Time Varying Biases and the State of the Economy

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Abstract

This paper is aimed at investigating whether the forecast is optimal given the available information when the forecast is made. Going beyond the papers that study forecast errors based on the model of Nordhaus (1987), we apply a time varying procedure to forecast revisions and further account for the possibility that the duration of the state may also affect the bias. Three testable hypotheses are presented to help researchers test the optimality of forecasts, with the ultimate goal of determining whether these biases depend on the underlying economic state and whether they are persistent with duration of the state. The corresponding bias-corrected forecasts can then be made based on these results. The empirical study finds that, the one-quarter-ahead official forecast of GDP growth in Taiwan indeed suffers from state-dependent biases-persistent under-estimation bias at the relatively good state and under-reaction bias which decays with duration at the relatively bad one. Eliminating these biases in the forecast can reduce over 44.0% variation of forecast errors, and pseudo out-of-sample experiments further support the fact that the resulting bias-corrected forecast is

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markedly better than that made by Taiwan's government and those made using other competing models.

Key words: optimality, economic growth rate, forecast error, over-reaction, under-reaction

JEL Classification: C53, C41, C22

1 Introduction

Given the available information, conventional theory suggests that the optimal forecast in the sense of minimized expected mean squared forecast errors (MSFE), must be the "rational expectation" of the target variable. The optimal forecast is unbiased because the induced forecast error has a zero mean, and it is efficient since the error is un-predictable and orthogonal to the components of the given information set when the forecast is made.

To investigate whether the forecast is optimal, there are two conventional types of regression models proposed in the literature. One directly focuses on the relationship between realized and forecasted values (e.g., Mincer and Zarnowitz, 1969); the other turns to the properties of forecast error (e.g., Nordhaus, 1987, and Holden and Peel, 1990). Note that, the parameters of these two types of models are (implicitly) assumed constant over time, and thus the implied bias (non-zero mean of the forecast error) of a forecast is time-invariant; see e.g., Mincer and Zarnowitz (1969), Nordhaus (1987), Holden and Peel (1990), Loungani (2001), Artis and Marcellino (2001), and Gavin and Mandal (2003).

However, many studies of analyzing survey data have shown that forecast performance is sometimes strongly correlated with underlying economic conditions; see, for example, Grunberg and Modigliani (1954), Döpke (2001), Fildes and Stekler (2002), Chauvet and Guo (2003), Swanson and van Dijk (2006), Sinclair *et al.* (2010), Sinclair *et al.* (2015), and Messina *et al.* (2015). These empirical evidences reveal that, the bias of a forecast could be time-varying, possibly depending on the underlying economic states and their durations.

To explain the possible time-varying bias of a forecast, Sinclair *et al.* (2010), Sinclair *et al.* (2015), and Messina *et al.* (2015) modify the models of Holden and Peel (1990), Mincer and Zarnowitz (1969), and Nordhaus (1987) by introducing a state variable whose values are *ex post* identified for the corresponding target

periods. In essence, they aim at investigating whether the forecasts incorporate the knowledge of "future state" of the economy at the time when the forecast is made. Therefore, their models can not be used to infer whether the forecast is optimal indeed. Moreover, they neglect the possibility that the duration of the state may also help to explain the forecast error.

Going beyond the studies mentioned above, this paper proposes a new framework to investigate whether the forecast is optimal by extending the regression model of Nordhaus (1987). Since the proposed model accounts for the possibility that the underlying economic state and its duration may also affect the bias, we demonstrate how to detect those time varying biases of a forecast given the available information set at the time when the forecast is made. The innovative features of the proposed model, which cannot be found in previous works when applying to a series of published forecasts are that it can help researchers to test the conventional rationality of a forecast; to tell whether the existing biases depend on the underlying state of the economy when the forecast is made; to determine whether these biases are persistent even though the duration of that economic state is getting longer; and further to construct a feasible bias-corrected forecast with lower MSFE once the forecast is published (which is useful for the real-time analysis in particular).

Based on the framework proposed herein, the real GDP growth (one-quarter-ahead) forecast of Taiwan government is analyzed. The analysis based on 105 real-time data vintages in Taiwan shows that this forecast suffers from time-varying biases; they depend on the underlying economic state and its duration. Specifically, the Taiwan government tends to under-estimate GDP growth during the relatively good states and this bias is persistent. On the contrary, when at the relatively bad economic states, the government under-reacts to the received news but this bias disappears with the duration of the state. Moreover, neither official forecast nor forecasts made using competing models substantially outperform the proposed bias-corrected forecast in pseudo out-of-sample experiments on Taiwan's GDP growth forecasts.

The rest of this paper is organized as follows. Section 2 introduces the methodology. Section 3 presents a relevant empirical study of forecast in Taiwan, and Section 4 draws conclusions.

2 Methodology

2.1 Optimal Forecast and Testable Models

Let y_t be the realized GDP growth rate at time t, and $y^f_{t|t-h}$ be its h-step-ahead forecast published at time $t-h$ for $h > 0$. Accordingly, define the (one-step-ahead) forecast error as $e_t \equiv y^f_{t|t-1} - y_t$, where $e_t > 0$ indicates an over-prediction while $e_t < 0$ indicates an under-prediction.

Let Ω_t denote the available information set at time t, then the conventional forecasting theory claims that the optimal one-step-ahead forecast should be $\mathbb{E}[y_t|\Omega_{t-1}]$, the conditional expectation of y_t given the information set at time $t-1$, while minimizing the expected MSFE (Patton and Timmermann, 2012; for example). It immediately follows that, if $y^f_{t|t-1}$ is optimal, the resulting forecast error e_t should satisfy the moment condition as

$$\mathbb{E}[e_t|\Omega_{t-1}] = 0, \tag{1}$$

which also implies that e_t should have zero mean ($y^f_{t|t-1}$ is thus unbiased, say) and be orthogonal to the variables in Ω_{t-1} ($y^f_{t|t-1}$ is thus efficient, say).

To test for the unbiasedness of a forecast, Holden and Peel (1990) propose the following simple regression:

$$e_t = \mu + u_t, \tag{2}$$

where u_t is the zero-mean un-predictable disturbance at time $t-1$. Given this model, $\mu = 0$ implies the forecast is unbiased while non-zero μ implies a biased forecast.[1] On the other hand, trying to introduce some available variables in Ω_{t-1} into the test for the unbiasedness and efficiency of $y^f_{t|t-1}$, Nordhaus (1987) considers the model for the forecast error as

$$e_t = \alpha + \beta \cdot FR_{t-1} + u_t, \tag{3}$$

where the variable $FR_{t-1} \equiv y^f_{t|t-1} - y^f_{t|t-2}$ is known as the forecast revision, which measures the forecaster's adjustment of the forecast in response to the new information received between time $t-2$ and $t-1$.[2] The intercept α indicates the

[1] Note that, however, the unbiased forecast does not necessarily imply the forecast is efficient if the residual of regression (2) is serially correlated, see Joutz and Stekler (2000), and Gavin and Mandal (2003).

[2] For example, a positive revision occurs when the forecaster receives some good news about the economy. If the forecaster expects the news to have a larger impact on the economy, then the public would see a larger revision in the forecast.

possibly systematic estimation error of the forecasts, and the slope β measures how all the new information is incorporated into the forecast made at time $t-1$; $\beta > 0$ ($\beta < 0$) thus reflects the forecaster's over-reaction (under-reaction) to the received information.[3] Throughout this paper, we thus interpret the positive α as the over-estimation bias and the negative one as the under-estimation bias while the positive (negative) β indicates the over-reaction (under-reaction) bias. If the forecast is optimal, then the null that $\alpha = \beta = 0$ should not be rejected significantly by the observed data. Besides, from the perspective of model selection, if the estimate of β is significantly non-zero, the model (2) of Holden and Peel (1990) may not be appropriate for modeling e_t since the variable FR_{t-1} can help to explain e_t statistically.

2.2 Time Varying Biases and State of the Economy

Since many studies of analyzing survey data have shown that forecast performance is sometimes strongly correlated with underlying economic conditions,[4] we may further consider the time-varying biases in different state of the economy by modifying above Nordhaus's model (3) as

$$e_t = [\alpha_0 + \beta_0 \cdot FR_{t-1}] \times (1 - \hat{s}_{t-1}) + [\alpha_1 + \beta_1 \cdot FR_{t-1}] \times \hat{s}_{t-1} + u_t, \qquad (4)$$

where $\hat{s}_{t-1}$ is the available estimate (or proxy) of the underlying state of the economy at time $t-1$ given the information set Ω_{t-1}. Throughout this paper, $\hat{s}_{t-1} = 0$ stands for relatively bad state and $\hat{s}_{t-1} = 1$ for the relatively good one. Based on this setting, we can capture the possible time-vary biases—state-dependent over-estimation/under-estimation biases and over-reaction/under-reaction biases. In practice, we would prefer using this model with time-varying biases to Nordhaus's model (3) if the data statistically rejects the null that $\alpha_0 = \alpha_1$ or/and the null that $\beta_0 = \beta_1$.

[3] In the literature, the intercept $\alpha > 0$ ($\alpha < 0$) can be interpreted as the behavioral bias of optimism (pessimism) while $\beta > 0$ ($\beta < 0$) captures the behavioral bias of over-reaction (under-reaction) to new information; see e.g., Ehrbeck and Waldmann (1996), Amir and Ganzach (1998), and Ashiya (2003). In particular, Ehrbeck and Waldmann (1996) establish a structural model and Amir and Ganzach (1998) propose a theory to support these existing behavioral biases.

[4] See, for example, Grunberg and Modigliani (1954), Zarnowitz (1992), Granger (1996), Döpke (2001), Loungani (2001), Fildes and Stekler (2002), Gavin and Mandal (2003), Chauvet and Guo (2003), Swanson and van Dijk (2006), and Ashiya (2007).

2.2.1 Proxy for state variable

When the relatively bad and relatively good states of the underlying economic conditions are of interest, given the information set Ω_{t-1}, a feasible proxy variable for $\hat{s}_{t-1}$ at time $t-1$ can be constructed as in Ang *et al.* (2007). This implementable proxy is modeled using deviations from D-quarter moving average (MA):

$$D\text{-quarter MA: } \hat{s}_{t-1} = \begin{cases} 0, & \text{if } y_{t-1} - \dfrac{1}{D}\displaystyle\sum_{i=1}^{D} y_{t-1-i} \leq 0, \\ 1, & \text{otherwise.} \end{cases} \tag{5}$$

The estimated underlying state of the economy at time $t-1$ is identified as a relatively bad state if y_{t-1} does not exceed its previous D-quarter moving average (y_{t-1} is decelerating); otherwise, the economy would be regarded as being a relatively good state at time $t-1$ (y_{t-1} is accelerating).

There are some remarks on this proxy. Firstly, the values of this proxy for the state of the underlying economy are not exactly equal to the ones identified *ex post* by the institute of the government such as National Bureau of Economic Research (NBER) in United States. The exact business cycle dates are not of our interest because they can not be used to test the optimality of the forecast since they are not observed at the time the forecast is made. This proxy in (5) tries to capture the relative momentum of the underlying economy based on the feasible information set instead. Secondly, in most cases, the finalized value of y_{t-1} also are unobserved at time $t-1$ when the forecast for y_t is made. However, some initial, preliminary or estimated values for y_{t-1} ($\hat{y}_{t-1}$ say) based on Ω_{t-1} can be obtained, this MA proxy $\hat{s}_{t-1}$ thus still can be computed when the value of y_{t-1} is replaced with $\hat{y}_{t-1}$ in (5). Finally, alternative proxy variable for the state of the economy may also be obtained by using approaches of band-pass filter proposed by Christiano and Fitzgerald (2003) and business cycle dating algorithm developed by Bry and Boschan (1971) and Harding and Pagan (2001), or derived from the well-known Markov-switching models or state-space framework in the literature. The induced estimation problems, however, are sometimes tedious.

2.3 Proposed Model

Besides the relative state of the economy at time $t-1$, the length of the duration of this state until time $t-1$ may also affect the released forecast such that the

potential biases are time-varying. Let $\hat{d}^0_{t-1}$ and $\hat{d}^1_{t-1}$ denote the duration of the relatively bad and of the relatively good states, respectively, until time $t-1$ given $\hat{s}_{t-1}$,[5] we thus propose a general model to test the moment condition (1) as

$$\begin{aligned} e_t = &\left[\alpha_0 \exp\left(-\gamma^0_\alpha \cdot \hat{d}^0_{t-1}\right) + \beta_0 \exp\left(-\gamma^0_\beta \cdot \hat{d}^0_{t-1}\right) FR_{t-1}\right] \times (1 - \hat{s}_{t-1}) \\ &+ \left[\alpha_1 \exp\left(-\gamma^1_\alpha \cdot \hat{d}^1_{t-1}\right) + \beta_1 \exp\left(-\gamma^1_\beta \cdot \hat{d}^1_{t-1}\right) FR_{t-1}\right] \times \hat{s}_{t-1} + u_t, \end{aligned} \tag{6}$$

where γ^0_α, γ^0_β, γ^1_α and γ^1_β are non-negative state-dependent parameters with superscript "0" standing for the relatively bad state and "1" for the relatively good one.

Some remarks must be made about this model. Firstly, for $i = 0, 1$ and $j = \alpha, \beta$, if the parameter γ^i_j is greater than zero, the function $\exp(-\gamma^i_j \cdot \hat{d}^i_{t-1})$ in (6) is nothing but the well-known exponential survivor function; that γ^i_j thus can be interpreted as the probability that the forecaster leaves the situation of suffering from the corresponding bias, and the expected duration is known to be $(\gamma^i_j)^{-1}$.[6] This implication is of interest because it helps to elucidate how long these biases from the forecast would take to die out on average. Secondly, if $\gamma^i_j = 0$, then it can be interpreted as the situation that the forecaster is not aware of the ongoing economy very well even though the duration of the underlying state is longer, this induced bias is thus persistent. Moreover, this proposed model (6) degenerates to the model (4) if all of γ^i_j, $i = 0, 1$, $j = \alpha, \beta$, are equal to zero; empirically, picking up either of these two models can thus be data-driven by testing whether the estimates of γ^i_j are significantly non-zero.

[5]Consider for example the case $\hat{d}^0_{t-1} = 3$, which means that $\hat{s}_{t-1} = \hat{s}_{t-2} = \hat{s}_{t-3} = 0$, and $\hat{s}_{t-4} = 1$, the economy has stayed in the relative bad state for three periods from $t-3$ to $t-1$. Likewise, $\hat{d}^0_{t-1} = 1$ means that the state at time $t-1$ (relatively bad state) differs from what at time $t-2$ (relatively good state).

[6]It is possible to consider other survivor functions such as Weibull or Log-logistic functions. However, these survivor functions are more general than exponential one because of introducing more parameters, it turns out that we may not easily obtain the convergent estimates of those general survivor functions in practice, especially when the sample period is not long enough. More properties and a further discussion of survivor functions can be found in, for example, Kiefer (1988).

2.4 Inferences and Implications

2.4.1 Three testable hypotheses concerning forecast error

Given the proposed model (6), determination of its value requires the validity (or otherwise) of the conventional theory of optimal forecast to be determined first.

Hypothesis 2.1 *(Optimal forecast)*
If the forecast is optimal in minimizing mean squared forecast error given the information set at the time that the forecast is made, then $\alpha_0 = \beta_0 = \alpha_1 = \beta_1 = 0$.

Other than separately considering four individual tests based on the corresponding estimates of these four parameters, three implied null hypotheses are also considered in our empirical study, they are

$$H_o : \alpha_0 = \alpha_1 = 0;$$
$$H_o : \beta_0 = \beta_1 = 0;$$
$$H_o : \alpha_0 = \beta_0 = \alpha_1 = \beta_1 = 0.$$

Rejecting any of them suggests that the forecast is either non-optimal or the minimum MSFE is not the only concern of the forecaster.

Besides, going beyond the model of Nordhaus (1987), the proposed model emphasizes the time-varying biases because of the important role of relative state of underlying economy in forming forecast errors. Whether the induced biases are independent of the underlying economic state when the forecast is made is of interest to determine. To this end, the following hypothesis may be tested.

Hypothesis 2.2 *(State-independent bias)*
If all the potential biases are independent of the state of the economy, then $\alpha_0 = \alpha_1$, $\beta_0 = \beta_1$, $\gamma_\alpha^0 = \gamma_\alpha^1$ *and* $\gamma_\beta^0 = \gamma_\beta^1$.

Three implied joint tests are of particular interest:

$$H_o : \alpha_0 = \alpha_1, \gamma_\alpha^0 = \gamma_\alpha^1;$$
$$H_o : \beta_0 = \beta_1, \gamma_\beta^0 = \gamma_\beta^1;$$
$$H_o : \alpha_0 = \alpha_1, \gamma_\alpha^0 = \gamma_\alpha^1, \beta_0 = \beta_1, \gamma_\beta^0 = \gamma_\beta^1.$$

Rejecting the first hypothesis implies that the over-estimation and/or under-estimation biases are state-dependent; likewise, rejecting the second one indicates

state-dependent over-reaction and/or under-reaction biases. The third test is the joint test of the first and the second ones, rejecting it means that not all of these biases are state-independent. Besides, any rejection among these three tests shows that employing the model of Nordhaus (1987) to analyze the forecast error is not supported by the data.

Furthermore, if the induced biases does not disappear with the duration of the state, they are persistent. This may come form the situation that the forecaster is not aware of the ongoing economy very well even though the duration of the underlying state is longer. Accordingly, if the bias in the forecast is present, whether this bias is persistent or not may be determined by testing the following hypothesis.

Hypothesis 2.3 *(Persistent bias)*

(i) If the bias caused by over-estimation or under-estimation of the forecast is persistent in relatively bad (good) state of the economy, then $\gamma_\alpha^0 = 0$ *(*$\gamma_\alpha^1 = 0$*).*

(ii) If the bias caused by over-reaction or under-reaction to news is persistent in relatively bad (good) state of the economy, then $\gamma_\beta^0 = 0$ *(*$\gamma_\beta^1 = 0$*).*

Regarding to the Hypothesis 2.3, some remarks are worthy of noting. First, the persistent bias during the relative state of the underlying economy can also be viewed as a kind of systematic bias documented by Fildes and Stekler (2002). Second, the proposed Hypothesis 2.3 is somewhat different from the recent work of Coibion and Gorodnichenko (2012, hereafter CG). In CG's model, the forecast error may occur owing to forecaster's limited information even he/she is rational. This information rigidities will be lessened as the underlying state has lasted for longer. Thus, they argue "forecast errors converge back to zero over time, as agent's information sets progressively incorporate the new information." On the contrary, the violation of Hypothesis 2.3, says that, provided that the forecast is non-optimal, the forecast errors will converge back to zero over time as the biases disappear with the duration of the state; that is, the biases of the forecast still exist with the finite length of the duration, which violates the implication of the optimal forecast.

2.4.2 Bias-corrected forecast

Recall that the main aim of this paper is to investigate whether the one-step-ahead forecast $y^f_{t|t-1}$ is optimal given the information set Ω_{t-1} at time $t-1$. We thus propose the model (6) induced from the moment condition (1) to detect the possible time-varying and state-dependent biases. Once the relevant parameters of this model for the forecast error can be consistently estimated, an interesting by-product at time $t-1$ is the so-called bias-corrected forecast, $y^{bc}_{t|t-1}$ say. It can be made as

$$y^{bc}_{t|t-1} = y^f_{t|t-1} - \hat{e}_t, \tag{7}$$

where $\hat{e}_t$ is an estimate of e_t based on model (6) provided the information set Ω_{t-1}. Notably, at time $t-1$, even though e_t $(= y^f_{t|t-1} - y_t)$ is determined *ex post*, the bias-corrected forecast $y^{bc}_{t|t-1}$ is still readily obtained because all the explanatory variables in model (6) are observable when the one-step-ahead forecast $y^f_{t|t-1}$ is published by the forecaster. This property is particularly useful in the real-time analysis because we can obtain the corresponding bias-corrected version of a forecast immediately once the forecast is published. The predictability of bias-corrected forecast $y^{bc}_{t|t-1}$ can not be worse than $y^f_{t|t-1}$ if the accuracy of a forecast is measured as the proportion of the volatility in the realization that the forecast correctly accounts for. Accordingly, the following always holds.

$$1 - \frac{\mathrm{var}\left(y_t - y^{bc}_{t|t-1}\right)}{\mathrm{var}(y_t)} \geq 1 - \frac{\mathrm{var}\left(y_t - y^f_{t|t-1}\right)}{\mathrm{var}(y_t)} \tag{8}$$

since $y_t = y^f_{t|t-1} - e_t = (y^f_{t|t-1} - \hat{e}_t) - (e_t - \hat{e}_t) = y^{bc}_{t|t-1} - (e_t - \hat{e}_t)$.

2.5 Remarks

While comparing above framework and the proposed model with the existing related works in the literature, there are some remarks.

Other than testing the optimality of $y^f_{t|t-1}$ via the moment condition (1) of forecast error, we may directly compare $y^f_{t|t-1}$ with the optimal forecast $\mathbb{E}[y_t|\Omega_{t-1}]$. If $y^f_{t|t-1}$ is optimal given Ω_{t-1}, then we would expect $(\lambda, \delta) = (0, 1)$ in the following regression model:

$$y_t = \lambda + \delta y^f_{t|t-1} + u_t.$$

This is known as the model of Mincer and Zarnowitz (1969, hereafter MZ) and is also popular in the literature. Section 3.3.1 will empirically compare the performances of this model and some of its variants with the proposed model.

For detecting the time varying biases, Sinclair *et al.* (2010), Sinclair *et al.* (2015) and Messina *et al.* (2015) also modify the Holden-Peel, MZ, and Nordhaus's models as followed:

$$e_t = \mu_0 \times (1 - s_t) + \mu_1 \times s_t + u_t,$$

$$y_t = \lambda_0 \times (1 - s_t) + \lambda_1 \times s_t + \delta y^f_{t|t-1} + u_t,$$

$$e_t = [\alpha_0 + \beta_0 \cdot FR_{t-1}] \times (1 - s_t) + [\alpha_1 + \beta_1 \cdot FR_{t-1}] \times s_t + u_t,$$

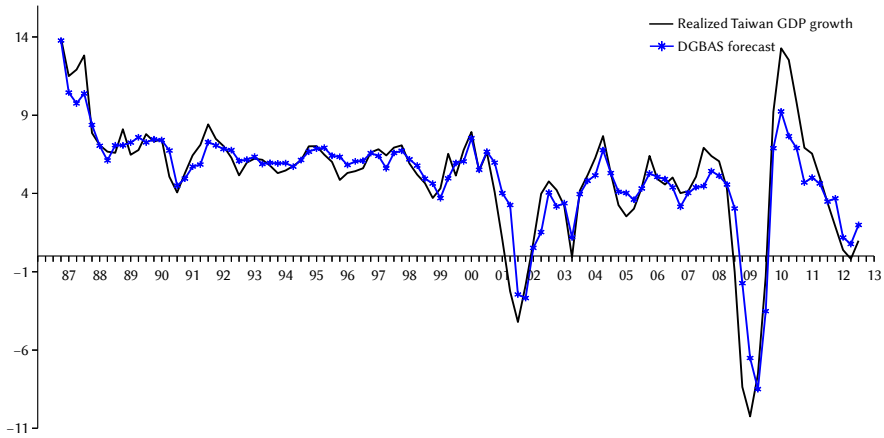
where the dummy variable s_t reflects the state of the economy at time "t". In essence, they investigate whether the forecasts incorporate knowledge about the "future state" of the economy at the time the forecast is made. However, this paper tests the moment condition (1) by introducing $\hat{s}_{t-1}$ in model (4) and thus (6) to infer if the forecast is optimal. Therefore, the spirit of these models is different from the proposed. Besides, the values of their state variable s_t are identified *ex post* and are infeasible at the time when the forecast is made; for example, the values of s_t are determined based on the NBER dated recession in Messina *et al.* (2015).

3 Empirical Study of Forecasts in Taiwan

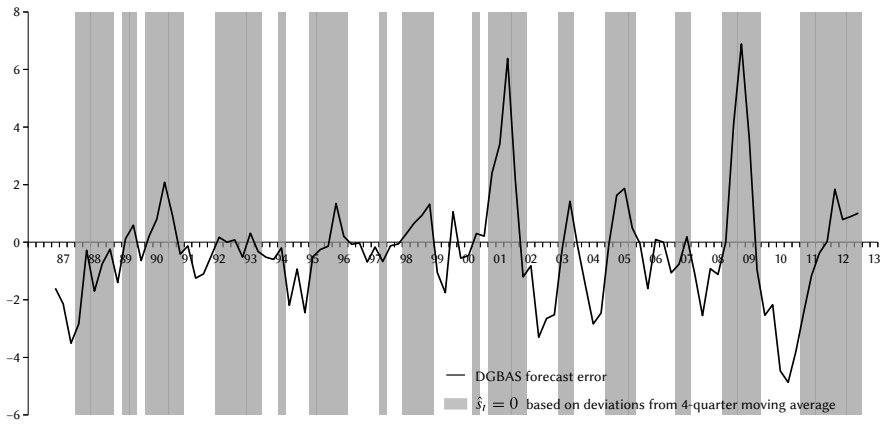
3.1 Data Description and Proxy for State Variable

In this section, an empirical study of forecasts in Taiwan is conducted based on the proposed framework. The target of interest is the quarterly real output growth rate of Taiwan. The GDP growth forecasts and realized values from 1986Q4 to 2012Q3 are sequentially taken from *Quarterly National Economic Trends*, routinely published in every February, May, August and November, by the Directorate General of Budget, Accounting and Statistics (DGBAS) in Taiwan; there are 105 real-time data vintages. Based on the identification of the multiple-quarter-ahead forecast in Chang *et al.* (2011), the DGBAS has roughly known the past realized quarterly GDP at the time when the one-quarter-ahead forecast is published; the realized data used here are released approximately 6 to 7 weeks after the quarter to which they refer.[7] This feature of Taiwan data

[7] According to the definition of Chang *et al.* (2011), if the target is the GDP growth in some quarter, the corresponding DGBAS's one-quarter-ahead forecast is identified as the forecast published in the second month of that quarter. For example, in February (the middle of Q1), the published forecast for Q1 is recorded as the "one-quarter-ahead" forecast for Q1 GDP



(a) Realized Taiwan GDP growth and its one-quarter-ahead forecast



(b) Forecast error and state of the economy

FIGURE 1: Time series plots of Taiwan real GDP growth, DGBAS forecast and forecast error. The shaded areas of (b) satisfy $\hat{s}_t = 0$ based on deviations from 4-quarter moving average.

makes it possible to construct the proxy for state variable $\hat{s}_{t-1}$ by using realized y_{t-1} directly without estimating y_{t-1} in advance; cf. (5).

Figure 1 plots the time series data. The upper figure (a) plots the realized GDP growth rate $\{y_t\}$ and its one-quarter-ahead forecast $\{y^f_{t|t-1}\}$ while the lower figure (b) plots its corresponding forecast error $\{e_t\}$ and the shaded area represents the periods of relatively bad state for which the proxy is constructed based on deviations from 4-quarter moving average, i.e., $D = 4$ in (5).

growth rate; likewise, the forecasts for Q2, Q3 and Q4 announced in Q1 are identified as the two-, three-, and four-quarter-ahead forecasts, respectively; see Chang *et al.* (2011, pp.1070) for details. By the way, at the time releasing the one-quarter-ahead forecast for Q1 GDP growth in February, DGBAS also publishes the initial realized GDP growth of Q4 in the previous year. That's why we say in this paper that the DGBAS has roughly known the past realized GDP while the one-step-ahead forecast is published.

From Figure 1(a), the Taiwan government seems unable to adjust its prediction immediately in response to the news since the forecast generally falls behind the realized Taiwan GDP growth rate, especially during volatile periods. Figure 1(b) demonstrates that the DGBAS forecast error is likely correlated with the state of the economy. That is, the DGBAS tends to over-predict the GDP growth rate during decelerating GDP environments and to under-predict it during accelerating ones. Moreover, the magnitude of DGBAS forecast error is observed to vary with the duration of the state; the absolute error initially increases and then decreases in most cases.

3.2 Results of Conventional Unbiasedness and Efficiency Analyses

To test whether the Taiwan forecast is optimal or not, we first employ some conventional models proposed in the literature; Holden-Peel regression model can be used to test for the unbiasedness while both of the MZ and Nordhaus regressions are considered for testing the optimality of a forecast.

Table 1 reports the estimation results. Estimation results of typical Holden-Peel regression, i.e., $\hat{\mu}$ in model (a1), show that the forecast is biased, and the Ljung-Box Q-statistics indicates that regression's residuals are significantly autocorrelated. This implies that the forecast is inefficient since the forecast error is predictable by its past values. Meanwhile, both the conventional MZ regression (b1) and Nordhaus model (c1) also indicate that the forecast is biased and inefficiency since the forecaster under-reacts to new information and thus fails to incorporate all new information in the forecast. Moreover, the results of state-dependent Nordhaus's model (c2) further show the time varying biases in different state of the economy.

3.3 Results of Proposed Model

The dynamics of time varying biases during different periods of the state of the economy now are analyzed using the proposed model (6).

Panel (A) in Table 2 presents the estimates, $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\gamma}^0_\alpha$, $\hat{\gamma}^1_\alpha$, $\hat{\gamma}^0_\beta$ and $\hat{\gamma}^1_\beta$, say, that are obtained using the nonlinear least squares (NLS) estimation method; Panel (B) presents the results of the testing of Hypothesis 2.1 and Hypothesis 2.2 with χ^2 statistics by Wald joint tests and of Hypothesis 2.3 with t statistics by individual tests.

Table 1: Conventional Unbiasedness and Efficiency Analyses

Panel (A) Holden-Peel Regression

Model (a1): $e_t = \mu + u_t$

Model	$\hat{\mu}$	$Q(1)$	$Q(2)$	$Q(3)$	$Q(4)$
(a1)	-0.303^{*}	0.000^{***}	0.000^{***}	0.000^{***}	0.000^{***}
	(-1.694)				

Note: Number in parentheses is robust t statistics based on heteroscedasticity-consistent estimates for corresponding variances; Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively; $Q(r)$ denotes the Ljung-Box Q-Statistic with its corresponding probability value that is used to test the null hypothesis of no autocorrelation of u_t for a specified order of autocorrelation lags r.

Panel (B) Mincer-Zarnowitz Regression

Model (b1): $y_t = \lambda + \delta y^{f}_{t|t-1} + u_t$

Model	$\hat{\lambda}$	$\hat{\delta}$	Joint Test	$\bar{R}^2$
(b1)	-0.696	1.197^{***}	0.000^{***}	0.819
	(-1.357)	(14.447)		

Note: Number in parentheses is robust t statistics based on heteroscedasticity-consistent estimates for corresponding variances; Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively; The p-value of joint test for the null hypothesis $H_o : \lambda = 0, \delta = 1$.

Panel (C) Nordhaus Regression

Model (c1): $e_t = \alpha + \beta \cdot FR_{t-1} + u_t$

Model (c2): $e_t = [\alpha_0 + \beta_0 \cdot FR_{t-1}] \times (1 - \hat{s}_{t-1}) + [\alpha_1 + \beta_1 \cdot FR_{t-1}] \times \hat{s}_{t-1} + u_t$

Model	$\hat{\alpha}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\bar{R}^2$
(c1)	-0.433^{***}			-0.642^{***}			0.319
	(-3.064)			(-4.620)			
(c2)		0.050	-0.886^{***}		-0.499^{***}	-0.546^{**}	0.361
		(0.261)	(-4.299)		(-2.814)	(-2.519)	

Note: Number in parentheses is robust t statistics based on heteroscedasticity-consistent estimates for corresponding variances; Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively.

Panel (A) of Table 2 shows the errors in the DGBAS forecast are attributable to biases of under-estimation and under-reaction to news in different state of the economy. In particular, with respect to the bias that is caused by over/under-reaction to new information, insignificant $\hat{\beta}_1$ but significant $\hat{\beta}_0$ indicate that the forecaster reacts to news more accurately during accelerating periods than during decelerating ones; under-reaction to the news is thus an important source of forecast error during decelerations. With respect to the bias that is generated by over-/under-estimation, the estimate $\hat{\alpha}_1$ is significantly negative whereas $\hat{\alpha}_0$ differs insignificantly from zero. This indicates that the DGBAS under-predicts GDP growth only in accelerating GDP environments. It is also worth noting that the goodness-of-fit measure $\bar{R}^2$ is 0.440, indicating that 44.0% of the variation in

Table 2: Estimation and Hypothesis Testing Results

Panel (A) Estimation Results

Decelerating periods			Accelerating periods
(I) *Bias due to over-estimation or under-estimation*			
$\hat{\alpha}_0$	0.929	$\hat{\alpha}_1$	-1.061^{**}
	(0.402)		(-2.154)
$\hat{\gamma}^0_\alpha$	0.861	$\hat{\gamma}^1_\alpha$	0.071
	(0.394)		(0.466)
(II) *Bias due to over-reaction or under-reaction*			
$\hat{\beta}_0$	-2.913^{**}	$\hat{\beta}_1$	-0.267
	(-2.356)		(-0.862)
$\hat{\gamma}^0_\beta$	0.631^{***}	$\hat{\gamma}^1_\beta$	-0.197
	(3.165)		(-0.866)
$\bar{R}^2$	0.440		

Note: Number in parentheses is robust t statistics based on heteroscedasticity-consistent estimates for corresponding variances; Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively.

Panel (B) Hypothesis Testings

	Statistics	p-value
(I) *Hypothesis 2.1 (Optimal forecast)*	χ^2	
(a) $H_o : \alpha_0 = \alpha_1 = 0$	4.800^{*}	0.091
(b) $H_o : \beta_0 = \beta_1 = 0$	6.296^{**}	0.043
(c) $H_o : \alpha_0 = \alpha_1 = 0, \beta_0 = \beta_1 = 0$	14.446^{***}	0.006
(II) *Hypothesis 2.2 (State-independent bias)*	χ^2	
(a) $H_o : \alpha_0 = \alpha_1, \gamma^0_\alpha = \gamma^1_\alpha$	1.729	0.421
(b) $H_o : \beta_0 = \beta_1, \gamma^0_\beta = \gamma^1_\beta$	7.484^{**}	0.024
(c) $H_o : \alpha_0 = \alpha_1, \gamma^0_\alpha = \gamma^1_\alpha, \beta_0 = \beta_1, \gamma^0_\beta = \gamma^1_\beta$	8.940^{*}	0.063
(III) *Hypothesis 2.3 (Persistent bias)*	t	
(a) $H_o : \gamma^1_\alpha = 0$	0.466	0.641
(b) $H_o : \gamma^0_\beta = 0$	3.165^{***}	0.002

Note: Joint Wald test with statistics χ^2 is employed for testing Hypothesis 2.1 and 2.2, while individual test with t statistics is for Hypothesis 2.3; Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively.

forecast errors can be eliminated by considering the proposed biases.

The testing results in part (I) and (II) of Panel (B) show the DGBAS forecast is not optimal and suffers from state-dependent biases, which also suggests that the original model (3) of Nordhaus (1987) may not be suitable to official forecasts in Taiwan. Since the biases significantly revealed in Panel (A), whether they are persistent is further tested in part (III) of Panel (B). The p-values for the corresponding t statistics suggest that $\gamma^0_\beta = 0$ is significantly rejected based on

the data while $\gamma_\alpha^1 = 0$ is not. It implies that (i) the DGBAS persistently publishes an average under-estimate of GDP growth of 1.061% during accelerating periods; (ii) the DGBAS under-reacts to new information during decelerating periods and this bias reveals, on average, only for 1.585 (= 1/0.631) quarters,[8] the degree of under-reaction is deminishing with duration.

3.3.1 Bias-corrected Forecasts

Since the forecast errors of DBGAS, which are attributable to possible over-/under-estimation or over-/under-reaction of biases across the state of the economy, have been identified after estimating the model (6), the bias-corrected forecasts can now be made via the formula (7),

$$y_{t|t-1}^{bc} = y_{t|t-1}^{f} - \hat{e}_t,$$

where $y_{t|t-1}^{f}$ is the forecast made by the DGBAS at time $t-1$, and $\hat{e}_t$ is the proposed estimate of e_t made using the model (6). As mentioned, this bias-corrected forecast is particularly useful for the real-time analysis because a better forecast can be obtained immediately once a biased forecast is released.

Figure 2 plots the time-series patterns of the bias-corrected forecast $y_{t|t-1}^{bc}$, along with the realized Taiwan GDP growth and the DGBAS forecasts from 1986Q4 to 2012Q3. Roughly, bias-corrected forecast $y_{t|t-1}^{bc}$ is much closer to the realized data than the DGBAS forecast $y_{t|t-1}^{f}$, especially during decelerating periods after 2000. On average, an 89.497% of the variation of realized GDP growth can be explained by $y_{t|t-1}^{bc}$ whereas the DGBAS forecast explains only 79.857%; cf. (8).

Beyond the above (in-sample) performance of the proposed bias-corrected forecast, the effectiveness of this corrected forecast in pseudo out-of-sample experiments is now determined. Two competing classes of models are compared below. The first class contains three MZ bias-adjusted models based on modifi-

[8] Recall that γ_β^0 in exponential survivor functionals measures the fixed probability that the forecaster leaves the state of suffering from the under-reaction during decelerating periods, the expected duration of under-reaction equals $1/\gamma_\beta^0$ quarters.

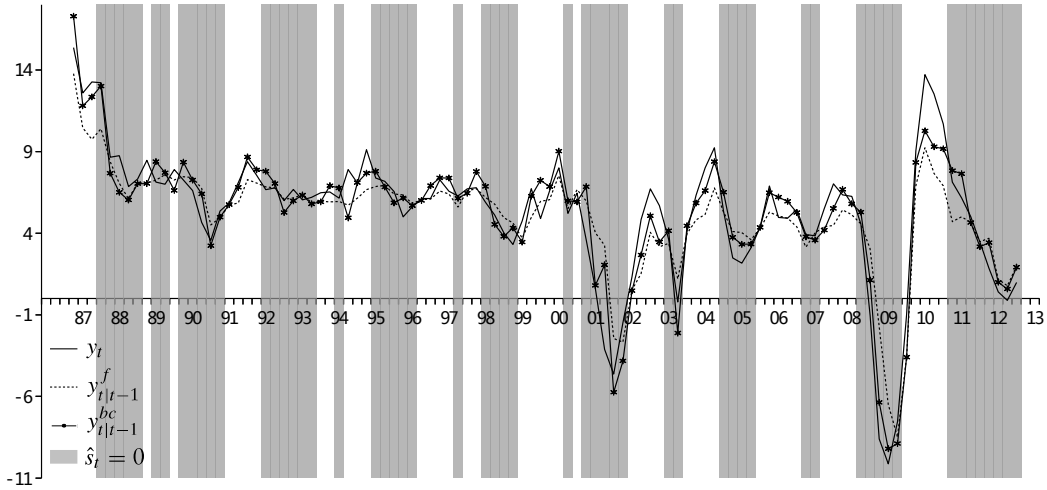


FIGURE 2: Time series plots of realized Taiwan GDP growth, DGBAS forecast and bias-corrected forecast $y^{bc}_{t|t-1}$. The shaded area represents decelerating periods where $\hat{s}_t = 0$ for all t.

cation of the work of Ang *et al.* (2007). They are

$$\text{MZ1: } y_t = \lambda + \delta y^f_{t|t-1} + \varepsilon_t;$$

$$\text{MZ2: } y_t = \lambda_{10} \times (1 - \hat{s}_{t-1}) + \lambda_{11} \times \hat{s}_{t-1} + \delta_1 y^f_{t|t-1} + \varepsilon_t;$$

$$\text{MZ3: } y_t = \left[\lambda_{20} + \delta_{20} y^f_{t|t-1}\right] \times (1 - \hat{s}_{t-1}) + \left[\lambda_{21} + \delta_{21} y^f_{t|t-1}\right] \times \hat{s}_{t-1} + \varepsilon_t,$$

where MZ1 is the conventional MZ regression, while MZ2 and MZ3 are two generalizations that depend on the proxy for the economic state $\hat{s}_{t-1}$.

The second class is composed of the linear/nonlinear autoregressive time series models. They are the typical linear autoregressive time series model:

$$\text{yAR}(p)\text{: } y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t,$$

where $1 \leq p \leq 5$, and the nonlinear exponential smooth transition autoregressive (ESTAR) model:

$$\text{yESTAR}(q)\text{: } y_t = a_0 + \sum_{j=1}^{q} \varphi_{0j} y_{t-j} + \left(a_1 + \sum_{j=1}^{q} \varphi_{1j} y_{t-j}\right) G(\gamma, \mu, y_{t-1}) + \varepsilon_t,$$

where

$$G(\gamma, \mu, y_{t-1}) = \left[1 + \exp\{-\gamma(y_{t-1} - \mu)\}\right]^{-1},$$

and $1 \leq q \leq 5$.

Fifty-one pseudo out-of-sample periods from 2000Q1 to 2012Q3 (about a half of whole sample period) are considered. Recursive estimation scheme is utilized for all regressions on the samples, beginning in 1986Q4 but ending from 1999Q4 to 2012Q2. That is, the first estimation begins with data from 1986Q4 to 1999Q4, and a one-step-ahead forecast for 2000Q1 is made based on these estimates; the second estimation and the resulting one-step-ahead forecast for 2000Q2 is based on the data from 1986Q4 to 2000Q1, and so on and so forth. Hence, the final forecast for 2012Q3 is made based on the data ends in 2012Q2. Notably, the optimal lag lengths p for yAR(p) model and q for yESTAR(q) are also recursively determined by applying Baysian information critiron (BIC) in each sample period. The performance is measured in terms of mean square error (MSE) and mean absolute error (MAE), and Diebold and Mariano's (1995) test, (the DM test herein) is implemented further to compare the performance of these six models to that of DGBAS. The null hypothesis of the DM test claims no difference between the forecast accuracy of the competing forecast with that of the DGBAS forecast, while the alternative is that the competing forecast is superior. Two test statistics, one for the squared errors difference (DM_S) and another for the absolute errors difference (DM_A) are computed.

Table 3 summarizes the results. First, DM tests do not support the hypothesis that the three MZ-type adjusted forecasts and the two autoregressive-type forecasts are significantly superior to the DGBAS forecast, even though some of their MAEs and MSEs are slightly lower than those of DGBAS. Second, the proposed bias-corrected forecast is better than those made using all other models because it has the lowest MSE and MAE. The bias-corrected forecast has a 38.67% lower MSE than the DGBAS forecast and a 16.46% lower MAE as it eliminates possible biases of over-/under-estimation and over-/under-reaction in different state of the economy. Furthermore, both DM tests significantly reject the null hypothesis in favor of the alternative hypothesis that the bias-corrected forecast is superior to the official forecast of Taiwan.

3.4 Robustness Checks for State Proxy and Long Horizon Forecasts

We also conduct robustness checks by using two alternative state proxies where 6-quarter and 8-quarter MA in (5) are adopted. In essence, the resulting inferences are the same.

On the other hand, longer horizon forecast errors are also considered in ad-

Table 3: Out-of-sample Performance Comparisons

	DGBAS	Bias-corrected model	MZ bias-adjusted models			Autoregressive models	
	$y^f_{t\|t-1}$	$y^{bc}_{t\|t-1}$	MZ1	MZ2	MZ3	yAR(p)	yESTAR(q)
MSE	5.669	3.477	5.602	4.738	5.283	5.952	7.964
MAE	1.762	1.472	1.772	1.615	1.676	1.925	2.036
DM_S		-2.454^{***}	-0.087	-1.175	-0.396	0.252	1.108
(p-value)		(0.007)	(0.465)	(0.120)	(0.346)	(0.600)	(0.866)
DM_A		-2.115^{***}	0.073	-1.009	-0.508	0.731	0.939
(p-value)		(0.017)	(0.529)	(0.156)	(0.306)	(0.768)	(0.826)

Note: The null and alternative hypotheses for DM tests are H_0 : "there is no difference in forecast accuracy of the competing forecast and DGBAS forecast" and H_1 : "the competing forecast is superior to DGBAS forecast", DM_S is the corresponding statistics calculated by square errors difference while DM_A is based on absolute errors difference. Symbols ***, **, and * denote rejection at the significance level of 1%, 5%, and 10%, respectively.

dition to the one-step-ahead forecast errors. The model for the h-step-ahead forecast is established as

$$e_{t,h} = \left[\alpha_0 \exp\left(-\gamma^0_\alpha \cdot \hat{d}^0_{t-h}\right) + \beta_0 \exp\left(-\gamma^0_\beta \cdot \hat{d}^0_{t-h}\right) FR_{t-h}\right] \times (1 - \hat{s}_{t-h})$$
$$+ \left[\alpha_1 \exp\left(-\gamma^1_\alpha \cdot \hat{d}^1_{t-h}\right) + \beta_1 \exp\left(-\gamma^1_\beta \cdot \hat{d}^1_{t-h}\right) FR_{t-h}\right] \times \hat{s}_{t-h} + u_{t,h},$$

where $e_{t,h} = y^f_{t|t-h} - y_t$ and $FR_{t-h} = y^f_{t|t-h} - y^f_{t|t-h-1}$. We analyzed the 2- and 3-step-ahead forecasts. In brief, the estimation and testing results show that the state-dependent biases in 2-step-ahead DGBAS forecast are weak while those of in 3-step-ahead DGBAS forecasts are statistically significant and persistent. Owing to space limitations, we do not report results in detailed.

4 Concluding Remarks

This paper is aimed at investigating whether the forecast is optimal given the available information when the forecast is made. Going beyond the papers that study forecast errors based on the model of Nordhaus (1987), we apply a time varying procedure to forecast revisions and further account for the possibility that the duration of the state may also affect the bias. Three testable hypotheses are presented to help researchers test the optimality of forecasts, with the ultimate goal of determining whether these biases depend on the underlying economic state and whether they are persistent with duration of the state. The corresponding bias-corrected forecasts can then be made based on these results. Briefly, this framework is novel and implementable using conventional estimation and hypothesis methods.

In the empirical part, we apply the proposed framework to investigating Taiwan DGBAS forecasts for GDP growth rates. We find that the one-quarter-ahead forecast is not optimal; it indeed suffers from state-dependent biases—persistent under-estimation bias at the relatively good state and under-reaction bias which decays with duration at the relatively bad one. Eliminating these biases in DGBAS forecast can reduce over 44.0% variation of forecast errors, and pseudo out-of-sample experiments further support the fact that the resulting bias-corrected forecast is markedly better than that made by Taiwan's government and those made using other competing models.

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