行政院國家科學委員會專題研究計畫　成果報告

分紅保單之實質存續期間

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中文摘要

既有文獻忽略了壽險保單的一個特質：淨現值在保單生命期中通常會變號。淨現值的變號會導致壽險保單的存續期間出現不尋常的值。第一，不同保單年度的保單其存續期間的正負號可能不同。在準備金大於 0 的保單年度中，存續期間是正的；但在淨現值大於 0 的年度中，存續期間會是負的。第二，壽險保單的存續期間可能很大。超大值會出現在淨現值接近 0 的時候，因為淨現值是計算存續期間的分母。

在本計畫中，我們以 Cox, Ingersoll, and Ross (1985)的利率期間結構來模擬利率，計算不同年度的保單的實質存續期間。為了檢視分紅條款與利率的隨機性如何地影響保單的存續期間，我們還計算另外三個存續期間：不分紅保單的麥考利存續期間，分紅保單的麥考利存續期間，以及隨機利率下不分紅保單的存續期間。

我們的結果和我們對保單有特殊存續期間的猜想一致。我們的結果屬於新發現，將對壽險公司的資產負債管理有實際的意涵。

關鍵詞：實質存續期間，分紅保單，壽險

Abstract

We spot a feature of life insurance policies not recognized by the above papers: net present value (NPV) usually incurs sign changes during the policy life. The feature of sign-changing NPV would result in unusual duration figures for life insurance policies. First, the durations of life insurance policies in different policy years might have different signs. Policies in the policy years when reserves are positive have positive durations, but policies in the policy years when NPVs are positive would have negative durations. Second, life insurance policies could have enormous durations. These durations emerge at the times when NPV is close to zero, since NPV is a denominator in effective duration.

In this project we calculate the effective durations of participating policies in different policy years, using a Cox, Ingersoll, Roll (1985) term structure (abbreviated as EDP) to simulate interest rate. We then analyze how participation and the randomness of interest rate affect policy duration by calculating three additional duration measures: the modified duration of a non-participating policy, the modified duration of a participating policy, and the effective duration of a non-participating policy.

Our results confirm our speculations about the unusual effective durations for life insurance policies. They are new to the literature and would be useful in the asset-liability management of a life insurance company.

Keywords: effective duration, participating policy, life insurance

Life insurance businesses are subject to significant interest rate risk because the contracts usually last for long periods and have guaranteed minimal credit rates. High leverage ratios of life insurance companies aggravate the threat from interest rate risk. Small changes in interest rates could cause large losses in life insurers' capital and surplus. Managing interest rate risk is therefore essential to life insurance companies.

Duration is a useful tool in managing interest rate risk. Interest rate risk can be managed by appropriately matching the durations of asset and liability. Life insurers should therefore be aware of their asset and liability durations. Since a life insurer's liability mainly comprises reserves of sold policies and a portfolio's duration is a weighted average of the components' durations, durations of individual policies are building blocks for a life insurer to manage interest rate risk.

Few papers investigate policy duration. Babbel (1995) estimates the effective durations for several types of life insurance policies using the PTS Shane Chalke software. Santomero and Babbel (1997) list the effective durations of several life insurance products in the paper about their on-site investigation into the risk management of the insurance industry. Briys and Varenne (1997; 2001, chapter 5) calculate the effective duration of a single-premium participating contract with a guaranteed minimal credit rate. Compared with the literature about the durations of major assets held by life insurance companies, e.g., Boquist, Racette, and Schlarbaum (1975), Bierwag, Kaufman, and Toevs (1983), Bierwag (1987), Bierwag, Corrado, and Kaufman (1992), and Babbel, Merrill, and Panning (1997), studies about policy duration are sparse.

We spot a feature of life insurance policies not recognized in the above papers: net present value (NPV) usually incurs sign changes during the policy life. The NPV of a life insurance policy is defined as the difference between the present value of expected cash inflows (premiums paid by the policyholder in the future) and that of expected cash outflows (e.g., death/survival benefits, surrender payments, interest rate dividends, commissions, and expenses)[1]. Selling a life insurance policy is a positive NPV project to the insurer if the pricing is correct. As time goes by, expected premium income decreases while expected benefit payment increases. The NPV of a life insurance policy thus decreases with time and turns from positive to negative at some point of time during the policy life.

The feature of sign-changing NPV would result in unusual durations for life insurance

[1] Notice that NPV and reserve have the same magnitude but have opposite signs. While NPV is the difference between the present value of future cash inflows and that of future cash outflows, policy reserve equals the present value of expected cash outflows minus the present value of expected inflows.

policies. First, the durations for policies in different policy years might have opposite signs. Policies in the policy years when reserves are positive have positive durations, but policies in the policy years when NPVs are positive would have negative durations[2]. Second, life insurance policies could have enormous duration figures. Huge figures emerge at the times when NPV is close to zero since policy NPV is a denominator in duration calculation.

In this project, we conduct simulation to calculate the effective durations for participating level-premium life insurance policies that are in different policy years to confirm the above speculations. We pick level-premium policies not only because they are more common than single-premium ones but also because their NPVs usually remain positive during the first several policy years. The NPV of a single-premium policy becomes negative right after receiving the premium, which hinders us from observing the effect of sign-changing NPV on duration. Participating policies are chosen because participating makes the cash flows of life insurance policies sensitive to interest rate and the interest rate sensitivities of cash flows are imperative determinants to duration[3]. We use a Cox, Ingersoll, Roll (1985) term structure model to capture the randomness of interest rate. The calculations for the effective durations of participating policies differentiate this project from Tsai, Ho, and Tsou (2002). Analyzing policies in different policy years to unveil the effect decreasing/increasing NPV/reserve on effective duration distinguishes the current study from the previous papers about the effective duration of the life insurance policy.

We compare four durations to analyze the impacts of randomizing interest rate and participating on duration. The simplest one is the modified duration of a non-participating policy (abbreviated as MDNP) whereas the most comprehensive one is the effective duration of a participating policy (abbreviated as EDP)[4]. Two other durations are the modified duration of a participating policy (MDP) and the effective duration of a non-participating policy (EDNP). The comparison of MDNP vs. MDP and that of EDNP vs. EDP would reveal how participating affects duration. The comparisons of MDP vs. EDP and MDNP vs. EDNP would demonstrate the effect of randomizing interest rate on duration.

We summarize all our results in the following four tables.

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[2] Positive NPV is equivalent to negative reserve. Policies profitable to life insurers indeed have negative reserves, even though negative reserves are not admitted in accounting standards and regulation.

[3] Policyholders may participate in insurers' favorable experiences in mortality rate, investment earnings, expense ratios, etc. We focus on the participation in the spread between market interest rate and credit rate in this project.

[4] The major difference between effective duration and modified duration is that the former one employs stochastic interest rate while the latter one assumes deterministic interest rate.

Table 1: The modified duration of a non-participating policy vs. that of a participating policy

	Deterministic r without dividend			Deterministic r with dividend		
Policy Year	2%	4%	8%	2%	4%	8%
1	82.94	(162.20)	(10.86)	82.94	(74.68)	12.74
2	51.22	183.20	(18.50)	51.22	85.15	(5.65)
3	37.54	63.30	(37.80)	37.54	29.59	(2.32)
4	29.82	39.66	(191.65)	29.82	18.66	(1.43)
5	24.97	29.76	90.05	24.97	14.09	(1.02)
6	21.49	24.07	40.46	21.49	11.46	(0.77)
7	18.72	20.14	26.86	18.72	9.64	(0.59)
8	16.42	17.19	20.29	16.42	8.27	(0.47)
9	14.55	14.96	16.53	14.55	7.23	(0.37)
10	12.95	13.18	14.03	12.95	6.39	(0.29)
11	11.32	11.38	11.68	11.32	5.54	(0.23)
12	9.86	9.83	9.85	9.86	4.80	(0.17)
13	8.53	8.46	8.35	8.53	4.15	(0.13)
14	7.30	7.21	7.05	7.30	3.55	(0.09)
15	6.15	6.05	5.88	6.15	2.99	(0.07)
16	5.05	4.96	4.80	5.05	2.46	(0.04)
17	4.00	3.92	3.79	4.00	1.95	(0.03)
18	2.97	2.92	2.81	2.97	1.45	(0.02)
19	1.97	1.93	1.86	1.97	0.96	(0.01)
20	0.98	0.96	0.93	0.98	0.48	(0.00)

Table 2: The modified duration of a non-participating policy vs. the effective duration of a non-participating policy

	Deterministic r without dividend			Stochastic r without dividend		
Policy Year	2%	4%	8%	2%	4%	8%
1	82.94	(162.20)	(10.86)	6.39	(7.18)	1.20
2	51.22	183.20	(18.50)	4.89	11.33	0.64
3	37.54	63.30	(37.80)	4.26	5.63	(0.89)
4	29.82	39.66	(191.65)	3.92	4.44	(19.01)
5	24.97	29.76	90.05	3.72	3.97	7.67
6	21.49	24.07	40.46	3.60	3.72	4.90
7	18.72	20.14	26.86	3.50	3.55	4.07
8	16.42	17.19	20.29	3.42	3.42	3.66
9	14.55	14.96	16.53	3.37	3.34	3.46
10	12.95	13.18	14.03	3.34	3.30	3.36
11	11.32	11.38	11.68	3.27	3.21	3.20
12	9.86	9.83	9.85	3.19	3.12	3.08
13	8.53	8.46	8.35	3.11	3.04	2.97
14	7.30	7.21	7.05	3.01	2.93	2.85
15	6.15	6.05	5.88	2.87	2.80	2.71
16	5.05	4.96	4.80	2.69	2.63	2.54
17	4.00	3.92	3.79	2.46	2.40	2.32
18	2.97	2.92	2.81	2.13	2.08	2.01
19	1.97	1.93	1.86	1.66	1.62	1.57
20	0.98	0.96	0.93	0.98	0.96	0.93

Table 3: The modified duration of a participating policy vs. the effective duration of a participating policy

	Deterministic r with dividend			Stochastic r with dividend		
Policy Year	2%	4%	8%	2%	4%	8%
1	82.94	(74.68)	12.74	5.30	9.39	(2.66)
2	51.22	85.15	(5.65)	4.28	4.21	3.36
3	37.54	29.59	(2.32)	3.85	3.56	2.86
4	29.82	18.66	(1.43)	3.62	3.35	2.81
5	24.97	14.09	(1.02)	3.51	3.28	2.83
6	21.49	11.46	(0.77)	3.44	3.25	2.85
7	18.72	9.64	(0.59)	3.39	3.22	2.86
8	16.42	8.27	(0.47)	3.34	3.18	2.84
9	14.55	7.23	(0.37)	3.31	3.16	2.84
10	12.95	6.39	(0.29)	3.29	3.15	2.84
11	11.32	5.54	(0.23)	3.23	3.09	2.80
12	9.86	4.80	(0.17)	3.17	3.03	2.75
13	8.53	4.15	(0.13)	3.09	2.96	2.70
14	7.30	3.55	(0.09)	2.99	2.87	2.63
15	6.15	2.99	(0.07)	2.86	2.76	2.54
16	5.05	2.46	(0.04)	2.69	2.60	2.40
17	4.00	1.95	(0.03)	2.45	2.38	2.22
18	2.97	1.45	(0.02)	2.12	2.06	1.94
19	1.97	0.96	(0.01)	1.66	1.62	1.54
20	0.98	0.48	(0.00)	0.98	0.96	0.93

Table 4: The effective duration of a non-participating policy vs. that of a participating policy

	Stochastic r without dividend			Stochastic r with dividend		
Policy Year	2%	4%	8%	2%	4%	8%
1	6.39	(7.18)	1.20	5.30	9.39	(2.66)
2	4.89	11.33	0.64	4.28	4.21	3.36
3	4.26	5.63	(0.89)	3.85	3.56	2.86
4	3.92	4.44	(19.01)	3.62	3.35	2.81
5	3.72	3.97	7.67	3.51	3.28	2.83
6	3.60	3.72	4.90	3.44	3.25	2.85
7	3.50	3.55	4.07	3.39	3.22	2.86
8	3.42	3.42	3.66	3.34	3.18	2.84
9	3.37	3.34	3.46	3.31	3.16	2.84
10	3.34	3.30	3.36	3.29	3.15	2.84
11	3.27	3.21	3.20	3.23	3.09	2.80
12	3.19	3.12	3.08	3.17	3.03	2.75
13	3.11	3.04	2.97	3.09	2.96	2.70
14	3.01	2.93	2.85	2.99	2.87	2.63
15	2.87	2.80	2.71	2.86	2.76	2.54
16	2.69	2.63	2.54	2.69	2.60	2.40
17	2.46	2.40	2.32	2.45	2.38	2.22
18	2.13	2.08	2.01	2.12	2.06	1.94
19	1.66	1.62	1.57	1.66	1.62	1.54
20	0.98	0.96	0.93	0.98	0.96	0.93

The above results confirm our speculations about the unusual effective durations for life insurance policies. They are new to the literature and would be useful in the asset-liability management of a life insurance company.

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