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An Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies: The Immunization Theory Approach

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Abstract

This article investigates the natural hedging strategy to deal with longevity risks for life insurance companies. We propose an immunization model that incorporates a stochastic mortality dynamic to calculate the optimal life insurance–annuity product mix ratio to hedge against longevity risks. We model the dynamic of the changes in future mortality using the well-known Lee–Carter model and discuss the model risk issue by comparing the results between the Lee–Carter and Cairns–Blake–Dowd models. On the basis of the mortality experience and insurance products in the United States, we demonstrate that the proposed model can lead to an optimal product mix and effectively reduce longevity risks for life insurance companies.

Introduction

In the past decade, annuity premiums in the United States have accounted for more than 50 percent of life insurers' premium income, with an average growth rate of 10.2 percent from 1988 to 2005 (American Council of Life Insurers, 2005). However, longevity risk[1]—or uncertainty about long-term trends in mortality rates and the impact on the long-term probability of survival of an individual—represents a critical threat to private insurers because it increases the payout period and the liability costs of providing annuities. In particular, human mortality has declined globally

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[1]To differentiate mortality risk from longevity risk, Cairns, Blake, and Dowd (2006a) define the latter as uncertainty in the long-term trend in mortality rates and the former as uncertainty in future mortality rates.

over the course of the twentieth century.[2] As Willets (2004) points out, mortality improvements do not occur in a smooth upward fashion but rather exhibit a "cohort effect."[3] Recent medical discoveries may increase human life spans even beyond the currently projected mortality table used by insurance companies. These issues have made it far more difficult for insurance actuaries to price annuity products correctly, and the resulting inaccurate mortality assumptions lead to major risks. Therefore, hedging longevity risks has taken on an increasingly important role for life insurance companies.

When considering how to hedge longevity risks, most prior research investigates mortality risk and pricing issues for annuity products (Friedman and Warshawsky, 1990; Frees, Carriere, and Valdez, 1996; Brown, Mitchell, and Poterba, 2000; Mitchell et al., 2001). More recent studies focus on the impact of stochastic mortality changes on life insurance and annuities (Marceau and Gaillardetz, 1999; Wilkie, Waters, and Yang, 2003; Cairns, Blake, and Dowd, 2006a). In addition, many financial vehicles, such as mortality derivatives and survivor bonds, have been proposed to reduce or hedge the longevity risks of annuity. Blake and Burrows (2001) first proposed that issuing survivor bonds[4] could help a pension fund insure against the longevity risk, and more recent studies also extend the issue of securitization of longevity risk (e.g., Dowd, 2003; Blake, Cairns, and Dowd, 2006; Blake, Cairns, Dowd, and MacMinn, 2006; Lin and Cox, 2005; Cox, Lin, and Wang, 2006; Denuit, Devolder, and Goderniaux, 2007). Dowd et al. (2006) suggest that a survivor swap can serve as a more advantageous survivor derivative than a survivor bond, because it can be arranged at a lower transaction cost and does not require a liquid market.[5]

Similar to the concept of survivor swap, insurers can hedge longevity risks internally between their own business products (life insurance and annuity), which are sensitive in opposing ways to the changes in mortality rates. This approach provides a so-called natural hedging strategy. If the future mortality of a cohort improves relative to current expectations, life insurers gain a profit because they can pay the death benefit later than initially expected, whereas annuity insurers suffer losses because they must pay annuity benefits for longer than they initially expected. Therefore, life insurance can serve as a dynamic hedge vehicle against unexpected mortality risk. Yet relatively few academic papers have investigated the issue of natural hedging. Cox and Lin (2007) suggest that natural hedging is appealing but may be too expensive to be effective in the context of internal life insurance and annuity products. They instead propose a pricing model for a mortality swap and provide empirical evidence to show that insurers that exploit natural hedging by using a mortality swap can charge a lower

[2]Benjamin and Soliman (1993), McDonald (1997), and McDonald et al. (1998) confirm that an unprecedented improvement in population longevity has occurred both in the United States and worldwide.

[3]Willets (2004) suggests mortality improvements exhibit cyclical patterns, in which some cohorts enjoy better improvements than others.

[4]Unlike the mortality bond that links the bond payments to mortality deviations, survivor bonds (or longevity bonds) link coupon payments to the survivor index (Cairns, Blake, and Dowd, 2006b).

[5]Survivor swaps are more flexible and can be tailored to suit diverse circumstances for counterparties (usually life insurance companies), which enables them to transfer their death exposure without heavy regulations.

risk premium than others. However, no discussion so far addresses product strategies for natural hedging. This article attempts to fill this gap.

To help life insurers achieve a better natural hedging effect, we propose an immunization model that incorporates a stochastic mortality dynamic to calculate the optimal level of a product mix that includes both life insurance and annuities and thereby effectively reduces longevity risks for insurance companies. Our article differs from Cox and Lin's (2007) in two aspects. First, whereas Cox and Lin propose a pricing model for a mortality swap for natural hedging, we propose an immunization model and calculate the optimal product mix ratio for natural hedging. Second, Cox and Lin illustrate the natural hedging strategy by using the 1995 U.S. Society of Actuaries (SOA) basic age last table for life and annuity to measure the market price of risk. However, their model does not consider the future dynamic of mortality. To better capture the stochastic mortality pattern, we estimate the parameters in both the Lee–Carter (LC) and Cairns–Blake–Dowd (CBD) models to reflect the age effect by fitting historical U.S. mortality experience. Our numerical results demonstrate that the proposed model indicates the optimal product mix and thus can effectively reduce longevity risks.

The remainder of this article is organized as follows: In the section "Immunization for Natural Hedging" section we introduce our proposed immunization models for a natural hedging strategy for life insurers. In the next section "Modeling Longevity Risk" section we discuss drivers of the mortality curve and model the future mortality using the LC model with U.S. mortality experience. We demonstrate how to implement our proposed model to calculate natural hedging product mix ratios for various products and analyze some related issues with numerical examples in the subsequent section. We discuss the model risk by comparing the results between the LC model and the CBD model for natural hedging strategies in the following section, and then we conclude in the last section.

IMMUNIZATION STRATEGY FOR NATURAL HEDGING

Natural hedging employs the interaction of life insurance and annuities in response to a change in mortality to stabilize the cash flow for insurers. Compared with external hedging tools, natural hedging does not require the insurer to find counterparties and demands no transaction costs. As an internal vehicle, natural hedging not only helps insurance companies diversify their mortality risk but also provides other advantages. Cox and Lin (2007) show that insurers that utilize natural hedging can charge a lower risk premium, which may increase their market share. However, in practice, it may be difficult to implement natural hedging strategies for several reasons. First, the durations of these two products cannot be matched easily to achieve proper hedges because annuities generally are purchased by older consumers, whereas life insurance policies are purchased by the young. Second, effective hedging may require insurance companies to change their business composition, which could introduce additional expenses or increase operational risks. To achieve an optimal hedging strategy, insurers may need to reduce or increase the price of their annuity or life insurance products to make them more or less attractive.[6] The result of this price adjustment also could

[6]We thank the referees for pointing out this important issue.

reduce the effect of natural hedging. In addition, some specialized insurers may be induced to change their business composition internally by switching production lines between life insurance and annuities.[7] To help life insurers achieve a better natural hedging effect, we propose an immunization model that incorporates a stochastic mortality dynamic to calculate the optimal level of a product mix that includes both life insurance and annuities.

Assume an insurer sells two types of products: life insurance and annuity. The total liability (V) of the insurer equals the sum of the liabilities for the different businesses, as in Equation (1)

$$V = V^{life} + V^{annuity}, \tag{1}$$

where V^{life} is the expected liability of life insurance policies, and $V^{annuity}$ is the expected liability of annuity policies.

On the basis of the concept of modified duration,[8] we posit that the effect of a mortality rate change on insurance total liability (V), under a constant force of mortality rate (μ) assumption, can be measured by mortality duration, as in Equation (2)

$$D_{\mu}^{V} = \frac{dV}{d\mu} \cdot \frac{1}{V}. \tag{2}$$

We extend the immunization theory proposed by Redington (1952) to deal with longevity risk because the effect of mortality changes on the liability of life insurers is similar to that of an interest rate change. The effect of mortality changes on total liability can be expressed by the Taylor expansion, as follows:

$$\Delta V = \left(\frac{dV^{life}}{d\mu} + \frac{dV^{annuity}}{d\mu}\right)\Delta\mu + \frac{1}{2}\left(\frac{d^2V^{life}}{d\mu^2} + \frac{d^2V^{annuity}}{d\mu^2}\right)(\Delta\mu)^2 + \cdots \tag{3}$$

To achieve the immunization strategy for the mortality change, we can obtain the optimal product mix by setting Equation (3) equal to zero, such that $\Delta V = 0$. When considering the first-order approximation, the immunization strategy can be achieved by Equation (4):

$$\left(\frac{dV^{life}}{d\mu} + \frac{dV^{annuity}}{d\mu}\right)\Delta\mu = 0. \tag{4}$$

We further denote the mortality duration of liability for life insurance as $D_{\mu}^{life} = \frac{dV^{life}}{d\mu} \cdot \frac{1}{V^{life}}$ and the mortality duration of liability for annuity as $D_{\mu}^{annuity} = -\frac{dV^{annuity}}{d\mu} \cdot \frac{1}{V^{annuity}}$.

[7]With regard to this limitation, Dowd et al. (2006) propose the possible use of survivor swaps.

[8]To hedge the interest rate risk, under a constant force of interest rate (δ) assumption, the effect of an interest rate change on the liability (V) of the insurer can be measured by modified duration as $D^V = -\frac{dV}{d\delta} \cdot \frac{1}{V}$.

Therefore, Equation (4) can be rewritten as

$$D_{\mu}^{life} \cdot \omega_{life} - D_{\mu}^{annuity} \cdot \omega_{annuity} = 0, \tag{5}$$

where $\omega_{life} = \frac{V^{life}}{V}$ and $\omega_{annuity} = \frac{V^{annuity}}{V}$ are the proportions of liability in life insurance and annuity, respectively, and $\omega_{life} + \omega_{annuity} = 1$.

However, when hedging interest rate risk, the weakness associated with using duration is the assumption of a flat yield curve, which Ahlgrim, D'Arcy, and Gorvett (2004) clearly imply is not suitable. To overcome this problem, Kalotay, Williams, and Fabozzi (1993), Babbel, Merrill, and Panning (1997), and Gaiek, Ostaszewski, and Zwiesler (2005) propose effective duration as an alternative risk measure, which can also apply to measuring mortality risk. Similar to the problem of hedging interest rate risk, the definition of mortality duration in Equation (2) assumes a constant force of mortality and ignores changes or improvements in future mortality. Therefore, we propose the effective mortality duration to measure the mortality risk. The main advantage of the effective mortality duration is that it can capture the mortality dynamic more precisely. Equations (6) and (7) show the calculations of effective mortality durations for life insurance and annuity ($D_{e\mu}^{life}$ and $D_{e\mu}^{annuity}$)

$$D_{e\mu}^{life} = \frac{V^{life+} - V^{life-}}{2 \times V^{life} \times \Delta\mu}, \tag{6}$$

and

$$D_{e\mu}^{annuity} = -\frac{V^{annuity+} - V^{annuity-}}{2 \times V^{annuity} \times \Delta\mu}, \tag{7}$$

where V^{life+} and $V^{annuity+}$ represent the liability values at higher mortality $\mu + \Delta\mu$, and V^{life-} and $V^{annuity-}$ represent the liability values at lower mortality $\mu - \Delta\mu$.

Using the concept of effective duration to calculate the immunization strategy, we can replace Equation (5) with the effective mortality durations by Equation (8), as follows:

$$D_{e\mu}^{life} \cdot \omega_{life} - D_{e\mu}^{annuity} \cdot \omega_{annuity} = 0. \tag{8}$$

From Equation (8), we can obtain the product mix of life insurance liability proportion ω_{life}^{*}, which is also the optimal product mix strategy for natural hedging, as in Equation (9)

$$\omega_{life}^{*} = \frac{D_{e\mu}^{annuity}}{D_{e\mu}^{annuity} + D_{e\mu}^{life}}. \tag{9}$$

As long as the insurers maintain a liability proportion of life insurance at ω_{life}^{*}, they are immunized from longevity risk.

In addition, the effect of convexity plays an important role in dealing with interest rate risk or mortality risk, especially when it encounters a big shock. In other words, the second-order approximation is needed. When considering the second-order approximation, the calculation of optimal product mix includes the effect of convexity. The effect of mortality changes on insurance liability can be measured by mortality convexity, as in Equation (10)

$$C_{\mu}^{V} = \frac{d^2 V}{d\mu^2} \cdot \frac{1}{V}. \tag{10}$$

Therefore, the effective mortality convexity for life insurance and annuity ($C_{e\mu}^{life}$ and $C_{e\mu}^{annuity}$), as in Equations (11) and (12), should be utilized

$$C_{e\mu}^{life} = \frac{V^{life-} + V^{life+} - 2V^{life}}{V^{life}(\Delta\mu)^2}, \tag{11}$$

and

$$C_{e\mu}^{annuity} = \frac{V^{annuity-} + V^{annuity+} - 2V^{annuity}}{V^{annuity}(\Delta\mu)^2}. \tag{12}$$

Thus, the change of total liability (ΔV) by considering both effective mortality duration and effective mortality convexity is as follows:

$$\Delta V = \big(D_{e\mu}^{life} \cdot \omega_{life} - D_{e\mu}^{annuity} \cdot \omega_{annuity}\big)\Delta\mu + \frac{1}{2}\big(C_{e\mu}^{life} \cdot \omega_{life} + C_{e\mu}^{annuity} \cdot \omega_{annuity}\big)(\Delta\mu)^2. \tag{13}$$

By setting Equation (13) equal to 0,[9] we can obtain the product mix of life insurance liability proportions, $\omega_{life}^{*} = \frac{D_{e\mu}^{annuity} + \frac{\Delta\mu}{2} C_{e\mu}^{annuity}}{D_{e\mu}^{annuity} + D_{e\mu}^{life} + \frac{\Delta\mu}{2}(C_{e\mu}^{annuity} - C_{e\mu}^{life})}$, which is the optimal product mix strategy for natural hedging after considering both effects of duration and convexity.

We conduct further numerical analyses in the section "Numerical Analysis for Natural Hedging Strategy." For the case of a small change in mortality (10 percent mortality shift), we use the concept of effective duration for the immunization strategy and calculate the optimal product mix $\omega_{life}^{*} = \frac{D_{e\mu}^{annuity}}{D_{e\mu}^{annuity} + D_{e\mu}^{life}}$ under the first-order approximation. For the case of a big change in mortality (22.5 percent and 25 percent shifts), we further include the concept of effective convexity for the immunization strategy and calculate the optimal product mix, $\omega_{life}^{*} = \frac{D_{e\mu}^{annuity} + \frac{\Delta\mu}{2} C_{e\mu}^{annuity}}{D_{e\mu}^{annuity} + D_{e\mu}^{life} + \frac{\Delta\mu}{2}(C_{e\mu}^{annuity} - C_{e\mu}^{life})}$, under

[9]For hedging longevity risk, we assume that the insurers' objective is to maintain the total liability unchanged, which is the conception of immunization.

second-order approximations. In summary, as long as the insurers maintain a liability proportion of life insurance at ω^{*}_{life}, they are immunized from longevity risk.

MODELING LONGEVITY RISK

In traditional insurance pricing and reserve calculation, an actuary treats mortality rates as constant over time, which means unanticipated mortality improvements can cause serious financial burdens or even bankruptcy for life insurers. In actuarial literature, the question of how to model mortality rates dynamically continues to represent an important issue. Earlier developments of stochastic mortality modeling rely on the one-factor model and pioneering work by Lee and Carter (1992), whose LC model is easily applied and provides fairly accurate mortality estimations and population projections. Renshaw, Haberman, and Hatzoupoulos (1996) and Renshaw and Haberman (2003) offer further analysis of the LC model.

In addition, more recent works develop two-factor mortality LC models. The most distinguishing feature of previous models is the consideration of a cohort effect in mortality modeling. Renshaw and Haberman (2003) apply a cohort effect, and Currie (2006) introduces an Age-Period-Cohort (APC) model. More recently, Cairns, Blake, and Dowd (2006b) allow not only for a cohort effect but also for a quadratic age effect in their CBD model. Cairns et al. (2007) extend Cairns, Blake, and Dowd (2006b) by comparing an analysis of eight stochastic models based in England and Wales with the U.S. mortality experience.

Other developments in two-factor models (e.g., Mileysky and Promislow, 2001; Dahl, 2004; Dahl and Møller, 2005; Miltersen and Persson, 2005; Luciano and Vigna, 2005; Biffis, 2005; Schrager, 2006) employ a continuous-time framework and thus offer an important means to understand pricing of mortality-linked securities. Following the most recent literature on mortality modeling, we employ stochastic mortality to implement the optimal natural hedging strategy for life insurers.

Stochastic Mortality Models

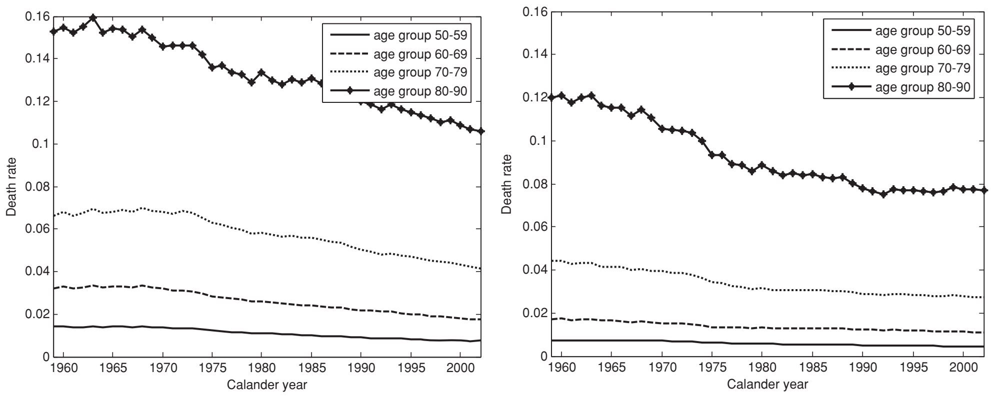
We adopt the most cited stochastic mortality model, the LC model, as our primary method to analyze natural hedging strategies. Moreover, we discuss model risk by comparing the results of the LC model with the recent developed mortality model of CBD model in the section "Impact of Model on Natural Hedging Strategy."

We first give a brief overview of the LC model. Lee and Carter (1992) propose the following mortality model for the central death rate $m_{x,t}$ for a person aged x at time t

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t}, \tag{14}$$

where the parameter α_x describes the average age-specific mortality, k_t represents the general mortality level, and the decline in mortality at age x is captured by β_x. The term $\varepsilon_{x,t}$ denotes the deviation of the model from the observed log-central death rates and should be white noise with zero and relatively small variance (R. D. Lee, 2000). Once the parameters are estimated, we are able to forecast age-specific death rates using extrapolated k_t and fixed α_x and β_x. In this article, we assume that the force of mortality remains constant over each year of integer age and over each calendar year,

FIGURE 1
Trend of Probabilities of Death for 10-Year Age Group, 1959–2002



(Left: male; Right: female)

not that the force of mortality is constant for all ages. Thus, our prediction captures the mortality improvements among different age cohorts (i.e., the effect of mortality changes as people get older). Let $\mu_{x,t}$ denote the force of mortality for a person aged x at integer time t. Thus, the survival probability ($p_{x,t}$) for a person aged x at time t for the next year can be calculated as $p_{x,t} = \exp[-\mu_{x,t}] = \exp[-m_{x,t}]$.

Empirical Patterns of Mortality

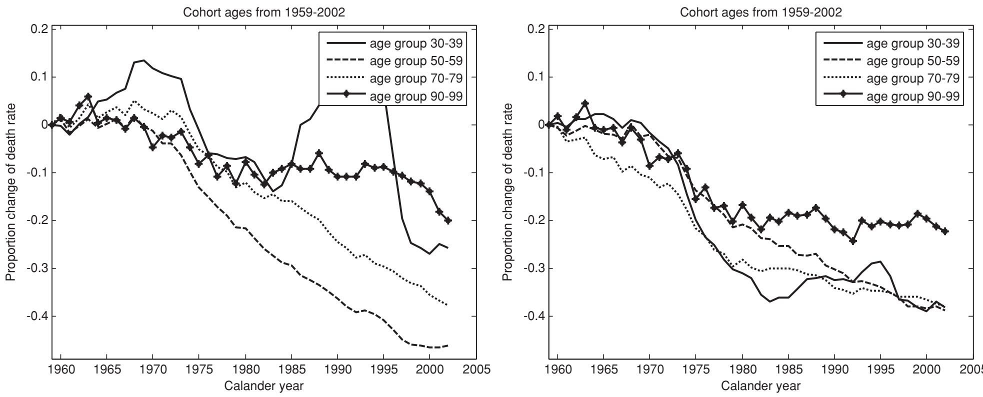
To generate appropriate future mortality dynamics of the LC model, we analyze U.S. mortality experience and the relevant data provided by the human mortality database (HMD, 2005).[10] We provide the empirical patterns of U.S. mortality data in Figures 1 and 2. Figure 1 depicts the average probability of death for each 10-year age group of men and women aged 50–90 years from 1959 to 2002. The patterns clearly show a decreasing trend in death probabilities for each age group, and the mortality improvement trend is more significant for older age groups. For example, in the group of men aged 80–89 years, the probability of death is 0.1526 in 1959 and 0.1061 in 2002. Among men in the 50–59 age group, the probability of death is 0.0140 in 1959 and 0.0075 in 2002. For women 80–89 years of age, the probability of death declines from 0.1202 in 1959 to 0.0770 in 2002, and for those aged 50–59 years, the probability of death is 0.0073 in 1959 and 0.0045 in 2002.

We further investigate the pattern of mortality improvement on the cohort basis[11] in Figure 2 for 1959 to 2002. We recognize patterns that suggest mortality improvement rates increase over time among the 30–39, 50–59, 70–79, and 90–99 year age

[10]The HMD was created to provide detailed mortality and population data to researchers, students, journalists, policy analysts, and others interested in the history of human longevity. The Web address is http://www.mortality.org/.

[11]The improvement rate calculated on a cohort basis is $\frac{q_{x+1,year+1} - q_{x,year}}{q_{x,year}}$, where $q_{x,t}$ denotes the probability that a person aged x in year t dies next year.

FIGURE 2
Average Mortality Change Rate for 10-Year Age Cohorts, 1959–2002



(Left: male; Right: female)

cohorts, with the exception of the male, 30–39 group. We also find that the mortality improvement in recent years is more significant for the male, 50–59 group.

Estimation of Parameters

To better capture future stochastic mortality, we estimate the parameters in the LC model by fitting historical U.S. mortality data from 1959 to 2002 with the HMD data.[12] We estimate the parameters in Equation (14) using the singular value decomposition method. The estimated parameters of α_x, β_x, and k_t for different age groups of men and women appear in Figures 3 and 4. From Figure 3, we can determine that the mortality improvement pattern in the United States differs for men and women, especially for middle (50–75) ages (see parameter estimates of β_x).

Forecasting Survival Probability

Using the parameters estimated in the section "Estimation and Parameters," we forecast age-specific death rates by modeling k_t as a stochastic time series process. In this article, we follow R. D. Lee (2000) to forecast k_t from ARIMA process and use ARIMA(1,1,0)[13] to project k_t. The ARIMA(1,1,0) process of k_t can be expressed as $\Delta k_t = b_0 \Delta k_{t-1} + e_t$, where e_t denotes the random term with $e_t \sim N(0,\sigma_t)$. According the estimates of k_t in Figure 4, we find the parameter of b_0 is 0.3686 for men and 0.4335 for women. We simulate k_t and calculate the corresponding survival probability for both men and women, aged 25 in the year 2003, for the future 70 years and depict the results in Figure 5. We compare the survival probability calculated according to the historic data with our simulated results. The dashed line represents the survival

[12]Due to data limitations, we cannot obtain long-term data pertaining to both life insurance and annuity mortality experiences to generate the parameters for the proposed stochastic mortality models for different products.

[13]R. D. Lee (2000) uses ARIMA(0,1,0) to project k_t but notes that different data sets might be preferable for other ARIMA process.

FIGURE 3
Parameter Estimates in LC model: α_x (Left) and β_x (Right)

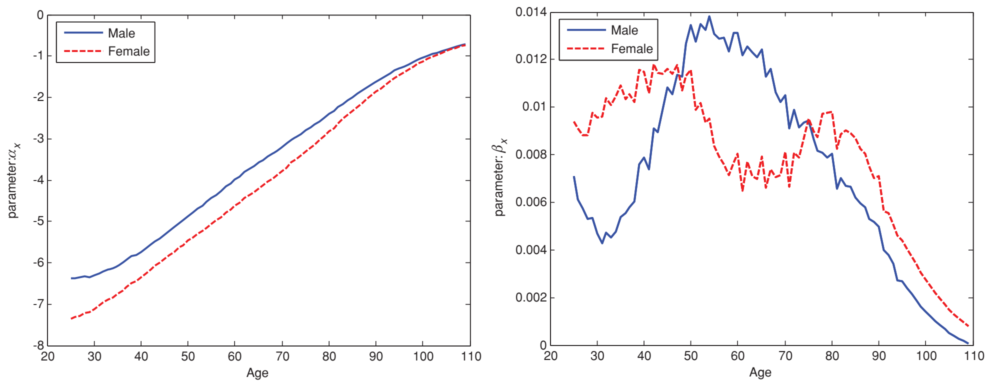
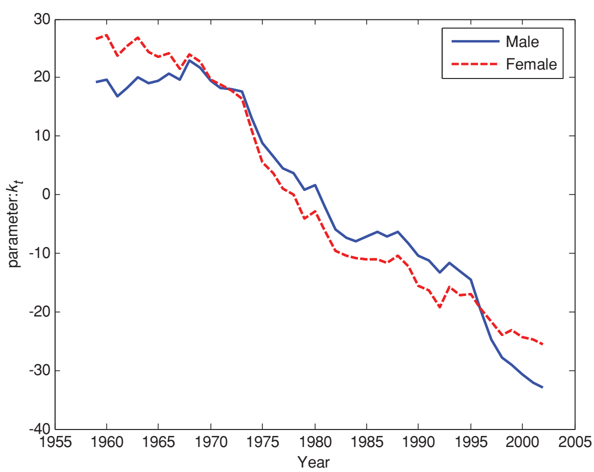


FIGURE 4
Parameter Estimates of k_t in LC Model

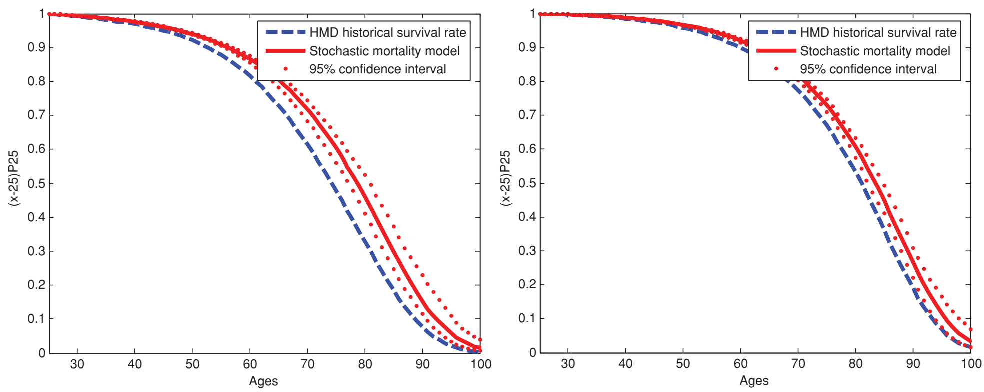


probability for a person aged 25 years, based on the mortality rate of the year 2002, according to the HMD. The solid line shows the mean survival probability from the simulated results, and the dotted line represents the 95 percent confidence interval. The simulated results show that survival probability has increased significantly, and the magnitude of improvement is greater for men. For example, the probability that a person aged 25 years will survive to age 80 changes from 0.3592 to 0.4912 for men and from 0.5635 to 0.6352 for women.

NUMERICAL ANALYSIS FOR NATURAL HEDGING STRATEGY

To demonstrate our proposed natural hedging strategy, we construct a numerical analysis for insurance companies using the LC model. We subsequently discuss model risk by comparing the results of the LC model and CBD model. By applying the methodology proposed in the section "Immunization for Natural Hedging," we can

FIGURE 5
Estimated Confidence Interval of Simulated Survival Probability for Age 25



(Left: male, Right: female)

calculate the optimal product mix liability proportion (ω^*_{life}) and find the natural hedging strategy for longevity risk. To clarify our analysis, we define the equivalent K-ratio to explain the natural hedging strategy in different product mix settings, as follows:

$$K = \frac{\omega^*_{life}}{\omega^*_{annuity}} \cdot \frac{UV_{annuity}}{UV_{life}}, \tag{15}$$

where UV_{life} is the liability per life insurance policy, and $UV_{annuity}$ is the liability per annuity policy. The K-ratio implies that if an insurance company sells one unit of an annuity policy, it should sell K units of life insurance policy to achieve the hedging effect and immunize itself against longevity risk.

Basic Assumptions

Assume an insurance company sells only two products: life insurance and annuities. The natural hedging strategy depends on the policy condition, such as issuance age, gender, coverage period, payment method, and so on. Thus, we set up a model with these basic assumptions in Table 1. We assume a portfolio of single premium deferred annuity (SPDA) and whole-life insurance policies written for male customers at age 45 for annuity and 35 for life insurance.[14] We also assume the payout benefit is US$1,000,000 for life insurance policies and US$10,000 per year, paid 20 years from the issue date, for annuity policies. For each type of product, the insurance premium is paid by a single premium. We also conduct sensitivity analyses of these model assumptions.

[14]These ages match the average issuance ages for these products in real practice.

Table 1
Basis Assumptions for Insurance Products

Product	Single Premium Deferred Annuity (SPDA)	Life Insurance
Age of insured	45	35
Gender	Male	Male
Coverage/payout benefit	US$10,000 (per year)	US$1,000,000
Coverage/payout benefit period (years)	Whole life	Whole life
Method of paying premium	Single premium	Single premium
Interest rate	4%	4%
Deferred period	20	None
Pricing mortality basis	HMD, 2002	HMD, 2002
Forecasted mortality basis	LC model	LC model

Both the interest rate and the mortality rate affect the product mix ratio. However, for the purpose of our demonstration, we focus on the mortality dynamic[15] and assume that the interest rate of the insurance product is constant, say 4 percent. Assume the actuary uses the mortality experience in year 2002 (HMD) for the pricing assumption and wants to hedge both systematic and unanticipated mortality improvements. If the actuary could foresee the future dynamic of mortality and capture the real mortality pattern in pricing, there would be no longevity risk for insurers. However, in the real world, the actuary may not be able to capture real longevity risk in pricing, mainly because of market competition[16] or misestimations of the mortality pattern. In our analysis, we assume that the actuary uses 2002 U.S. mortality experience in HMD for pricing mortality basis and relies on the LC model (forecasted mortality basis) to forecast longevity risk. The corresponding actuarial present values (the single pure premiums[17]) for different type of insurance policies with different mortality basis are presented in Table 2.

In Table 2, we compare the single premium of 10 percent mortality down shock with that of pricing mortality basis and find that the price change accounts for −5.27 percent for a whole life policy, −13.47 percent for 30-year term life, and −15.56 percent for 20-year term life. In addition, the price change accounts for 6.08 percent for a whole-life annuity policy, 5.87 percent for a 30-year term life annuity, and 4.89 percent for a 20-year term life annuity. The results imply that the price changes caused by unexpected mortality shock can be large; therefore, hedging longevity risks is important for life insurance companies.

[15]In reality, the interest rate dynamic likely influences the natural hedging strategy, but we do not consider it to simplify the discussion. However, we recognize that the sensitive effect caused by the yield curve changes. Thus, an analysis of the combined effects of mortality and interest rates would offer more realistic results and should be considered in further research.

[16]Pricing competition in the market often forces actuaries to lower prices to match the goals of the sales department, such as attaining a certain growth rate, market share, or premium income.

[17]The calculation of the single pure premium is based on the actuarial present value (see Bower et al., 1997).

TABLE 2
Single Premium Prices of Illustrated Insurance Products for Different Mortality Bases and Model Assumptions

Coverage/ Payout Benefit Period	(1) Pricing Mortality Basis (HMD Data)		(2) Forecasted Mortality Basis by Lee–Carter		(3) Forecasted Mortality With 10% Down Shock		(4) = [(3) − (1)]/(1) Price Change	
	20-Year Deferred Annuity	Life Insurance	20-Year Deferred Annuity	Life Insurance	20-Year Deferred Annuity	Life Insurance	20-Year Deferred Annuity	Life Insurance
20-year term	40,679	48,237	41,042	45,112	42,670	40,732	4.89%	−15.56%
30-year term	43,911	86,671	44,307	82,695	46,489	74,997	5.87%	−13.47%
Whole life	44,212	221,734	44,598	219,171	46,902	210,045	6.08%	−5.27%

Note Issuing age is 35 years for life insurance policy and 45 years for annuity policy.

TABLE 3
Product Mix Proportion (ω^*_{life}) and K-Ratio: Men, Single Premium

Coverage/Payout Benefit Period	20-Year Term Life	Whole Life
20-year term SPDA	30.1%	49.5%
	(0.364)	(0.180)
30-year term SPDA	34.6%	54.6%
	(0.481)	(0.238)
Whole-life SPDA	35.5%	55.6%
	(0.504)	(0.249)

Note: Figures in parentheses represent the K-ratios.

Hedging Strategy for Two Products

In this section, we illustrate how insurers can use our proposed product mix strategy to hedge against unexpected mortality shock. For demonstration purposes, we discuss three different cases of mortality shocks: (1) the base case with a 10 percent mortality shift for both annuity and insurance policies, (2) a sudden big shock with a 25 percent mortality shift for both annuity and insurance policies, and (3) a sudden big shock with a 25 percent mortality shift for annuity and 22.5 percent mortality shift for insurance policies. According to our numerical findings, the effect of convexity is not significant for a small mortality shock. Therefore, in the case of the 10 percent mortality shift, we use only the effective durations (i.e., Equations (6) and (7)) to measure mortality risk and ignore the effect of convexity.[18] However, for the 22.5 percent and 25 percent mortality shifts, we include the effect of convexity by using Equation (13) to measure mortality risk in our analyses.

We calculate both the optimal product mix proportion (ω^*_{life}) and the K-ratio in different product settings under our basic assumptions and report the results in Table 3. According to Table 3, the optimal product mix proportion for natural hedging of a whole-life insurance product is 55.6 percent for men. Therefore, as long as the liability for life insurance accounts for 55.6 percent of the total liability, the insurer can achieve a natural hedging effect. The corresponding K-ratio for such product mix is 0.249. In other words, if the insurance company sells one unit of whole-life SPDA to a cohort of men, it should sell 0.249 units of whole life insurance to hedge its longevity risk.

In Table 3, we observe that the optimal product mix proportion becomes smaller and the K-ratio higher as the coverage period of a life insurance become shorter. For example, for a product mix that contains a whole-life SPDA and 20-year term life, the corresponding optimal product mix proportion is 35.5 percent (smaller than 55.6 percent for whole life), and the K-ratio is 0.504 (higher than 0.249 for whole life). As mentioned in the section "Immunization Strategy for Natural Hedging,"

[18]The impact of ignoring the effect of convexity on the value of both product mix proportion and K-ratio under the case of 10 percent mortality shift in Table 3 is less than 0.002.

the optimal product mix of the life insurance proportion is $\omega_{life}^{*} = \frac{D_{e\mu}^{annuity}}{D_{e\mu}^{annuity}+D_{e\mu}^{life}}$. Thus, when the coverage period of life insurance becomes shorter, the optimal product mix proportion decreases because the effective duration of the life insurance increases and the effective duration of the annuity remains the same. The results of the K-ratio in our numerical analysis imply that compared with its sale of whole-life policies, the insurer will require more units of term-life policies to hedge against the longevity risk associated with whole-life annuity contracts.

In contrast, both the optimal product mix proportion and the K-ratio decrease as the coverage period of SPDA becomes shorter in Table 3. For example, for a product mix with a 20-year term SPDA and whole-life product, the optimal product mix proportion is 49.5 percent (smaller than 55.6 percent for whole-life SPDA), and the K-ratio is 0.18 (lower than 0.249 for whole-life SPDA); with a 30-year term SPDA and whole-life product, the optimal product mix is 54.6 percent and the K-ratio is 0.238, again smaller than the whole-life SPDA. In these cases, the optimal product mix proportion decreases because the effective duration of life insurance remains the same and the effective duration of the annuity becomes smaller. The results of the K-ratio in our numerical analysis imply that compared with a short-term annuity policy, a long-term policy requires more life insurance policy units to achieve a natural hedging effect.

In Table 4, we report the corresponding calculated results for women, whose patterns of optimal product mix proportion and K-ratios are similar. In most cases,[19] the corresponding optimal product mix proportion for women is smaller and the K-ratio higher than those for men. For example, for a product mix containing a whole-life SPDA and whole-life insurance, the optimal product mix proportion for women is 49.3 percent (less than 55.6 percent for men), and the corresponding K-ratio for women is 0.279 (higher than 0.249 for men). From the section "Modeling Longevity Risk," we know that the survival probability of both men and women has increased significantly but the magnitude of improvement is greater for men. Thus, compared with those for men, the optimal product mix proportions of women are smaller because the effective duration of life insurance is larger, and the effective duration of the annuity smaller than those for men.[20] The K-ratio ($K = \frac{\omega_{life}^{*}}{\omega_{annuity}^{*}} \cdot \frac{UV_{annuity}}{UV_{life}}$) of women in most cases is higher because the liability ratios of annuity to life insurance per policy ($\frac{UV_{annuity}}{UV_{life}}$) for women are generally higher than those of men. We also find that the K-ratio for women in the case of a term-life policy is almost 30–50 percent higher than that of men. The result implies that women have longer life expectancies, so the insurer may need to sell more life insurance policies to hedge for its longevity risks for women in our numerical examples.

[19] For the product mix with 20-year term annuity and whole life, both the corresponding optimal product mix proportion and the K-ratio for women are smaller than those for men.

[20] In the setting of our example, the mortality improvement has greater impact on the liability of annuity than on life insurance. Thus, the optimal product mix proportion of women is smaller than that of men.

TABLE 4
Product Mix Proportion (ω^*_{life}) and K-Ratio: Men and Women Comparison

	20-Year Term Life for Men	20-Year Term Life for Women	Whole Life for Men	Whole Life for Women
20-year term SPDA	30.1%	22.3%	49.5%	40.2%
	(0.364)	(0.478)	(0.180)	(0.171)
30-year term SPDA	34.6%	27.6%	54.6%	47.2%
	(0.481)	(0.705)	(0.238)	(0.253)
Whole-life SPDA	35.5%	29.3%	55.6%	49.3%
	(0.504)	(0.776)	(0.249)	(0.279)

Note: Figures in parentheses represent the K-ratios.

TABLE 5
Product Mix Proportion (ω^*_{life}) and K-Ratio: Deferred Period Effect

Product Mix	Men	Women
Whole-life SPDA (deferred 10 years)	41.5%	36.2%
	(0.300)	(0.320)
Whole-life SPDA (deferred 20 years)	55.6%	49.3%
	(0.249)	(0.279)
Whole-life SPDA (deferred 30 years)	69.5%	63.5%
	(0.165)	(0.204)

Note: Figures in parentheses represent the K-ratios.

In the sensitivity analysis, we further analyze the impacts of deferred period, age, and coverage effects. For simplicity and without loss of generality, we illustrate these effects only for whole-life insurance policies and display the simulation results in Tables 5 and 6. In Table 5, we investigate the impact of a deferred period of a whole-life SPDA on the optimal product mix proportion and K-ratio. For both men and women, the optimal product mix proportion increases as the deferred period becomes longer because the effective duration of the life insurance remains the same and the effective duration of the annuity becomes larger. In Table 5, the K-ratio decreases with respective to an increase in the deferred age because the liability ratio of annuity to life insurance per policy decreases as the deferred period of a whole-life SPDA becomes longer. The longer the period before the insured receives the annuity payment, the shorter is the annuity payout period. Thus, longevity risk is considered less significant as the deferred period of a whole-life SPDA becomes longer. The results of the K-ratio imply that in our numerical example, insurers require fewer life insurance policy units to achieve a natural hedging effect.

We provide the age effects on the optimal product mix proportion and K-ratio in Table 6. The optimal product mix proportion becomes smaller and the corresponding K-ratio higher as the issue age of SPDA increases. For example, for men as their issue age of SPDA increases from 35 to 55, the K-ratio increases from 0.173 to 0.349;

TABLE 6
Product Mix Proportion (ω^*_{life}) and K-Ratio: Age Effect

Product Mix	Men	Women
Whole-life SPDA (issued at age 35, attend age 65)	56.8%	50.2%
	(0.173)	(0.192)
Whole-life SPDA (issued at age 45, attend age 65)	55.6%	49.3%
	(0.249)	(0.279)
Whole-life SPDA (issued at age 55, attend age 65)	52.8%	47.2%
	(0.349)	(0.390)

Note: Figures in parentheses represent K-ratios.

for women, it increases from 0.192 to 0.39. In these cases, the optimal product mix proportion decreases because the effective duration of life insurance remains the same and the effective duration of the annuity becomes smaller as the issue age increases. However, the K-ratio increases because the liability ratio of annuity to life insurance per policy increases as the issue age of SPDA increases (Table 6). Thus, compared with that for younger groups, the K-ratio is larger for older consumers, which implies that in our numerical example, the insurer needs to sell more life insurance policies to hedge its longevity risk with regard to its annuity business for older customers because the deferred period of annuity is shorter.

In our previous analyses, we assume insurance premium is paid by a single premium. Next, we investigate the effect for policies involving a level premium. When we compare the results of Table 7 with those in Table 3 for single-premium products, we find that both the optimal product mix proportion and the corresponding K-ratio are smaller for level-premium products than are those for single-premium products. The optimal product mix proportion decreases because the ratio of effective duration of life insurance to annuity ($\frac{D^{life}_{e\mu}}{D^{annuity}_{e\mu}}$) is smaller with the level-premium calculation. The K-ratio results demonstrate that in our numerical example, compared with a single-premium policy, a level-premium policy requires fewer life insurance policy units to achieve natural hedging. That is, longevity risk can be diversified through various deposits of premiums, similar to the coupon effect for a coupon bond.

We further demonstrate the impact of big unexpected mortality shocks on the K-ratio.[21] In our previous numerical analyses, we assume the mortality curve shifts 10 percent for all ages, but to demonstrate the impact of unexpected mortality shocks on K-ratio, we also examine the sensitivity of the mortality curve shift and assume that it changes from 10 percent to 25 percent in Table 8. In the case of a 25 percent mortality shift, we measure mortality risks by including the effect of convexity. From Table 8, we also observe the same pattern as in Table 3: both the optimal product mix proportion and the K-ratio increase as the coverage period of SPDA becomes longer. That is, the insurer must sell more units of term-life than whole-life policies to

[21] To hedge longevity risk, insurers mainly worry about big unexpected mortality shocks and less about small mortality deviations.

TABLE 7
Product Mix Proportion (ω^*_{life}) and K-Ratio: Men, Level Premium

	(1) 20-Year Term Life	(2) Difference Between Single and Level Premium	(3) Whole Life	(4) Difference Between Single and Level Premium
20-year term SPDA	30.1%	0.0%	49.3%	−0.2%
	(0.357)	(−0.007)	(0.178)	(−0.002)
30-year term SPDA	34.5%	0.0%	54.4%	−0.2%
	(0.472)	(−0.009)	(0.236)	(−0.002)
Whole-life SPDA	35.5%	0.0%	55.4%	−0.2%
	(0.496)	(−0.008)	(0.247)	(−0.002)

Note: Figures in parentheses represent the K-ratios. All numbers are rounded to the third digit after zero. The differences of product mix the K-ratios between Tables 7 and 3 appear in the second and fourth columns.

TABLE 8
Product Mix Proportion (ω^*_{life}) and K-Ratio: 25% Mortality Shift

	(1) 20-Year Term Life	(2) Difference in Mortality Curve Shift (10% to 25%)	(3) Whole Life	(4) Difference in Mortality Curve Shift (10% to 25%)
20-year term SPDA	30.3%	0.0%	49.4%	0.0%
	(0.367)	(0.000)	(0.179)	(0.001)
30-year term SPDA	34.9%	0.0%	54.5%	0.0%
	(0.487)	(0.002)	(0.237)	(0.001)
Whole-life SPDA	36.0%	0.2%	55.7%	0.1%
	(0.513)	(0.004)	(0.250)	(0.000)

Note: Figures in parentheses represent the K-ratios. All numbers are rounded to the third digit after zero. The differences of product mix and the K-ratios calculated by considering the effects of duration and convexity for mortality curve shift between 10 percent and 25 percent appear in the second and fourth columns.

hedge its longevity risk. We compare the results of the product mix and the K-ratios calculated by considering the effects of duration and convexity in both cases and find that both the optimal product mix proportion and the K-ratio in the second and forth columns change insignificantly (the values of difference are less than 0.004 for all cases) because the mortality shift has been immunized by the proposed model to hedge against unexpected mortality shocks.

However, in practice, mortality experiences for annuity products differ from those for life insurance because of the underwriting effect and adverse selection. Therefore, we next consider a case that includes a different significant mortality shift between annuity and insurance policies. That is, we imagine the mortality curve shift is

TABLE 9
Product Mix Proportion (ω^*_{life}) and K-Ratio: 25 Percent Mortality Shift for Annuity and 22.5 Percent for Life Insurance

	(1) 20-Year Term Life	(2) Difference in Mortality Curve Shift	(3) Whole Life	(4) Difference in Mortality Curve Shift
20-year term SPDA	32.7%	2.3%	52.1%	2.6%
	(0.408)	(0.041)	(0.199)	(0.019)
30-year term SPDA	37.4%	2.5%	57.2%	2.6%
	(0.542)	(0.057)	(0.265)	(0.027)
Whole-life SPDA	38.5%	2.7%	58.3%	2.7%
	(0.571)	(0.062)	(0.279)	(0.029)

Note: Figures in parentheses represent the K-ratios. All numbers are rounded to the third digit after zero. The differences of product mix and the K-ratios calculated by considering the effects of duration and convexity for mortality curve shift between 10 percent and 25 percent appear in the second and fourth columns.

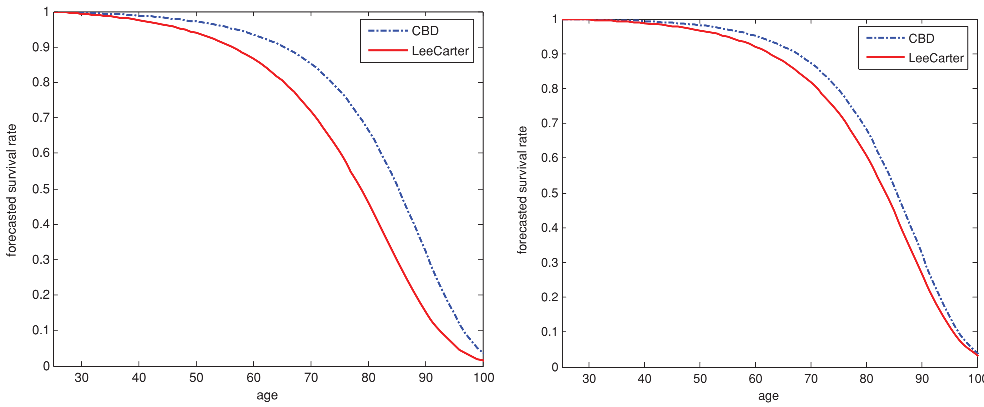
25 percent for annuity and 22.5 percent for life insurance products.[22] Comparing the results of Table 8 with those of Table 9, we find that both the optimal product mix proportion and the K-ratio in the second and fourth columns now have slightly greater proportion changes because the shift in each policy type of mortality experience differs. The optimal product mix proportion increases and is slightly higher for the annuity products with longer duration because the mortality curve shift for life insurance is greater than that of the annuity. However, the impact of this shock on insurers is still relatively small because our proposed model can help hedge against unexpected mortality shocks.

IMPACT OF MODEL RISK ON NATURAL HEDGING STRATEGY

To achieve the optimal natural hedging strategy, insurers must use an appropriate model to estimate longevity risk. However, the existing literature proposes various models to calculate the mortality rate, with different stochastic mortality paths for each model. Alternatives to the mortality model appear in Cairns et al. (2007), who offer a comparative analysis of eight stochastic models based on English, Welsh, and U.S. mortality data. Melnikov and Romaniuk (2006) and Koissi and Hognas (2006) suggest that model risk and parameter risk are crucial when dealing with longevity risk. To explore this issue further, we discuss the impact of model risk on a natural hedging strategy by comparing the results derived from the one-factor LC model with those from the two-factor CBD model. Although the CBD model is similar to the LC model, it adopts a cohort effect in mortality modeling that is not discussed in LC model. As we noted previously, the cohort effect is an important factor in stochastic mortality modeling. We summarize the details of the CBD model and parameter estimates in the Appendix.

[22]Due to data limitations; otherwise, we could make the assumption more realistic.

FIGURE 6
Simulated Survival Probability for Age 25 With CBD and LC Models



(Left: male, Right: female)

For comparison, we use the same U.S. population mortality experience to generate the parameter estimates in the CBD model. We present the future survival probability for both the LD and CBD models in Figure 6. The mortality improvement forecasted by the CBD model is more significant than that using the LC model for both men and women because the cohort effect of the CBD model captures the greater mortality rate dynamics for older compared with younger consumers.

On the basis of these two models, we present the optimal product mix ratios in Table 10; specifically, the insurer may need to sell a different amount of life insurance to hedge longevity risk. In most cases,[23] the optimal product mix proportion becomes smaller and the corresponding K-ratio higher in the CBD model. For example, in a product mix featuring whole-life insurance and whole-life SPDA, the LC model indicates a K-ratio of 0.249, whereas the CBD model reveals a ratio of 0.298. In other words, according to the CBD model, the insurer needs to sell more whole-life insurance policies to hedge against its risk of long-term annuity. This pattern is even more significant in a product mix featuring 20-year term-life insurance and whole-life SPDA. For example, the LC model indicates a K-ratio of 0.504, whereas the CBD model reveals a ratio of 1.307. The mortality improvement forecasted by the CBD model is more significant than that using the LC model because the cohort effect of the CBD model captures the greater mortality rate dynamics for older compared with younger consumers.

Insurers therefore should first work to understand the characteristic of the various models and choose the appropriate one, depending on their objectives and the time period allowed for hedging strategies. Asset-liability managers cannot ignore the

[23]For the product mix with a whole-life insurance and a 20-year term SPDA products, the K-ratios are 0.180 (LC) and 0.159 (CBD). The result implies that to hedge against the longevity risk for a shorter-term annuity, the effect of mortality improvement based on the CBD model is less significant than that based on the LC model. That is, according to the calculation of the CBD model, the insurance company can sell less whole-life insurance and still hedge against its longevity.

TABLE 10
Product Mix Proportion (ω^*_{life}) and K-Ratio: Two Models Comparison

Coverage/Payout Benefit Period	20-Year Term Life		Whole Life	
	LC	CBD	LC	CBD
20-year term SPDA	30.1%	24.9%	49.5%	38.4%
	(0.364)	(0.552)	(0.180)	(0.159)
30-year term SPDA	34.6%	32.5%	54.6%	47.6%
	(0.481)	(0.893)	(0.238)	(0.257)
Whole-life SPDA	35.5%	33.6%	55.6%	51.0%
	(0.504)	(1.307)	(0.249)	(0.298)

Note: Figures in parentheses represent the K-ratios.

possible impacts of model risks, regardless of when they undertake short- or long-term natural hedging strategies associated with longevity risk.

CONCLUSION AND DISCUSSION

This research extends existing literature of hedging longevity risk by proposing an immunization model that incorporates stochastic mortality dynamics to calculate an optimal product mix for natural hedging. Our numerical analyses thus provide new insights into insurance literature. First, to our knowledge, we are the first to address product mix strategies for natural hedging. Second, to capture the stochastic mortality pattern, we estimate the parameters in both the LC and CBD models by fitting real historical U.S. mortality experience data with the HMD to analyze the natural hedging strategy. Third, in recognition of the problems of model risk raised by many recent studies, we compare the results of the LC and CBD models and thus integrate model risk into our discussion.

The numerical analyses strongly support the use of our proposed model as a tool to obtain the optimal balance that will effectively reduce longevity risks for life insurance companies. The proposed natural hedging strategy immunizes longevity risk, once the total liability ratio is set according to the optimal hedging proportion, no matter how the mortality curve shifts. Of course, the dynamics of the mortality pattern must be captured precisely; insurers must adjust the sales volume of life insurance and annuity products to regain an optimal liability proportion. Our numerical analyses also show that the K-ratio for women with a term-life policy is higher than that of men. In noting the impacts of deferred, age, and coverage effects, we recognize that the K-ratio decreases as the deferred period increases but increases as ages increase. In addition, K-ratios are smaller for level-premium products than for single premiums. For the purpose of hedging longevity risk, it is particularly important to consider the effects of big unexpected mortality shifts, and we demonstrate that the impact of longevity risk is more critical for policies of longer duration.

In addition, our numerical analyses show that in terms of hedging longevity risk for a shorter-term annuity, the effect of mortality improvements based on the CBD model is less significant than that based on the LC model. Thus, it is very important for insurers to understand the characteristic of the various models and choose an appropriate one, according to their objectives and the time period of their hedging strategies. Asset-liability managers also cannot ignore the possible impacts of model risks.

Is natural hedging really cheaper than other external hedging instruments, such as mortality derivatives and survivor bonds? Due to a lack of suitable price data for various insurance products, we are unable to analyze the actual natural hedging cost in real practice for insurance companies. Furthermore, mortality experiences may differ for life insurance and annuity products, but because of our data limitations, we cannot obtain sufficiently long life insurance and annuity mortality experiences to generate the parameters for the proposed stochastic mortality models from the U.S. mortality data in the HMD. However, we believe insurers can make use of their own mortality experience to revise the parameters in the mortality models and apply our proposed mythology to find their natural hedging strategy. Finally, some ongoing important issues for insurance research and practice clearly deserve more investigation. For example, studies of alternative mortality models could use the insurer's actual mortality experience with different products and immunization strategies with certain assumptions about the future dynamics of nonparallel mortality shifts. We illustrate the hedging strategy with a term structure force of mortality, but we do not consider the effect of dynamic interest rates. Therefore, an analysis of the combined effects of mortality and interest rates would offer more realistic results and should be considered by further research.

Appendix: CBD Mortality Model

The Model

The CBD mortality model we use was proposed in Cairns, Blake, and Down (2006b). They suggest a two-factor model for modeling initial mortality rates instead of central mortality rate. The mortality rate for a person aged x in year t ($q(t,x)$) is modeled as follows:

$$\text{logit}\, q(t,x) = \beta_t^1 k_t^1 + \beta_t^2 k_t^2 \tag{A1}$$

where k_t^1 and k_t^2 reflect period-related effects.

This model can be presented in a simple parametric form by setting β_t^1 equal to 1 and $\beta_t^2 = x - \bar{x}$. Thus, the mortality rate can be modeled as in Equation (A2)

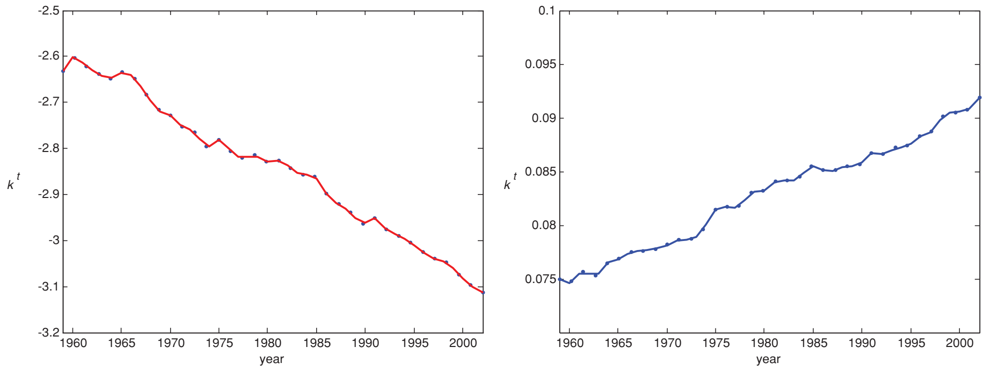
$$\text{logit}\, q(t,x) = k_t^1 + k_t^2(x - \bar{x}), \tag{A2}$$

where $\bar{x}$ is the mean age. For other generalizations of CBD model, see Cairns et al. (2007).

The Parameter Estimates

To investigate the model risk, we also estimate the parameters in the CBD model by fitting historical U.S. mortality data from 1959 to 2002 with the HMD data. The

FIGURE A1
Parameter Estimates in CBD Model for Men (Left: k_t^1, Right: k_t^2)



estimated parameters of k_t^1 and k_t^2 for men are depicted in Figure A1. k_t^1 shows a downward trend, and k_t^2 indicates an upward trend.

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