$

Definition. Let $(\Omega, \mathcal{A}, P)$ be a probability space. By *a random closed set* on $\mathbb{R}^d$, we mean a map $X : \Omega \to \mathcal{F}$ which is $\mathcal{A} - \mathcal{B}(\mathcal{F}) -$ measurable. The probability law of X is the probability $P_X = PX^{-1}$ on $\mathcal{B}(\mathcal{F})$, i.e., for $\mathbb{A} \in \mathcal{B}(\mathcal{F}), P_X(\mathbb{A}) = P(X \in \mathbb{A})$.

For an elementary exposition on the whole theory of random closed sets, the reader can consult Nguyen (2006). Here, we just indicate the counterpart of Lebesgue-Stieltjes theorem for random closed sets. Observe that, if we define $T : \mathcal{K} \to [0, 1]$ by

$$T(K) = P(\mathcal{F}_K) = P(F \in \mathcal{F} : F \cap K \neq \emptyset\}$$

then T satisfies the following axioms

(1) $T(\emptyset) = 0$

(2) T is alternating of infinite order, i.e., for any $n \geq 2$ and $K_1, K_2, \ldots, K_n$ in $\mathcal{K}$,

$$T(\cap_{i=1}^n K_i \leq \sum_{\emptyset \neq I \subseteq \{1,2,\ldots,n\}} T(\cup_{i\in I} K_i)$$

(3) If $K_n \searrow K$ in $\mathcal{K}$ then $T(K_n) \searrow T(K)$

Any function $T : \mathcal{K} \to [0, 1]$ satisfying the above three axioms is called *a capacity functional*. Capacity functionals play the role of distribution functions of random variables. The collection of closed sets $\mathcal{F}_K = \{F \in \mathcal{F} : F \cap K \neq \emptyset\}$ plays the role of intervals $(-\infty, y]$ on the real line, in the determination of the distribution function of a real-valued random ble $:F_Y(y) = P(Y \leq y) = P_Y((-\infty, y])$.

Like Lebesgue-Stieltjes theorem, the following result simplifies the search for probability laws governing random evolution of random sets.

Choquet Theorem. If $T : \mathcal{K} \to [0, 1]$ is a capacity functional, then there exists a unique probability P on $\mathcal{B}(\mathcal{F})$ such that $P(\mathcal{F}_K) = T(K)$ for all $K \in \mathcal{K}$.

We turn now to *random fuzzy (closed) sets*. First, a fuzzy subset of $\mathbb{R}^d$ say, is a function $A(.) : \mathbb{R}^d \to [0, 1]$. Since a crisp subset A is a closed set if and only if it indicator function $1_{A(.)}$ is upper semicontinuous (u.s.c.), i.e., for any $\alpha \geq 0$, its level set $\{y : 1_{A(y)} \geq \alpha\}$ is a closed set, we say that the fuzzy set $A(.)$ is *a fuzzy closed subset* of $\mathbb{R}^d$ if it is u.s.c., i.e., for any ≥ 0, $\{y : A(y) \geq \alpha\} \mathcal{F} \in \mathbb{R}^d$), the set of closed subsets of $\mathbb{R}^d$. We denote by $\mathcal{F}^*\mathbb{R}^d$ the set of fuzzy closed subsets of $\mathbb{R}^d$.

How to extend Matheron's hit-or-miss topology to $\mathcal{F}^*\mathbb{R}^d$?

Remark. We take this opportunity to say an important thing. Fuzzy sets are "new" mathematical objects. If we are going to talk about topology, it should be (ordinary) topology of fuzzy sets, and not "fuzzy topology", i.e., a "new" concept of topology generalizing ordinary topology in mathematics! A kind of "topology" where ordinary neighborhoods of "points" in $\mathbb{R}^d$ become fuzzy. This should be so since the objects under considerations are fuzzy sets and not points in $\mathbb{R}^d$. This applied also to the wrong approach by trying to treat fuzzy data in a context of "fuzzy statistics"! Fuzzy statistics should be (like fuzzy logics) ordinary statistics of fuzzy data, and not "fuzzifying ordinary statistics"!

Now, again, a direct extension of Matheron's topology for (crisp) closed sets seems difficult. We need an equivalent way of looking at his topology for the purpose of extension. Note that this is a "routine" in mathematical investigation, as far as extensions of concepts, theories, are concerned. This includes the extension of Black-Scholes option pricing formula in financial econometrics, based on PDE: it is the equivalence in terms of martingales which allows extensions.

Another look at the hit-or-miss topology of closed sets is this. If we consider the set containment $\supseteq$ as a partial order relation on $\mathcal{F}$, i.e., for $A, B \in \mathcal{F}$, we say that B is "less informative" than A if $B \supseteq A$ (B contains A, the reverse of standard partial order relation among sets), then $(\mathcal{F}, \supseteq)$ happens to be a continuous lattice (see, [38]). As such, there is a canonical topology, called

Lawson topology, generated by $\supseteq$. Without going into technical details, we simply say that this Lawson topology is precisely the Matheron topology, see however, [37]. Thus, the $\sigma-$ field, for defining random closed sets, is the Borel $\sigma-$field of the Lawson topology.

Now, on $\mathcal{F}^*\mathbb{R}^d$, if we consider the partial order relation among u.s.c. functions: f is "less" than g if $f(.) \geq g(.)$ (as extension of $\supseteq$ among sets), then, $(\mathcal{F}^*\mathbb{R}^d, \geq)$ is also a continuous lattice, and as such we simply take the Borel $\sigma-$field $\mathcal{B}(L)$ of its Lawson topology to define random fuzzy (closed) sets.

Definition. A random fuzzy (closed) set is then a map $\Omega \rightarrow \mathcal{F}^*, \mathcal{A}-\mathcal{B}(L)-$ measurable.

For the extension of Choquet theorem to random fuzzy sets, see [39].

In summary, random fuzzy data are defined as observations from bona fide random elements, termed random fuzzy (closed) sets, in the framework of probability theory, from which theoretical results can be derived leading to standard statistical inference with fuzzy data.

6. Conclusions

This paper focused on engineering economics in which we discussed various situations where fuzziness and randomness coexist naturally. Special attention is put on how fuzzy technology can help the statistical analysis of engineering economic problems as exemplified by the need to use fuzzy data in problems such as regression with seemingly unobservable covariates, statistical quality control, and regression discontinuity designs in causal inference. Fuzzy rule-based systems, which are useful in fuzzy systems designs, provide an additional method for handling fuzzy data in engineering economics. The rationale and foundations for a statistical theory of fuzzy data is provided via the theory of random fuzzy sets which is based upon a natural generalization of the well-known theory of random sets by exploiting the continuous lattice structure of membership functions of fuzzy closed sets. As a result, the use of fuzzy technology in engineering economics is firmly established in a rigorous framework. Within such a rigorous formulation of random fuzzy sets, fuzzy rule-based systems form an addition to the repertoire of statistical tools to investigate larger classes of useful applied problems. Developing new statistical procedures from random fuzzy sets for estimation, testing and prediction will be our future work.

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