行政院國家科學委員會補助專題研究計畫成果報告
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※　在預算限制下網路頻寬的最佳化過程(I)　※
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行政院國家科學委員會專題研究計畫成果報告
在預算限制下網路頻寬的最佳化過程(I)
Network Bandwidth Optimization Under Budget Constraints

計畫編號: NSC 96-2221-E-004-001
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Abstract. We present an integer programming approach to offer optimal bandwidth allocation and QoS routing for network management problems under budget constraints. The bandwidth allocation policy in such communication networks with different traffic classes is determined in accordance with the proportionally fair rule, which can be expressed by utility functions of network users. The objective of this utility maximization scheme is to prepare a routing table for future user request. This routing table identifies the allocated bandwidth and the suitable end-to-end path under QoS guarantees for each user.

1 Introduction

Because of rapid growth of Internet traffic, aggressive deployment of broadband fiber optic network, advance of Voice over IP (VoIP) technology, and the global standardization of IP technology, the telecommunication industry is moving toward a converged network, which uses a single global IP based packet switching network to carry all types of network traffics, to replace the traditional separated packet switching and circuit switching networks. The international telecommunication standard organizations have decided to adopt this new All-IP network as the base transport network for future development. A backbone network (core network) realizes the connections between local networks, to which most of the traffic generators are connected. Traffic from different users in a local network is concentrated by a switch which connects to more distant places, through the backbone.

While several Internet Service Providers (ISPs) have proposed their architecture and detailed specifications to support multiple heterogeneous applications in next generation network [1], [2], [4], they all have to deal with a critical issue that how to schedule traffic and allocate bandwidth for different traffic classes at both backbone networks and access links. The rules for bandwidth allocation in networks carrying elastic traffic can be defined in several ways, meeting different overall network goals. In Internet, it is desired to avoid poor users, meaning users possessing an unfairly small share of bandwidth. The available bandwidth can be divided between the users according to their needs. Therefore, a problem of network dimensioning with elastic traffic requires to allocate bandwidth to maximize flows fairly [5].

We focus on allocating resources with proportional fairness [3] and finding a Quality of Service (QoS) routing scheme on communication networks. An approach is presented for the fair resource allocation problem and QoS routing in networks with multiple classes, offering multiple services to users. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the sum of the users' utility. The main function of QoS routing is to find appropriate paths between a source and destination node satisfying one or more QoS constraints.

2 Problem Definition

The most convenient way of representing a communication network is by the notions provided by graph theory. Specifically, a network formed by nodes and links is modelled by a graph of vertices and edges. A link is regarded as a connection between the two nodes to which it is attached. In the general setting, a link facilitates transfer of various entities, that is, it may carry flows associated with several demands simultaneously. We consider all links to be directed. Furthermore, each link is usually assigned a capacity, limiting the amount of flow that it can carry.

Consider a directed network topology $G = (V, E)$, where V and E denote the set of nodes and the set of links in the network respectively. We assume that there are m different service classes in the network. Let $I = \{1, \ldots, m\}$ be an index set consisting of m different service classes. For each class i, the specific QoS requirements include maximal end-to-end delay constraint D_i and minimal bandwidth requirement b_i, below which gains no utility at all. We denote the total number of users, for each class i, by K_i. Let J_i, for each class i, be an index set consists of K_i users, that is, $J_i = \{1, \ldots, K_i\}$. Every user of the same class is allocated the same bandwidth and has the same QoS requirement, which imply that bandwidth is allocated such that each user is offered with equalized utility to guarantee fairness. The parameters K_i are usually uncertain at the beginning of planning periods. We solve this in the parameters by their best point estimator, i.e., expected values. Here, we assume the expected number K_i is known for the discussion under this precomputation-based scheme.

Let $E_o \subseteq E$ and $E_d \subseteq E$ be subsets of links connected with the source o and destination d respectively. All users are delivered between the same source o and destination d in the core network. We denote $E_\nu^{in} \subseteq E$ a subset of incoming links to the node $\nu \in V \setminus \{o, d\}$, and we also denote $E_\nu^{out} \subseteq E$ a subset of outgoing links from the node $\nu \in V \setminus \{o, d\}$. The maximal possible link capacity is U_e on each link $e \in E$. Suppose, for each link e, we have a mean delay ℓ_e related to the link's speed, propagation delay, and maximal transfer unit. We also have the link cost κ_e for using one unit bandwidth. In general, link cost κ_e is inversely proportional to mean delay ℓ_e.

Let $\chi_{i,j}(e)$ denote the binary variable which determines whether the link e is chosen for user j in class i. The decision variable θ_i is the bandwidth allocated to each user in class i. We use $A_{i,j}(e)$ to represent the bandwidth allocated to link $e \in E$ for user j in class i.

A user j in each class i should be routed through specific path $p_{i,j}$ between o and d. Under a limited available budget B, we plan to allocate the bandwidth in order to provide each class with maximal possible QoS and determine the optimal path from end to end under guaranteed service. The purpose of this work is to show that a methodology that allows the decision maker to explore a set of solutions could satisfy preferences with fairness, and choose the solution optimally.

The following definitions given by us will be used throughout this paper.

Definition 1 *A **feasible path** $p_{i,j}$ between o and d, for a user j of class i, is defined as a (routing) path from o to d such that $D(p_{i,j}) \leq D_i$ and each link along $p_{i,j}$ satisfies the capacity constraint.*

Definition 2 *A feasible path $p_{i,j}$ is called a **Pareto optimal path**, for a user j in class i, if no other feasible path is as less as $p_{i,j}$ with respect to two evaluation, path cost and ene-to-end delay, and strictly less than $p_{i,j}$ with respect to at least one evaluation.*

Definition 3 *The set of all Pareto optimal paths is called the **routing tables** P. That is, $P = \{p_{i,j} | \; p_{i,j}$ is the Pareto optimal path from o to d, $\forall\, j \in J_i,\; i \in I\}$.*

Definition 4 *A link e is called **bottleneck link** if the usage of bandwidth achieves its link capacity, that is, $\sum_{i \in I} \sum_{j \in J_i} A_{i,j}(e) = U_e$.*

3 Utility Function

Traditional TCP elastic users, including those relying FTP or P2P to download files, implements traditional TCP protocol with built-in congestion control mechanisms. The utility function of these users features an increasing, strictly concave and continuously differentiable curve, which has decreasing marginal increment as bandwidth increases [6]. Kelly et al. [3] advocated proportional fairness characterized by $\log(\theta_i)$, which is often used to quantify the utility functions of TCP elastic users. TCP interactive user mainly includes web users and Telnet users who concern packet delays. When web users are surfing in Internet, he/she may be impatient for waiting long time before retrieving information. The utility function of TCP interactive user has a minimum tolerable bandwidth, below which the utility drops directly to zero.

An allocation policy can be expressed in terms of a utility function, as a function of allocated bandwidth, in the sense that the desired bandwidth allocation maximizes aggregate utility subject to constraints. In this work, we assume the relationship between the utility and the bandwidth is linear in a proper (sufficiently small) region. Depending on the specified aspiration and reservation levels for each class i, a_i and r_i, respectively, we construct our utility function $f_i(\theta_i)$ of bandwidth θ_i as a piecewise linear function. Between r_i and a_i, we have break points $r_i = k_{i,0} < k_{i,1} < \ldots < k_{i,n-1} < k_{i,n} = a_i$. We define $f_i(\cdot)$ over the range $[0, M_i]$, where M_i is the upper bound of bandwidth θ_i. Depending on the specified reference levels, this utility function can be interpreted as a measure of the decision maker's satisfaction with the value of the i-th criteria [7]. It is a strictly increasing function of θ_i, having value 1 if $\theta_i = a_i$, and value 0 if $\theta_i = r_i$. Denote $\alpha_i = \frac{a_i}{r_i}$, we have parameters $\mu_i(M_i) = \log_{\alpha_i} M_i/r_i$ and $\mu_i(k_{i,l}) = \log_{\alpha_i} k_{i,l}/r_i$ for $l = 1, \ldots, n-1$. Moreover, the parameters $\rho_0 = M_i/(r_i - b_i)$, $\rho_M = (\mu_i(M_i) - 1)/(M_i - a_i)$ and

$$\rho_l = \frac{n \log_{\alpha_i}(k_{i,l}/k_{i,l-1})}{a_i - r_i},$$

represent a slope on the l-th line segment for $l = 0, 1, \ldots, n$. For $k_{i,l-1} \le \theta_i < k_{i,l}$, we define

$$f_i(\theta_i) = \rho_l(\theta_i - k_{i,l}) + \mu_i(k_{i,l}) \tag{1}$$

For $0 \le \theta_i < b_i$, we define $f_i(\theta_i) = -M_i$, and we define $f_i(\theta_i) = \rho_M(\theta_i - M_i) + \mu_i(M_i)$ whenever $a_i \le \theta_i \le M_i$.

Proposition 5 *The utility function (1) is continuous, increasing, and concave.*

4 Mixed-Integer Linear Programming Model

By using the utility function (1), the utility function for each class i is expressed. Our goal is to maximize the total utility of all competing classes. The mixed-integer linear programming (MILP) is formulated as follows:
Maximize

$$\sum_{i \in I} w_i f_i(\theta_i) \tag{2}$$

subject to

$$\sum_{e \in E} \sum_{i \in I} \sum_{j \in J_i} \kappa_e A_{i,j}(e) \le B \tag{3}$$

$$\sum_{i \in I} \sum_{j \in J_i} A_{i,j}(e) \le U_e, \ \forall\, e \in E \tag{4}$$

$$\sum_{e \in E} \ell_e \chi_{i,j}(e) \le D_i, \ \forall\, j \in J_i, \ i \in I \tag{5}$$

$$A_{i,j}(e) \le M \chi_{i,j}(e), \ \forall\, e \in E, \ j \in J_i, \ i \in I \tag{6}$$

$$\theta_i - A_{i,j}(e) \le M(1 - \chi_{i,j}(e)), \ \forall\, e \in E, \ j \in J_i, \ i \in I \tag{7}$$

$$A_{i,j}(e) - \theta_i \le M(1 - \chi_{i,j}(e)), \ \forall\, e \in E, \ j \in J_i, \ i \in I \tag{8}$$

$$\theta_i \ge b_i, \ \forall\, i \in I \tag{9}$$

$$\sum_{e \in E_o} A_{i,j}(e) = \theta_i, \ \forall\, j \in J_i, \ i \in I \tag{10}$$

$$\sum_{e \in E_\nu^{in}} A_{i,j}(e) = \sum_{e \in E_\nu^{out}} A_{i,j}(e), \ \forall\, \nu \in V, \ j \in J_i, \ i \in I \tag{11}$$

$$\sum_{e \in E_d} A_{i,j}(e) = \theta_i, \ \forall\, j \in J_i, \ i \in I \tag{12}$$

$$A_{i,j}(e) \ge 0, \ \forall\, e \in E, \ j \in J_i, \ i \in I \tag{13}$$

$$\theta_i \ge 0, \ \forall\, i \in I \tag{14}$$

$$\chi_{i,j}(e) = 0 \text{ or } 1, \ \forall\, e \in E, \ j \in J_i, \ i \in I, \tag{15}$$

where $w_i \in (0, 1)$ is the weight assigned to each class i, $\sum_{i \in I} w_i = 1$, and $M = \sum_{i \in I} w_i M_i$ is a constant.

Theorem 6 *The maximization model, MILP, is bounded.*

Theorem 7 *The maximization model, MILP is NP-hard.*

5 Analysis of Model Solutions

Given a limited available budget B, we determine the optimal bandwidth allocation $A^*_{i,j}(e)$, θ^*_i and the optimal choices of links $\chi^*_{i,j}(e)$ by solving the maximization model MILP. The optimal bandwidth θ^*_i allocated to each class i, is unique and it can provide the proportional fairness to each class. Moreover, we also attain the maximal bandwidth $R_{i,e}$ by which the link e can offer for class i, i.e.,

$$R_{i,e} = \sum_{j \in J_i} A^*_{i,j}(e). \tag{16}$$

Proposition 8 *A link e is the bottleneck link if $\sum_{i \in I} R_{i,e} = U_e$.*

After applying the optimization model, we obtain a network $G = (V, E')$, where V is the original set of nodes and $E' \subseteq E$ is the subset of links belonging to each end-to-end path $p \in P$. Bandwidth are allocated along less expensive paths that connect the origin o and the destination d. From the optimization of these precomputation schemes, we obtain the Pareto optimal bandwidth allocation and introduce a routing table with end-to-end QoS guarantees.

Proposition 9 *If $p_{i,j} = \{e \in E |\ \chi^*_{i,j}(e) = 1\}$ for user j in class i, then path $p_{i,j}$ is the Pareto optimal path from the source o to the destination d.*

Proposition 10 *The Pareto optimal path $p_{i,j}$ is unique for each user j in class i.*

Proposition 11 *The end-to-end delay on the optimal path $p_{i,j}$ is*

$$D(p_{i,j}) = \sum_{e \in E} \ell_e \chi^*_{i,j}(e). \tag{17}$$

The optimal path $p_{i,j}$ between o and d is feasible for a user j in class i, that is, $D(p_{i,j}) \leq D_i$.

Proposition 12 *The end-to-end unit cost for bandwidth on the optimal path $p_{i,j}$ is*

$$\sum_{e \in p_{i,j}} \kappa_e \chi^*_{i,j}(e)$$

for user j in class i.

Proposition 13 *If link e belongs to the optimal path $p_{i,j}$, then the bandwidth by which the link e can offer for user j in class i is the same. That is, $A^*_{i,j}(e) = A^*_{i,j}(e')$ for all $e,\ e' \in p_{i,j}$.*

Proposition 14 *Let $\theta_{i,p} \geq 0$, for each class i, be the bandwidth allocated to each optimal path $p \in P$. Then we have*

$$\sum_{p \in P} \theta_{i,p} = K_i \theta^*_i \tag{18}$$

and

$$0 \leq \sum_{i \in I} \theta_{i,p} \leq \min_{e \in p} U_e. \tag{19}$$

Next, we study the sensitivity to the maximal number K_i of users for each class i. Let

$$c_i = \frac{\sum_{j \in J_i} \sum_{e \in E} \kappa_e A^*_{i,j}(e)}{K_i} \tag{20}$$

be a mean budget allocated to each user in class i. If π_i denotes the reserved budget for each class i, then the budget constraint (3) may be represented as

$$\sum_{i \in I} (K_i c_i + \pi_i) = B, \tag{21}$$

where $\pi_i \geq 0$, $\forall\ i \in I$.

Definition 15 *The ratio $\sum_{j=1}^{K_i} \sum_{e \in E} \kappa_e A^*_{i,j}(e)/B$ is called a* ***budget ratio*** *that is allocated to class i.*

Each class is given a percentage, budget ratio, of the total budget B. It must further be noted that the budget ratio can be computed from $c_i K_i / B$.

Proposition 16 *Suppose $\pi_i = 0$ for some class i. If the shadow price of (21) is ς, then the impact on the value of (2) is decreased by $\epsilon \varsigma \sum_{i=1}^{m} c_i$ as K_i is increased by ϵ.*

The results show a striking effect of the number of users on budget allocation. If there is no reservation for class i, it will decrease the total satisfaction level by snatching others' bandwidth. If the number, K_i, of users in some class i increases when the others are fixed, then the mean budget c_i will decrease. However, the product $K_i c_i$ may increase or decrease depending on the budget and other factors. To evaluate the effect of K_i, we check it by the budget ratio.

Theorem 17 *Let c_p be the unit path cost along the Pareto optimal path $p \in P$, i.e., $c_p = \sum_{e \in p} \kappa_e$. If the budget B satisfies*

$$B \geq \sum_{i \in I} \pi_i + \sum_{p \in P} c_p \min_{e \in p} U_e, \qquad (22)$$

then there exists one Pareto optimal path p which contains at least one bottleneck link. Moreover, link e is the bottleneck link if $U_e = \sum_{p \ni e} \sum_{i \in I} \theta_{i,p}$.

Theorem 18 *The routing table P includes the path of minimal cost from the source node to the destination node on the network.*

6 Conclusions

We present an approach for the fair bandwidth allocation and QoS routing in communication networks. This utility maximization scheme determines end-to-end paths with QoS guarantees under the network constraints. Solving the maximization model, we can find the optimal bandwidth allocation under a limited budget, and this allocation can provide the so-called proportional fairness for every traffic class. The proposed utility maximization scheme can be implemented in both the core and the edge routers serving heterogenous services. This specially designed traffic scheduler can be equipped in both the input and the output port of the router to function as the traffic-class sub-scheduler in its multi-tiered packet scheduler.

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