行政院國家科學委員會補助專題研究計畫期中報告

※※※※※※※※※※※※※※※※※※※※※※※
※　於預算控制下計算M/G/K/K和GI/M/K/K　※
※　　　　　　　　的滿載機率　　　　　　　　※
※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：□v 個別型計畫　　□整合型計畫
計畫編號：NSC 98-2221-E-004-001
執行期間：　98 年 8 月 1 日至　100 年 7 月 31 日

計畫主持人：陸行

共同主持人：

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位：國立政治大學應用數學系

中　華　民　國　99　年　6　月　30　日

行政院國家科學委員會專題研究計畫期中報告

於預算控制下計算M/G/K/K和GI/M/K/K的滿載機率

Blocking Probability of $M/G/K/K$ and $GI/M/K/K$ under Budget Constraints

計畫編號: **NSC 98-2221-E-004-001**
執行期限: 98年08月01日至100年07月31日

Abstract. We address the issue of bandwidth allocation on end-to-end communication networks with multi-class traffic, where bandwidth is determined optimally under the budget and network constraints. We derive the blocking probabilities with respect to bandwidth, traffic demand and the available number of end-to-end paths based on Erlang loss formula for all service classes. Depending upon the blocking probability, the project presents different performance metrics, such as budget ratio, utilization level and bandwidth elasticity of blocking. Monotonicity and convexity of blocking probabilities with allocated bandwidth, traffic demand and the number of end-to-end paths are also discussed.

1 Introduction

For a communication network providing performance guarantees, it has to reserve resources and exercise call admission control [43]. Network users are mainly interested in obtaining good quality connections whenever they place requests. It is the network providers' mission to have an end-to-end path with suitable bandwidth. Clearly, it is too costly for the network providers to have a 100% guaranteed availability for all connections under the budget constraint at any time. This is also not necessary since demand for connections or bandwidth capacity varies over time. Traffic flow fluctuates with time, and connections do not last forever but occur at random times and vanish in the network once the corresponding digital document has been transferred completely. This results in a random dynamic set of active connections. Moreover, the bandwidth assigned to each connection would determine how long that connection will stay active and thus impacts the evolution of the set of active connections. The network chooses an optimal sharing scheme for the different users under the total budget to fulfill connection requirements. In addition, the risk (probability) of rejecting connection requests due to lack of resources is supposedly kept below a negotiated level.

In this work, we aim to analyze the relationship among blocking probability, bandwidth, traffic demand and the available number of end-to-end paths on communication networks with service from ISPs, where requests for connections represent customers arriving at the system. As soon as requests are accepted by the system, the service begins. The installed bandwidth allocation is used to maintain a guaranteed connection availability where the blocking probability is kept below certain negotiated levels. Our intention is to analyze the sensitivity of the blocking probability with respect to these system parameters, where the parameters for the system change one at a time.

We derive the relationship between the blocking probability and allocated bandwidth under the budget constraint, which has received relatively little attention in the literature. The blocking probability of connections for each QoS class is formulated as a function of allocated bandwidth, traffic demand and the available number of end-to-end paths. Monotone and convex properties of the blocking probability are shown in both theoretical construction and numerical examples. The results of this work can be helpful in the operational processes involved in the efficient set-up and usage of a core network under the budget constraint, e.g., network design and provisioning purposes.

The closed-form expression of the blocking probability in terms of bandwidth can be used to investigate the optimal buffer size in capacitated communication systems so that the blocking probability is kept below a specific threshold [35]. One application of the relationship between blocking probability and bandwidth allocation may be referred to designing network pricing mechanisms for sharing bandwidth in terms of blocking/congestion costs, whose examples were given by Yacoubi et al. [43] and Anderson et al. [3], etc. Another application of this work is used to consider the admission control in end-to-end networks under different bandwidth sharing policies including throughput maximization, max-min fairness, proportional fairness and balanced fairness, etc. Interested readers may refer to Egorova et al. [10], Bonald et al. [7], Nilsson and Pióro [29], etc.

2 Problem definitions

Consider a directed network topology $G = (N, A)$, where N and A denote the set of nodes and the set of links in the network respectively. All connections are delivered through G from the source node o to the destination node d. There are m different Quality of Service (QoS) classes in this core network G [4], and $M = \{1, 2, \ldots, m\}$ denotes an index set consisting of m QoS classes.

We assume connections of class i occur at the source node o in accordance with independent Poisson processes at rate $\lambda_i(t)$ at period t, but the connection volume to be transmitted has an arbitrary distribution with mean $\sigma_i(t)$ [40]. At period t, we intend to allocate the bandwidth under a limited budget $B(t)$ in order to provide each class with maximal possible QoS. The number of virtual paths of class $i \in M$ is denoted by $s_i(t)$. Every virtual path of class $i \in M$ is allocated the same amount of bandwidth $x_i(t)$ at period t.

For each class $i \in M$, the mean sojourn time $1/\mu_i(t)$ of connections on virtual paths corresponds to the packet transmission time, and it is equal to average connection volume divided by bandwidth, i.e.,

$$\frac{1}{\mu_i(t)} = \frac{\sigma_i(t)}{x_i(t)}. \tag{1}$$

Suppose that connections occupy the virtual paths in the order they occur and that sojourn times are identically distributed and mutually independent.

In this article, we investigate the relationships between performance measures of interest and model parameters at period t, which is similar at other periods. To simplify the notation, we skip the notation (t), and the derivation is conducted in general format. The following definitions are given and will be used throughout the whole context of this article.

Definition 1 *The* ***traffic demand*** y_i *for class* $i \in M$ *is defined as the product of the mean occurrence rate* λ_i *and the average connection volume* σ_i*, i.e.,*

$$y_i = \lambda_i \sigma_i. \tag{2}$$

This communication system is analyzed as an Erlang loss model under assumptions of Poisson arrivals, general sojourn time, preset s_i virtual paths with identical bandwidth x_i, and no waiting space [28], [40]. For a traffic class $i \in M$, we derive the steady-state occupancy probabilities of n $(0 \leq n \leq s_i)$ connections, P_n. The unique steady-state probability exists for this stable system [40]. Hence, we have

$$P_n = \frac{P_0}{n!} \left(\frac{\lambda_i \sigma_i}{x_i} \right)^n, \ n = 1, 2, \ldots, s_i,$$

where λ_i is the mean occurrence rate of connections, σ_i is the average connection volume, x_i is the bandwidth allocation and s_i is the preset number of virtual paths. Solving for P_0 in the equation $\sum_{n=0}^{s_i} P_n = 1$, we can obtain P_0 and P_n for $n = 1, 2, \ldots, s_i$. Thus, the blocking probability of incoming connections is formulated as

$$P(x_i, s_i, y_i) = \frac{(y_i/x_i)^{s_i}}{s_i!} \left[\sum_{n=0}^{s_i} \frac{(y_i/x_i)^n}{n!} \right]^{-1}, \tag{3}$$

where x_i is the allocated bandwidth, s_i is the preset number of virtual paths in the off-line optimization, and $y_i = \lambda_i \sigma_i$ is the traffic demand from on-line traffic flow. Moreover, the expected path occupancy in the steady state is

$$L(x_i, s_i, y_i) = \sum_{n=1}^{s_i} \frac{(y_i/x_i)^n}{(n-1)!} \left[\sum_{j=0}^{s_i} \frac{(y_i/x_i)^j}{j!} \right]^{-1}. \quad (4)$$

Note that $L(x_i, s_i, y_i) = (y_i/x_i)(1 - P(x_i, s_i, y_i))$. The average throughput for class $i \in M$ can be determined by $x_i L(x_i, s_i, y_i)$.

In real world cases, the numbers of connections (or users) on networks are always huge, i.e., $s_i \gg 0$. If the traffic intensity $\rho_i = y_i/s_i x_i < 1$, equation (3) can be rewritten as

$$P(x_i, s_i, y_i) \approx \frac{(y_i/x_i)^{s_i} e^{-y_i/x_i}}{s_i!}, \text{ as } s_i \to \infty. \quad (5)$$

Moreover, we can conclude that

$$L(x_i, s_i, y_i) \approx \frac{y_i}{x_i} \left(1 - \frac{(y_i/x_i)^{s_i} e^{-y_i/x_i}}{s_i!} \right), \quad (6)$$

as $s_i \to \infty$.

3 Network management schemes

Network managers may wish to maximize the average revenue of the system [26] when regulating the bandwidth allocation $\vec{x} = (x_1, \ldots, x_m)$ and the number of virtual paths $\vec{s} = (s_1, \ldots, s_m)$. Given traffic demand y_i for class $i \in M$, network managers would like to determine the values of $\vec{x}$ and $\vec{s}$ to optimal the system. As far as QoS is concerned, bandwidth allocation x_i and blocking probability $P(x_i, s_i, y_i)$ are the key elements of the network revenue management scheme [7], [10], [12], [14], [17], etc. The operating costs can be determined by the type of traffic transmitted (data, voice, video) and the QoS guaranteed for such transfer (delay constraint, bandwidth allocation and blocking probability, etc) [43]. When designing a network revenue management scheme, one can formulate an optimization model with the following average revenue function for traffic class $i \in M$ [43]:

$$f_i(x_i, s_i, y_i) = c_i^t L(x_i, s_i, y_i) + c_i^b \lambda_i x_i (1 - P(x_i, s_i, y_i)), \quad (7)$$

where users of class $i \in M$ are charged the cost c_i^b for using per unit of bandwidth and users of class $i \in M$ are charged the cost c_i^t per unit of time for the sojourn time $1/\mu_i = \sigma_i/x_i$ on those virtual paths. Note that c_i^b and c_i^t can possibly be varied according to the time of the day to serve with a congestion control mechanism. The total revenue is obtained by summing over (7) for all traffic classes.

Let $\Omega(\vec{s}, B, G))$ be the feasible set consisting of the network constraints under preset numbers of virtual paths $\vec{s} = (s_1, \ldots, s_m)$, limited budget B and network topology G. A network optimization scheme can be executed as follows [3], [9], [16], [38], [39], etc.

$$\max \quad \sum_{i \in M} w_i f_i(x_i, s_i, y_i) \quad (8)$$

$$\text{s.t.} \quad \vec{x} \in \Omega(\vec{s}, B, G), \quad (9)$$

where $w_i \in (0, 1)$ is a fixed weight assigned to each class i by network managers. Here, $\vec{x} = (x_1, \ldots, x_m)$ is the decision variable, and $\vec{s} = (s_1, \ldots, s_m)$, B, y_i are parameters. The goal is to determine the bandwidth allocation $\vec{x}$ under negotiated QoS level so that the revenue earned by the network access providers is maximized. The feasible set $\Omega(\vec{s}, B, G))$ is bounded. This result follows since the bandwidth allocated to each class i in (8), $\forall i \in M$, has a upper bound due to limited budget B. Moreover, the feasible set $\Omega(\vec{s}, B, G))$ decreases to an empty set if $||\vec{s}||_2 = (\sum_{i=1}^m s_i^2)^{1/2}$ increases to a sufficiently large number, where $|| \cdot ||_2$ denotes the well-known Euclidean norm on the vector space $\mathbb{R}^m$.

Given fixed network topology G and limited budget B, we can determine the optimal solutions $\vec{x}^* = (x_1^*, \ldots, x_m^*)$ under preset numbers of virtual paths $\vec{s} = (s_1, \ldots, s_m)$, where x_i^* represents the optimal bandwidth allocated to every virtual path of class $i \in M$. Note that the optimal bandwidth allocation $\vec{x}^*(\vec{s}, B, G)$ is a function of $\vec{s}$, B and G. Consequently, the maximal throughput of s_i virtual paths of class i is $s_i x_i^*$.

4 Monotonicity and convexity of blocking probability

The monotonicity and convexity properties of the blocking probability (3) are listed below.

Proposition 1 *The blocking probability $P(x_i, s_i, y_i)$ is a decreasing function of bandwidth x_i, given $s_i \geq 1$ and $y_i > 0$ fixed.*

Corollary 1 *In the case of large $s_i \gg 1$, if the traffic intensity $\rho_i = y_i/(s_i x_i) < 1$ holds, the first derivative of blocking probability $P(x_i, s_i, y_i)$ with respect to bandwidth x_i is always negative, i.e.,*

$$\frac{\partial P(x_i, s_i, y_i)}{\partial x_i} = (\frac{y_i}{x_i} - s_i)\frac{y_i^{s_i} e^{-y_i/x_i}}{s_i! x_i^{s_i+1}} < 0. \quad (10)$$

Proposition 2 *For each $s_i \geq 1$ and $y_i > 0$, there exists a subset (or region) $\mathbb{S}$ of positive real numbers such that the blocking probability $P(x_i, s_i, y_i)$ is convex (concave) in bandwidth x_i for all $x_i \in (\notin)\ \mathbb{S}$.*

It should be noted that, as $s_i \to \infty$, the limit of the sequence $\{s_i + \frac{5}{2} - \sqrt{s_i^2 + 4s_i + 2} \mid s_i \in \mathbb{N}\}$ is 0.5, where $\mathbb{N}$ is the set of positive integers. As the number of end-to-end paths s_i is huge in real-world communication systems, Proposition 2 implies that $P(x_i, s_i, y_i)$ is convex in bandwidth x_i if we have $0.5 < P(x_i, s_i, y_i) \leq 1$. Otherwise, there exist two inflection points x_i^* and x_i^{**} when $0 \leq P(x_i, s_i, y_i) < 0.5$.

Proposition 3 *If the traffic intensity $\rho_i = y_i/s_i x_i > 1$ holds in the case of large $s_i \gg 1$, the expected path occupancy $L(x_i, s_i, y_i)$ is a decreasing function of bandwidth x_i, given $y_i > 0$ fixed.*

Given $y_i > 0$ and $s_i \geq 1$ fixed, there exists an inflection point x_i^* such that for all $x_i \leq (\geq) x_i^*$ the expected path occupancy $L(x_i, s_i, y_i)$ is concave (convex) in bandwidth x_i.

It can also be observed that the utilization level U is a decreasing function of bandwidth x_i for given $y_i > 0$ and $s_i \geq 1$. This is because the utilization level U equals to the expected path occupancy $L(x_i, s_i, y_i)$ divided by s_i. Meanwhile, there exists an inflection point x_i^* such that for all $x_i \leq (\geq) x_i^*$ the utilization level U is concave (convex) in bandwidth x_i.

Proposition 4 *The blocking probability $P(x_i, s_i, y_i)$ is increasing in traffic demand y_i, given $x_i > 0$ and $s_i \geq 1$ fixed.*

5 Elasticity

For each traffic class, we investigate the elasticity of blocking probability with respect to bandwidth, traffic demand and the number of virtual paths individually. Based on the investigation of elasticity, one can develop distributed algorithms for network revenue management that takes user's elasticity into consideration [44], [3], etc. By using the concept of elasticity, we can define the bandwidth elasticity of blocking ε_i^b for class $i \in M$ as follows.

Definition 2 *The **bandwidth elasticity of blocking** is defined as*

$$\varepsilon_i^b = \frac{\triangle P(x_i, s_i, y_i)/P(x_i, s_i, y_i)}{\triangle x_i / x_i}, \quad (11)$$

where $\triangle x_i$ is the change in allocated bandwidth, and $\triangle P(x_i, s_i, y_i)$ is the change in blocking probability.

The elasticity ε_i^b represents the percent change in blocking probability in response to a percent change in bandwidth. Similarly, the demand elasticity of blocking ε_i^d and the capacity elasticity of blocking ε_i^c for class $i \in M$ are given below.

Definition 3 *The **demand elasticity of blocking** is defined as*

$$\varepsilon_i^d = \frac{\triangle P(x_i, s_i, y_i)/P(x_i, s_i, y_i)}{\triangle y_i / y_i}, \quad (12)$$

where $\triangle y_i$ is the change in the traffic demand.

Definition 4 *The **capacity elasticity of blocking** is defined as*

$$\varepsilon_i^c = \frac{\triangle P(x_i, s_i, y_i)/P(x_i, s_i, y_i)}{\triangle s_i / s_i}, \quad (13)$$

where $\triangle s_i$ is the change in the number of virtual paths.

Proposition 5 shows the phenomenon that the blocking probability will decrease as the allocated bandwidth increase. Proposition 6 infers that the blocking probability will increase as the traffic demand increases. Proposition 7 concludes that the blocking probability is decreasing as enlarging the number of virtual paths. Due to the limit of pages, the proofs of those propositions are skipped here. Those phenomena can also be observed in the numerical results.

Proposition 5 *The bandwidth elasticity of blocking ε_i^b is nonpositive and decreasing as bandwidth x_i increases.*

Proposition 6 *The demand elasticity of blocking ε_i^d is nonnegative as the traffic demand $y_i \geq 0$.*

Proposition 7 *The capacity elasticity of blocking ε_i^c is nonpositive and decreasing as the number of virtual paths s_i increases.*

6 Conclusions

We consider the bandwidth allocation problem on communication networks, where the network is modelled with multiple classes of traffic. This work concentrates on study of the blocking probability property of connections in terms of the available number of end-to-end paths and the allocated bandwidth under the budget constraint. We have presented important relations among the blocking probability, allocated bandwidth, traffic demand and the number of end-to-end paths.

The monotonicity and convexity relationships have been analyzed among model parameters and performance measures of interest, e.g., blocking probabilities and expected path occupancy. We also presented three elasticities to investigate the effect of varying model parameters on the average revenue in analysis of economic models. Those results are verified with numerical examples of the blocking probability and utilization level. One can use those monotone and convex properties to investigate the marginal revenue in capacitated communication systems so that the blocking probability is kept below a specific threshold. Future work will be conducted in the direction of further investigation for the network revenue management schemes.

References

[1] T. Aktaran-Kalayci, H. Ayhan, Sensitivity of optimal prices to system parameters in a steady-state service facility, *Eur. J. Oper. Res.*, Vol. 193, pp. 120–128, 2009.

[2] S. Alouf, E. Altman, C. Barakat, P. Nain, Optimal estimation of multicast membership, *IEEE Trans. Sign. Proces.*, Vol. 51, No. 8, pp. 2165–2176, 2003.

[3] E. Anderson, F.P. Kelly, R. Steinberg, A contract and balancing mechanism for sharing capacity in a communication network, *Manage. S-ci.*, Vol. 52, No. 1, pp. 39–53, 2006.

[4] N. Antunes, C. Fricker, P. Robert, D. Tibi, Analysis of loss networks with routing, *Ann. Appl. Probab.*, Vol. 16, No. 4, pp. 2007–2026, 2006.

[5] I. Atov, H.T. Tran, R.J. Harris, OPQR-G: Algorithm for efficient QoS partition and routing in multiservice IP Networks. *Comput. Commun.*, Vol. 28, pp. 1987–1996, 2005.

[6] T. Bonald., A. Proutière, On performance bounds for balanced fairness, *Perform. Eval.*, Vol. 55, pp. 25–50, 2004.

[7] T. Bonald, L. Massoulié, A. Proutière, J. Virtamo, A queueing analysis of max-min fairness, proportional fairness and balanced fairness, *Queueing Syst.*, Vol. 53, 65–84, 2006.

[8] D.M. Chiu, A.S.W. Tam, Fairness of traffic controls for inelastic flows in the Internet, *Comput. Netw.*, Vol. 51, pp. 2938–2957, 2007.

[9] H. Cho, A. Girard, C. Rosenberg, On the advantages of optimal end-to-end QoS budget partitioning, *Telecommun. Syst.*, Vol. 34, pp. 91–106, 2007.

[10] R. Egorova, S. Borst, B. Zwart, Bandwidth-sharing networks in overload, *Perform. Eval.*, Vol. 64, pp. 978–993, 2007.

[11] J.S. Esteves, J. Craveirinha, D. Cardoso, Computing Erlang-B function derivatives in the number of servers, *Commun. Stat. Stochastic Models*, Vol. 11, No. 2, pp. 311–331, 1995.

[12] A. Faragó, Efficient blocking probability computation of complex traffic flows for network dimensioning. *Comput. Oper. Res.*, Vol. 35, No. 12, pp. 3834–3847, 2008.

[13] Y. Guan, W. Yang, H. Owen, D.M. Blough, A pricing approach for bandwidth allocation in differentiated service networks, *Comput. Oper. Res.*, Vol. 35, pp. 3769–3786, 2008.

[14] T. Güven, R.J. La, M.A. Shayman, B. Bhattacharjee, A unified framework for multipath routing for unicast and multicast traffic, *IEEE/ACM Trans. Netw.*, Vol. 16, No. 5, pp. 1038–1051, 2008.

[15] A. Harel, Convexity properties of the Erlang loss formula. *Oper. Res.*, Vol. 38, No. 3, pp. 499–505, 1990.

[16] S. van Hoesel, Optimization in telecommunication networks. *Stat. Neerl.*, Vol. 59, No. 2, pp. 180–205, 2005.

[17] V.B. Iversen, S.N. Stepanov, Derivatives of blocking probabilities for multi-service loss systems and their applications, *LNCS*, Vol. 4712, pp. 260–268, 2007.

[18] A.A. Jagers, E.A. Doorn, On the continued Erlang loss function, *Oper. Res. Lett.*, Vol. 5, No. 1, pp. 43–46, 1986.

[19] R. Johari, D.K.H. Tan, End-to-end congestion control for the Internet: Delays and stability, *IEEE/ACM Trans. Netw.*, Vol. 9, No. 6, pp. 818–832, 2001.

[20] F.P. Kelly, A.K. Maulloo, D.K.H. Tan, Rate control for communication networks: Shadow prices, proportional fairness and stability, *J. Oper. Res. Soc.*, Vol. 49, pp. 237–252, 1998.

[21] H.S. Kim, N.B. Shroff, Loss probability calculations and asymptotic analysis for finite buffer multiplexers, *IEEE/ACM Trans. Netw.*, Vol. 9, No. 6, pp. 755–768, 2001.

[22] C. Maglaras, A. Zeevi, Pricing and capacity sizing for systems with shared resources: Approximate solutions and scaling relations, *Manage. Sci.*, Vol. 49, No. 8, pp. 1018–1038, 2003.

[23] C. Maglaras, A. Zeevi, Pricing and design of differentiated services: Approximate analysis and structural insights, *Oper. Res.*, Vol. 53, No. 2, pp. 242–262, 2005.

[24] I. Maoui, H. Ayhan, R.D. Foley, Optimal static pricing for a service facility with holding costs *Eur. J. Oper. Res.*, Vol. 197, pp. 912–923, 2009.

[25] E. Mingozzi, G. Stea, M.A. Callejo-Rodríguez, J. Enríquez-Gabeiras, G. García-de-Blas, F.J. Ramón-Salquero, W. Burakowski, A. Beben, J. Sliwinski, H. Tarasiuk, O. Dugeon, M. Diaz, L. Baresse, E. Monteiro, EuQoS: End-to-end quality of service over heterogeneous networks, *Comput. Commun.*, Vol. 32, pp. 1355–1370, 2009.

[26] D. Mitra, Q. Wang Stochastic traffic engineering, with applications to network revenue management, *Proc. IEEE INFOCOM 2003*, Vol. 22, No. 1, pp. 396–405, 2003.

[27] J. Mo, J. Walrand, Fair end-to-end window-based congestion control, *IEEE/ACM Trans. Netw.*, Vol. 8, No. 5, pp. 556–567, 2000.

[28] P. Nain, Qualitative properties of the Erlang blocking model with heterogeneous user requirements, *Queueing Syst.*, Vol. 6, No. 2, pp. 189–206, 1990.

[29] P. Nilsson, M. Pióro, Solving dimensioning tasks for proportionally fair networks carrying elastic traffic, *Perform. Eval.*, Vol. 49, pp. 371–386, 2002.

[30] I.Ch. Paschalidis, Y. Liu, Pricing in multiservice loss networks: Static pricing, asymptotic optimality, and demand substitution effects, *IEEE/ACM Trans. Netw.*, Vol. 10, No. 3, pp. 425–438, 2002.

[31] J.W. Roberts, A survey on statistical bandwidth sharing, *Comput. Netw.*, Vol. 45, pp. 319–332, 2004.

[32] S. Sengupta, S. Anand, M. Chatterjee, R. Chandramouli, Dynamic pricing for service provisioning and network selection in heterogeneous networks, *Phys. Commun.*, Vol. 2, pp. 138–150, 2009.

[33] S. Shakkottai, R. Srikant, Economics of network pricing with multiple ISPs, *IEEE/ACM Trans. Netw.*, Vol. 14, No. 6, pp. 1233–1245, 2006.

[34] S. Shenker, Fundamental design issues for the future Internet, *IEEE J. Sel. Areas Commun.*, Vol. 13, pp. 1176–1188, 1995.

[35] J.M. Smith, $M/G/c/K$ blocking probability models and system performance, *Perform. Eval.*, Vol. 52, pp. 237–267, 2003.

[36] The Cooperative Association for Internet Data Analysis. [Online]. Available: http://www.caida.org/home

[37] C.H. Wang, H. Luh, Two-phase modeling of QoS routing in communication networks, *Proc. 16th Int. Conf. Comput. Commun. Netw.*, pp. 1210–1216, 2007.

[38] C.H. Wang, H. Luh, A fair QoS scheme for bandwidth allocation by precomputation-based approach. *Int. J. Inf. Manage. Sci.*, Vol. 19, No. 3, pp. 391–412, 2008.

[39] C.H. Wang, H. Luh, A utility maximization scheme for end-to-end QoS routing in communication networks with heterogeneous service classes. *Proc. 2008 IEEE Int. Conf. Serv. Oper. Logistics Inf. (IEEE/SOLI'2008)*, pp. 488–493, 2008.

[40] C.H. Wang, H. Luh, Blocking probabilities of multiple classes in IP networks with QoS routing, in: *Advances in Queueing Theory and Network Applications*, W. Yue, Y. Takahashi, H. Takagi (Eds.), pp. 281–290, Springer, 2009.

[41] C.H. Wang, H. Luh, Measuring average revenue with bandwidth and traffic demand on communication networks, Proc. of International Conference on Computational Intelligence and Software Engineering (CiSE 2009), Wuhan, China, December 11-13, 2009.

[42] J.H. Wang, D.M. Chiu, J.C.S. Lui, A game-theoretic analysis of the implications of overlay network traffic on ISP peering, *Comput. Netw.*, Vol. 52, No. 15, pp. 2961–2974, 2008.

[43] M. Yacoubi, M. Emelianenko, N. Gautam, Pricing in next generation networks: a queuing model to guarantee QoS, *Perform. Eval.*, Vol. 52, pp. 59–84, 2003.

[44] M. Yuksel, S. Kalyanaraman, Elasticity considerations for optimal pricing of networks, *Proc. IEEE Symp. Comput. Commun.*, Vol. I, pp. 163–168, 2003.

[45] G. Zachariadis, J.A. Barria, Dynamic pricing and resource allocation using revenue management for multiservice networks, *IEEE/ACM Trans. Netw.*, Vol. 5, No. 4, pp. 215–226, 2008.